Fifty years of Finite Element Analysis of plates and shells (1960-2010)

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Abstract

Development of finite elements for thin, moderately thick as well as thick plates and shells is one of the active areas of the finite element technology for 50 years, followed by hundreds of publications. A variety of plate and shell elements exist in FE programmes, but “the best” finite element is still to be discovered. The work deals with a historical view into the subject from the first papers in early sixties of the last century until now.

Keywords: finite element method, plates, shells, history

1. Origins

The origins of development of plate and shell bending finite elements are dated for early sixties for research group of professor R.W. Clough in Berkeley, USA. Under the direction of Clough, graduate student Ari Adini, who shared an office with Edward Wilson, used a matrix algebra program to solve several problems. He continued his finite element research by using the matrix algebra program to solve thin plate bending problem using rectangular finite elements and demonstrated that this class of structures could be modeled accurately by the method. The resulting research paper [1] published in 1960 was demonstrated that plate bending problems could also be solved by the finite element method, however, it was not accepted for presentation at the ASCE Conference since two other papers from Berkeley had been accepted. The element had 12 degrees of freedom. It used a complete third order polynomial expansion in x and y, aligned with the rectangle sides, plus two additional x^3y and xy^3 terms. The element satisfied completeness as well as transverse deflection continuity but normal slope continuity was only maintained at the four corner points. Adini solved several simple shell structures using the matrix algebra approach and additional commands to form membrane and bending stiffness matrices for thin shells rectangular elements. In 1961 he completed his Ph.D. thesis on the Finite Element Analysis of Shell Structures [2].

In 1960 Jim Tocher, a Ph.D. student working under direction of Clough, started a search for a practical plate-bending element. After two years of work Tocher produced a dissertation that indicated real plate structures could be modeled by triangular elements where the normal displacements were approximated by a ten term polynomial [12]. However the element was too flexible and he could not prove that the results converged to the exact solution as the mesh was refined. In 1962-1963 Tocher spent a year as a post doctoral fellow in Norway and continued to work on the triangular plate element while in communication with Clough. A former student of Clough, T. K. Hsieh, suggested a complicated method of creating a triangular element which satisfied the displacement compatibility conditions. The resulting plate bending element was implemented and tested by Tocher while working at Boeing. The element produced excellent results and was named the HCT element, after Hsieh, Clough and Tocher. The results were published in 1965 at the Wright-Patterson Conference on Matrix Methods [6]. This element was used by the profession for over twenty years to solve thin plate bending problems.

2. Concepts

Historically two similar schemes of development shell/plate finite elements are created – Fig. 1. The *classical shell concept* and the *degeneration concept* are shown as two paths, both starting from a real shell-like 3-D structure and both arriving at the FEM shell element – not necessary equivalent. A very detailed analysis of similarities and differences between those two schools was recently presented by Bischoff et al. [5]. For years the degenerated isoparametric shell elements became a dominating strategy in the FEA of plates and shells.

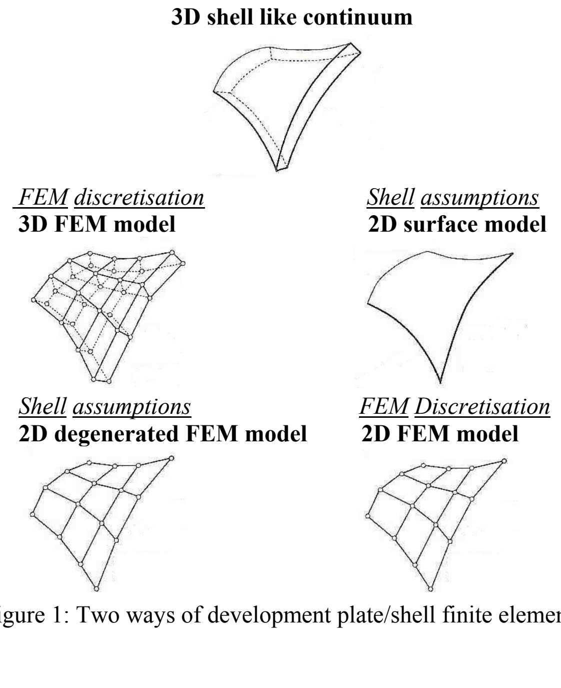

Figure 1: Two ways of development plate/shell finite elements

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3. Milestones

Looking for the history of the finite element formulations for plates and shells it is possible to define the most important papers as the milestones in the field. Beside the already mentioned papers one has to consider the following milestones.

