Inverse problem in structural damage identification

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Abstract

The paper presents definitions and theory of inverse problem and their application for damage detection, localisation and assessment. The methods based on vibration measurements as well as based on wave phenomena in structures are discussed. The damage in the structure is define as improper operations or change of material properties. The methods are classified into two groups; one which is based on identification of structural properties or materials properties changes, but the second is based on loads identification. Load identification problem based on structural response measurements is directly inverse identification problem in which the input to the system is identified based on knowledge about model of the structures and measured responses. But the problem of material properties changes and structural parameters changes is typical identification problem in which based on measured input and structural responses parameters of models are identified. Four case studies will be presented. The first one is identification of contact forces between rail track and wheel of rail vehicle during operation based on bearing boxes acceleration measurements. The second case study concerns identification of material parameters based on wave phenomena in materials. The identification of Young's modulus of composite structures is discussed. The third case study covers localization of damage in the structure based on FE models updating taking into account uncertainties in measurements and modelling. The last one concerns detection of structural damage taking into account changes of stiffness and damping during object operation. All four cases will be focused on numerical methods formulation, implementation and application.

Keywords: inverse problem, structural health monitoring, load identification, material parameters identification

1. Introduction

In mechanical engineering two main problems can be distinguish; direct problems and inverse problems [1]. General scheme of these problems is shown in the figure 1.

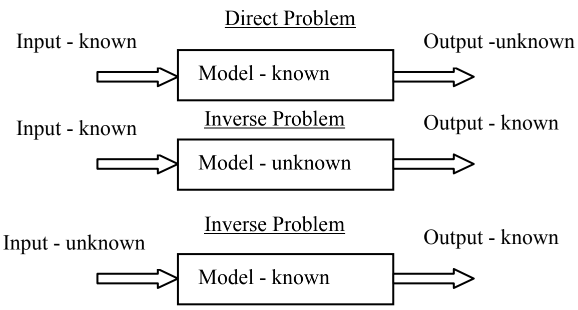

Figure 1. Direct and Inverse problem definition

The direct problem can be solved using common simulation tools depends of the area of interest. In structural health monitoring the direct problem solution can be applied in prediction of save life of the structure if model and loads are known. The direct problem is well – posed problem and has usually unique and stable solution using numerical methods. The inverse problems is commonly ill – posed and its solution is more difficult to obtain and it needs to be re-formulated for numerical treatment. Typically this involves additional assumptions, such as smoothness of solution. This process is known as *regularization* and Tikhonov regularization is one of the most commonly used for regularization of linear ill-posed problems [2]. Ill – posed problem includes loads identification and model parameters identification and they are often used in structural health monitoring [3].

Two main tasks for SHM related to inverse problem can be distinguish. The first is problem consists of detection of changes in load of monitored structure which can be a reason for damages, but the second one is tracking of model parameters changes during operation. The model changes are indicators of structural integrity.

The problem is more difficult if uncertainties of measurements and modeling are considered [4]. In some cases uncertainty of the model is obstacle for application of inverse problem solution for SHM because of confidence bounds of damage indicators are bigger the changes due to damage itself. If measurements uncertainties occur, their influence on SHM methods can be limited by measurements data filtering or averaging. The problem can be more difficult if non-stationary mechanical structure is considered, the some dedicated procedures for data processing have to be formulated and are presented in the paper.

Newest approach for damage detection is based on material properties identification using active methods, during a structure operation. Active methods are based on external excitation and measurements of structural responses. Many different methods using different phenomena can be applied. This approach can help to detect damage at early stage of its nucleation, but its application required more sophisticated measurement equipment mainly used for NDT techniques.

Four case studies of application of inverse problem formulation for SHM are presented in the paper.

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2. Rail track damage identification

Structural Health Monitoring is becoming more and more important for today's railway[5]. Higher requirements are set for safety and availability of both trains and tracks, that can be achieved using new type of SHM systems installed in infrastructural elements and vehicles.

Formulated track roughness identification is based on solution of an inverse identification problem, which is defined as follows; model of the system is known as well as the response of the system, but excitation in form of the rail irregularities is to be identified. Such defined problem is not easy to solve because a rail vehicle - rail track system is non-linear and non-collocated. The problem has to be solved in the time domain to find track irregularities (fig.2). The method bases on the minimization of the objective function: min J → f.

