Multi-scale analysis of masonry structures

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Abstract

This presentation describes a multi-scale analysis methodology for masonry structures. At the structural level a macro-scale model based on Cosserat theory is used. The constitutive model for the Cosserat material is derived from a micro-scale model using a representative volume element with a standard Cauchy form of the continuum. A continuum damage model is used as the constitutive model for the mortar and a linear elastic model for the blocks (bricks, stone or concrete). A energy based Hill-Mandel equivalence is used to bridge the scales. A finite element method is developed and implemented as a parallel extension to *FEAP*.

Keywords: Computational mechanics, finite elements, masonry structures, multi-scale

1. Introduction

Masonry is a common form of construction that has been employed for centuries. When subjected to seismic loading, however, they are prone to serious damage or collapse. Thus, an analytical assessment of their response and, for old structures, remedial reinforcement, is of considerable importance. Typical masonry structures are composed of millions of stones or bricks in which the constituents are bound together by mortar. In order to perform an assessment of the structural behavior of a masonry building using standard computational mechanics approaches, such as a finite element model, would lead to very large problem sizes. Thus an alternative approach is needed in order to reduce the problem to a manageable size.

Among several numerical procedures that offer capability to perform reliable structural analyses, multi-scale techniques are one of the most competitive choices. Multi-scale methods have been proposed in different ways as a computational procedure to determine constitutive behavior that includes the effects of micro-scale structural behavior. One procedure is to use a two-scale model in which a finite element analysis is used to solve boundary value problems at each scale. The coarse or macro-scale problem describes the structure to be analyzed and the fine or micro-scale problem is used to provide the constitutive model for the macro-scale analysis based on a solution on a representative volume element (RVE) subjected to imposed boundary displacements. This has often been named an FE^2 approach. Classically Cauchy models are used at both the macro-scale and micro-scale levels and the approach leads to a first order homogenization. For a description of the heterogeneous nature of masonry, an orthotropic continuum is needed at the macro-scale level that incorporates added size effects and texture from the masonry constituents.

In the present work we employ a Cosserat formulation for the macro-scale in order to incorporate size effects in the structural model. At the micro-scale a Cauchy model is employed and utilizes standard constitutive models for each of the constituents. Generally a linear elastic model is used for the bricks and a continuum damage model for the mortar. Finite element analysis is used to solve the boundary value problem at each scale. At both scales a standard finite element approach is used for each of the displacement components in the models. To bridge between the scales we use a Hill-Mandel energy equivalence approach to compute the constitutive properties for the macro-scale structural model.

The multi-scale model described above is implemented as an extension to the general finite element analysis program, *FEAP* (Ref. [6]). A parallel paradigm is formulated in which an RVE model is solved for the strains from each Gauss point of the structural model. To reduce compute times multiple processors are used with results passed between the scales by MPI messages.

2. Theory

A Cosserat material model is used in which, for a two-dimensional plane strain formulation, the three components of displacement are two translations (U, V) and one rotation (Φ). Upper case letters are used to denote quantities associated with the structural macro-scale model. Assuming a small displacement form, the strains for the model are given by

$$\mathbf{E} = \begin{Bmatrix} E_X \\ E_Y \\ E_{XY} \\ \Theta \\ K_X \\ K_Y \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial U}{\partial X} \\ \dfrac{\partial V}{\partial Y} \\ \dfrac{\partial U}{\partial Y} + \dfrac{\partial V}{\partial X} \\ \dfrac{\partial U}{\partial Y} - \dfrac{\partial V}{\partial X} - 2\Phi \\ \dfrac{\partial \Phi}{\partial X} \\ \dfrac{\partial \Phi}{\partial Y} \end{Bmatrix} \tag{1}$$

The first three components are standard Cauchy strains while the latter three denote added components for the adopted Cosserat model. The energy conjugate stresses to the strains are denoted by

$$\mathbf{\Sigma} = \begin{bmatrix} \Sigma_X & \Sigma_Y & \Sigma_{XY} & Z & M_X & M_Y \end{bmatrix}^T \tag{2}$$

For linear elastic behavior of an orthotropic material the constitutive model is given in principal directions by

$$\mathbf{\Sigma} = \mathbf{C}\,\mathbf{E} \text{ with } \mathbf{C} = \begin{bmatrix} C_{11} & C_{12} & 0 & 0 & 0 & 0 \\ C_{21} & C_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{33} & C_{34} & 0 & 0 \\ 0 & 0 & C_{43} & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & D_{11} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{22} \end{bmatrix} \tag{3}$$

For the Cauchy theory used in the RVE model standard linear continuum theory and finite element forms are used (Ref. [7]).