In 1970 Ahmad et al. [3], proposed to construct shell finite elements by a proper modification of isoparametric solid finite elements. That modification included mainly an implementation of the basic assumption of the Mindlin-Reissner kinematics. The name of the "*degenerated isoparametric shell elements*" was commonly accepted soon. Shortly after the publication it was found that the convergence of degenerated shell elements was very slow for thin plate/shell applications due to an over estimation of the element stiffness. In computational mechanics such a phenomenon is known as the "*locking*". Different types of *locking* were named and described in the FEM literature.

In 1971 Pawsey & Clough [9] concurrently with Zienkiewicz et al. [13] revealed an apparent paradox that behavior of degenerated elements can be significantly improved by lowering the order of numerical integration used for evaluation of stiffness matrices. The technique of a "*reduced integration*" became a very popular remedy for the *locking* of degenerated elements. However, it was also found that the reduced integration procedure lowered rank of the element stiffness matrices; some spurious hourglass (zero-energy) mechanisms appeared which, depending on the boundary conditions and an applied mesh of elements, could cause a singular solution.

A significant progress in the development of effective plate/shell elements is related to the application of special "*improved*" interpolation schemes for selected strain components. One of the first successful attempts in that field was reported by MacNeal [7] in 1978 for element QUAD4 with a special treatment of transverse shear strains.

In 1986, Bathe & Dvorkin [4] introduced the code name *MITC4* for the 4-node shell element based on *Mixed Interpolation of Tensorial Components* and *MITC8* for its 8-node counterpart. For the curved element *MITC8* it was also necessary to provide a special treatment of in-plane strains to avoid the membrane locking. Since then, the family of *MITCn* plate/shell elements (with *n* standing for varying number of nodes) served as a typical example of the *Assumed Strain* mixed variational approach application. In 1986 Park & Stanley [8] presented the shear and membrane locking-free 9-node shell element (9-ANS) formulated within the *Assumed Natural Strain*.

Aside from ANS, another popular technique of avoiding locking problems in FEA is the *Enhanced Assumed Strain* (EAS) method; Simo & Rifai [11]. While the ANS method lowered the polynomial order of interpolation for some selected strain components, in the EAS method the conventional strain fields resulting from differentiation of displacements are augmented with additional independent strain field which is incompatible. When those additional strain fields are orthogonal with the corresponding stress fields, they do not contribute to the element energy. Simo & Rifai started their derivation of the EAS method from the three field variational principle of Hu–Washizu; however, other researchers declared that "*the EAS formulation is not a mixed method; but it is rather a displacement model with a limited softening effect*".

Fundamentals of the mixed and hybrid formulations can be found e.g. in the paper of Pian [10]. A lot of mixed/hybrid formulations are presented in the literature to reduce the problem of locking observed in displacement models. Some of the formulations are equivalent to the displacement models with "numerical tricks" like reduced integration, for example.

In spite of 50 years of efforts all over the world "the best" finite element for plate/shell problems is still to be discovered.

4. Selected problems

Hundreds of references in the field of Finite Element formulation for plate/shell structures can be analysed from various points of view. The authors would like to present some selected aspects of the subject during the Conference:

- Thin versus moderately thick finite element models,
- Computational aspects of non-linear analysis,
- Laminated composites and sandwich shells,
- The problem of 6th d.o.f. for shell finite elements,
- "Physical" shape functions for plates and shells.

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