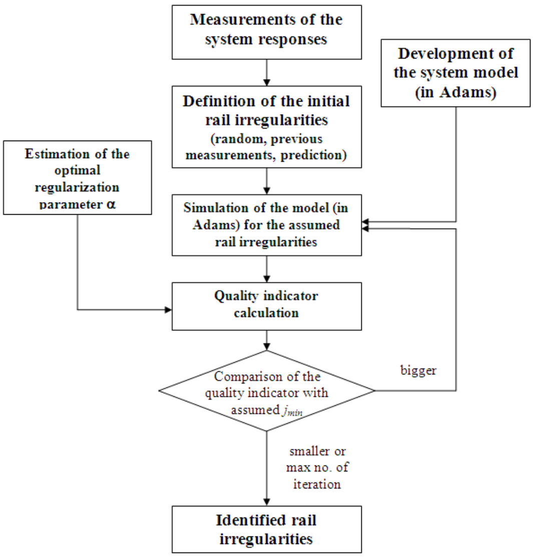

Figure 2. The scheme of inverse identification problem for rail track quality assessment

The objective function is defined as a difference between response of the system excited by the unknown track irregularities and response of the simulation of system model excited by the identified track irregularities:

$$J(x, f_j) = \sum_{j=1}^{N} (q_j - y_j) D (q_j - y_j)^T$$

where: c – initial conditions of the motion f_j, q_j, y_j – vectors of load (rail irregularities) and of the response calculated and measured in the time sample j, D – weight matrix.

Such a formulation of the problem is not sufficient because a mathematical solution that will minimize J will usually end up with the model exactly matching the data. The situation that is to be avoided. This is where the regularization method enters. By adding a term to the objective function one can control the amount of smoothness that occurs in the solution by varying the parameter a. This method is referred to as Tikhonov's method. With the regularization term the full formula of the objective function presents as follows:

$$J(c, f_j) = \sum_{j=1}^{N} (q_j - y_j) D (q_j - y_j)^T + \alpha f_j^T f_j \tag{2}$$

where: α – smoothing parameter.

Author decided to use L – curve method for α parameter selection as the one which requires less computations. To solve the optimization problem the dynamic programming has been used. Entire procedure was programmed in MATLAB. The model of the vehicle was build in MD ADAMS. The results of track irregularities identification are shown in figure 3 [6].

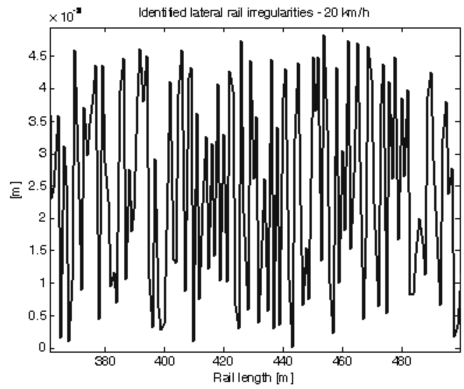

Figure 3. Identified track irregularities

Both the character of irregularities and their amplitude have reasonable value.

3. Identification of material properties based on Lamb waves phenomena analysis

Knowledge of materials mechanical properties and their changes during structure operation plays essential role in SHM. There is many different methods for material testing, but mostly methods are based on destructive testing, which cannot be applied for testing of material properties during structure operation [7]. The goal of this section is to formulate a method which can be applied during operation and gives estimation of Young modulus of materials on-line. If the method is applied for composite structures with anisotropic properties then results should be in a form of Young's modulus in function of direction [8]. The method presented in the paper is based on a solution of both direct and inverse problems [9]. The Lamba wave propagation phenomena is considered as experimental data in material parameters identification procedure. The time of flight (ToF) measurements and experimentally obtained dispersion curves for tested materials are employed for parameters identification. Analytical relations between material properties and wave velocities are directly use for identification. The first step of the procedure consists of sensitivity analysis of wave phase velocity on changes of material properties. The results indicated which parameters can be identified using velocity phase or group velocity measurements. The next step is to identify dispersion curves using broadband noise excitation and 2D Fourier analysis of measured response signals. The measured dispersion curves are compared with simulated one and the measure in form of least square of distances between these curves are to be minimized to obtain Young's modulus of tested materials but geometry of tested specimen has to be known.