The displacements for the boundary conditions of the micro-scale model ($\mathbf{u}$) are obtained from the structural Gauss point macro-scale Cosserat strains using the form from References [3], [1] and [2]. Accordingly,

$$\mathbf{u} = \mathbf{M}\,\mathbf{E} \tag{4}$$

where

$$\mathbf{M} = \begin{bmatrix} x & 0 & \frac{1}{2}y - \alpha_1 f_2 & -\beta_1 f_2 & -xy & -\frac{1}{2}y^2 \\ 0 & y & \frac{1}{2}x - \alpha_2 f_1 & -\beta_2 f_1 & \frac{1}{2}x^2 & xy \end{bmatrix}, \tag{5}$$

the coordinate functions are

$$\begin{aligned} f_1 &= x\left(x^2 - 3\,\rho_2\,y^2\right) \\ f_2 &= y\left(y^2 - 3\,\rho_1\,x^2\right) \end{aligned} \tag{6}$$

and geometric parameters of the RVE geometry are (see Fig. 1)

$$\begin{aligned} \alpha_1 &= \frac{5(b^2 - h^2)}{8b^4} \text{ and } \alpha_2 = \rho_1^2 \alpha_1 \\ \beta_1 &= \frac{5(b^2 + h^2)}{4b^4} \text{ and } \beta_2 = \rho_1^2 \beta_1 \end{aligned} \tag{7}$$

with

$$\rho_1 = \left(\frac{h}{b}\right)^2 \text{ and } \rho_2 = \left(\frac{b}{h}\right)^2 \tag{8}$$

In the above b and h are geometric size of the RVE (see Fig. 1).

3. Numerical solution

The two scale model for the masonry structure is solved using a finite element method in which displacement components at both scales are approximated by an isoparametric interpolation (e.g., see Ref. [7]). At each Gauss point of the structural model the Cosserat strains are computed and passed to the micro-scale RVE using MPI. These strains are used to compute the displacements at the restrained and periodic boundary nodes using Eq. (5). The constitutive behavior for the RVE uses a linear elastic model for the stiff bricks and a continuum damage model for the mortar (Ref.[5]). The non-linear RVE boundary value problem is solved using a Newton method. On an RVE the equations become

$$\delta\mathbf{u}_a^T \mathbf{r}_a = \sum_e \int_{V_e} \delta\epsilon^T \sigma\,\mathrm{d}V = \delta\mathbf{u}_a^T \sum_e \int_{V_e} \mathbf{B}_a^T \sigma\,\mathrm{d}V \tag{9}$$

which yields the solution for nodes not associated with boundary conditions $\mathbf{r}_a = \mathbf{0}$. Cosserat stresses and material moduli are computed using Hill-Mandel equivalence of energy and an extended approach to that presented in Reference [4]. The basic energy balance for the case where all boundary nodes are restrained is

$$\delta\mathbf{E}^T \mathbf{\Sigma}\,V_{RVE} = \sum_b \delta\mathbf{u}_b^T \mathbf{r}_b = \delta\mathbf{E}^T \sum_b \mathbf{M}_b^T \mathbf{r}_b \tag{10}$$

where b denotes the set of restrained boundary nodes, $\mathbf{M}_b$ denotes Eq. (5) evaluated at x_b, y_b and V_{RVE} is the volume of the RVE. The tangent moduli may also be deduced in a similar way. The computed stresses and material moduli are then passed back to the structural model using MPI. These form the constitutive behavior necessary to solve the structural model using standard solution methods for quasi-static and transient problems. All steps are implemented as user modules to the general finite element analysis program *FEAP*.

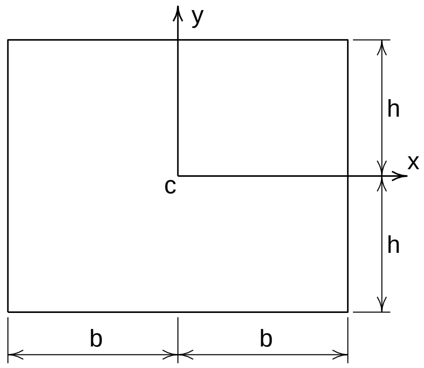

Figure 1: Representative volume element (RVE) geometry

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