The dispersion curve are obtained by simulation using LISA algorithm and its implementation using CUDA technology on Nvidia hardware platform. This software is very efficient and allows to simulate dispersion curves [10]. The geometry data is imported from commercial CAD software which simplify input data preparation and enables to use this method for structures with difficult shape. In presented case study aluminium and composite plate like structures are considered. For the simulation harmonic signal with 450 kHz frequency and one period of the signal has been analyzed.

Chosen simulation time of wave phenomena in the aluminium plate with thickness 2 mm is 125 ms to avoid reflections from boundary.

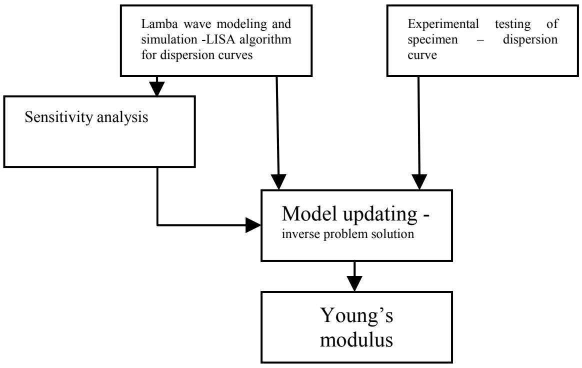

Figure 3. Simulated pattern of waves in the thin aluminium plate.

The model simulation results in a form of dispersion curve has been updated to fit experimental data. The least square method has been applied to estimate material parameters. The quality of dispersion curve fitting is shown in figure 4.

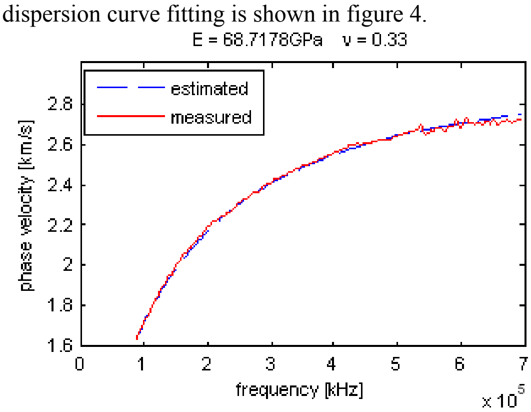

Figure 4. Quality of dispersion curve fitting during Young's modulus identification.

Obtained results in a form of Young modulus have maximum error about 7%, which is enough precise in SHM application. Theoretically S_0 waves gives better results but it is more difficult to excite this kind of waves in the structure.

4. Damage detection based on FE model updating

There exist several approaches for model based damage evaluation based on the dynamic characteristics of a structure and on numerical models [11]. Among others there are the approaches based on FE Model Updating (MU) that allow for two main things, namely: the improvement of the quality of an initial numerical model of a structure in order to obtain a reliable baseline reference and the evaluation of damage in that structure in a current state.

The presence of uncertainty and variability in the data should not be neglected as it may lead to false reasoning based on such data. To cope with the problem the issues of Uncertainty Quantification have been analyzed in this paper with the scope of application in model updating procedures. For the experimental data a framework for estimation of uncertainty in modal parameters has been developed based on the statistical bootstrap technique. For the numerical predictions the perturbation based and Quasi Monte Carlo procedures have been implemented to deal with uncertainties in both direct and inverse problems.

The general outline of model based damage evaluation approach is depicted in figure 5 [4]. The structure to be monitored is equipped with sensor network that measures the structural responses due to external excitation. Residual of these structural responses is then computed based on the reference responses dataset for the baseline undamaged structure. Evaluation of the residual, that is meant do yield damage parameters, is done by solving an inverse problem based on the numerical model of the structure.

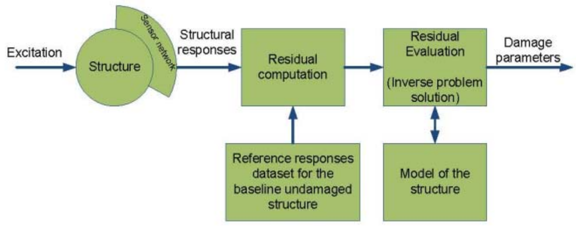

Figure 5. Scheme of model assisted damage detection

The solution of an inverse problem in model based damage evaluation as depicted in Figure 5 can be efficiently done with Model Updating techniques [4]. The concept of MU is shown in Figure 6 [12].

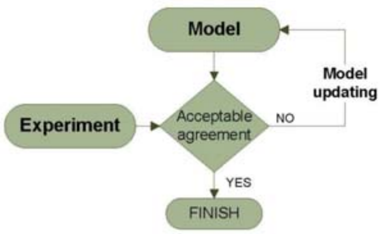

Figure 6. Scheme of model updating procedure.

In MU experimental results are compared with numerical predictions usually by means of some form of residual metric. If acceptable agreement is found there is no need to correct the numerical model. However in case where the agreement is not acceptable Model Updating procedure should be applied to correct the parameters of the model.

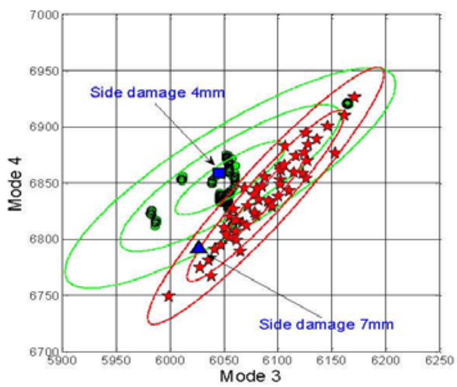

Figure 6. Differences between set of data for damaged and undamaged structures of brake's calliper [4].

Even more difficult task regards an inverse problem that has to be solved in order to identify damage and which also has to be considered with uncertainty.

The procedure has been applied for damage detection in brake's calliper. The results of damage detection using experimentally identified mode shapes as input data for model updating with uncertainties are shown in figure 6. As it can be noticed from figure 6, there is possibility to distinguish models between damaged structure and undamaged one. On –line model parameters monitoring based on acceleration measurements

In vibration based SHM methods, there are many new methods formulated in the literature [12]. The most popular are model assisted methods. Modal models of the structure are most popular in this application, because of relatively easy to identify during structure operation (Operational Modal Analysis – OMA) and physical interpretation of obtained identification results. The idea of application of OMA to SHM is based on on-line modal model parameters identification using output only measurements [13] and comparison of actual model parameters with reference one [14]. The health of the structure is related to value of identified model parameters. This section presents a recursive method of modal parameters identification with wavelet based data filtering [15]. The scheme of the method is presented in figure 7.

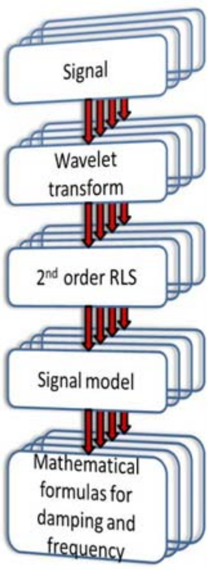

Figure 7. Scheme of modal parameters on-line identification

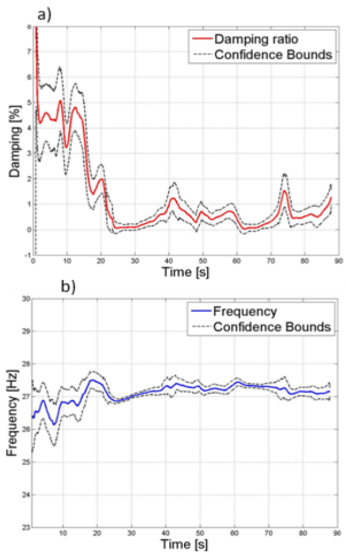

Figure 8. Results of natural frequency and modal damping tracking during airplane flight

This approach significantly reduces demand for computing power which have a direct impact on estimation time of modal parameters. The algorithm also allows assess of the quality of estimated parameters by the calculation of confidence bounds on - line. The method is tested on both numerical models and real systems [15]. Case study of method application for in-flight aviation structure monitoring is presented. Discussed approach is a new idea of application of inverse problem for in-flight flutter monitoring.

5. Conclusions

SHM problems are typical inverse problems. Damage detection, localization and damage assessment problem can solved employing known techniques of inverse problem solution. But presence of uncertainties in both models and experimental data are strong limit in this application of inverse problem.

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