A rectangular strain based finite element for plate bending analysis

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Abstract

A new four node rectangular finite element is developed using the strain based approach and the Reissner/Mindlin plate theory for the linear analysis of plate bending. This element contains three degrees of freedom (β_x , β_y and w) at each of the four corner nodes. The displacement field of the developed element satisfies the equilibrium equations within the element. The performance of this element is evaluated on several problems related to both thick and thin plates in bending. This element is found to be numerically more efficient than the corresponding displacement based element.

Keywords: finite element methods, numerical analysis, plates, structural mechanics, elasticity.

1. Introduction

The formulation of the first plate bending finite elements was based on the thin plate theory (Kirchhoff theory), which neglects the effects of transverse shear. Facing the difficulty of achieving C^1 continuity in the formulation of Kirchhoff plate bending finite elements, considerable research works have been oriented to the Reissner/Mindlin plate theory [1,2]. These Reissner/Mindlin plate elements take into account the shear effects so that they can be used for the analysis of both thick and thin plates. However, the Reissner/Mindlin elements with low order interpolation polynomials tend to cause undesirable shear locking phenomena when dealing with thin plates. As alternative for displacement models, many finite elements based on the strain approach were developed [3-7]. The advantages of using the strain based elements have been illustrated on several two-dimensional and three-dimensional elements [3-7]. In this paper a rectangular finite element based on the strain approach is presented for plate bending problems. The displacement functions of the present element satisfy both compatibility and equilibrium equations.

2. Formulation of the rectangular plate element SBPRE

Consider the Reissner/Mindlin plate element shown in Figure 1., The five components of strain in terms of the three components of the displacement (W, β_x , β_y) are given by:

$$\kappa_x = \frac{\partial \beta_x}{\partial x} \qquad \gamma_{xz} = \beta_x + \frac{\partial W}{\partial x}$$
$$\kappa_y = \frac{\partial \beta_y}{\partial x} \qquad \gamma_{yz} = \beta_y + \frac{\partial W}{\partial y} \tag{1}$$
$$\kappa_{xy} = \left(\frac{\partial \beta_x}{\partial y} + \frac{\partial \beta_y}{\partial x}\right)$$

Where κ_x ,κ_y ,κ_{xy} are the bending curvatures and ,γ_{xz} ,γ_{yz} represent the transverse shear strains. The strain field for the developed element (baptized **SBPRE**) is assumed as follow:

$$\kappa_x = a_4 + a_5 y + f(x) \quad ; \quad \kappa_y = a_6 + a_7 x + g(y) \quad ; \quad \kappa_{xy} = a_8$$
$$\gamma_{xz} = a_9 + a_{10} y - (a_7 y^2) \quad ; \quad \gamma_{yz} = a_{11} + a_{12} x - (a_5 x^2) \tag{2}$$

Where f and g are defined for the present element. The above five strains given by equation (2) must satisfy the compatibility equations and the three equilibrium equations. Then using the known strain displacement relationships given by equation (1), the displacement functions of W, β_x , β_y are derived.

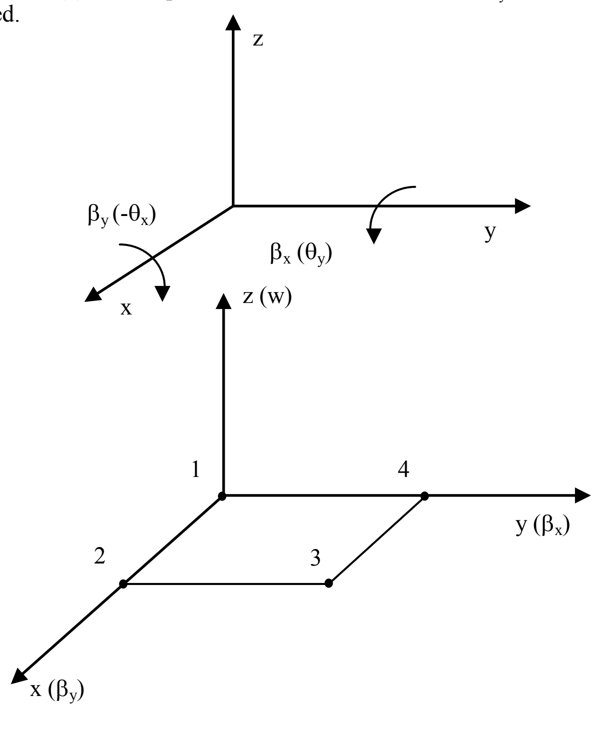

Figure 1: Rectangular plate element with w, β_x and β_y at each node of the four nodes

Table 1: Central deflection of a simply supported plate with a uniform load

	$\frac{WD}{qL^4}$ x100					
	L/h=10			L/h=100		
Mesh	R4	SBPRN	SBH8	R4	SBPRN	SBH8
2x2	0.23170	0.35268	0.35935	0.00446	0.06644	0.08817
4x4	0.36519	0.42907	0.43161	0.01727	0.30900	0.31235
6x6	0.41146	0.44542	0.44686	0.03689	0.37902	0.38061
8x8	0.43142	0.45196	0.45299	0.06128	0.39552	0.39628
10x10	0.44163	0.45524	0.45609	0.08819	0.40112	0.40157
12x12	0.44742	0.45711	-	0.11606	0.40362	-
14x14	0.45103	0.45828	-	0.14237	0.40497	-
16x16	0.45342	0.45905	-	0.16747	0.40580	-
Reference solution	0.46169			0.4062		

Table 2: Influence of L/h on the central deflection (W_c /W_{ref}) of plates

L/h	Simply-supported				Clamped			
	Uniform load		Concentrated load		Uniform load		Concentrated load	
	SBPRE	**R4**	**SBPRE**	**R4**	**SBPRE**	**R4**	**SBPRE**	**R4**
5	1.361	1.358	1.898	1.899	1.718	1.717	2.650	2.654
10	1.130	1.116	1.252	1.241	1.188	1.171	1.410	1.399
20	1.047	0.992	1.071	1.016	1.046	0.971	1.095	1.026
40	1.014	0.826	1.016	0.822	1.009	0.755	1.014	0.711
50	1.008	0.744	1.008	0.736	1.004	0.657	1.004	0.668
100	0.999	0.412	0.996	0.400	0.996	0.318	0.989	0.324
W_{ref}	$0.4062x10^{-2}qL^4/D$		$1.16x10^{-2}PL^2/D$		$0.126x10^{-2}qL^4/D$		$0.56x10^{-2}PL^2/D$	

3. Numerical examples

The present element **SBPRE** is to be compared with the corresponding rectangular bilinear element based on displacement model (named **R4**), with analytical solutions and with numerical results of other elements in order to show its performance for the linear analysis of thick and thin plates in bending. Many tests have been carried which are plate patch tests and square plate. The tests of the simply supported square plates and the clamped square plates are examined with either a uniform loading (q=1) or with a concentrated load (P=1) at the centre. The convergence tests are carried out on two different L/h ratios of 10 and 100 for thick and thin plates respectively (table 1). The effect of L/h ratio on the deflection at the centre w_C for a plate is studied. The results presented in table 2 are given for the 12x12 meshes in terms of w_C/w_k where w_k is the reference Kirchhoff solution [8] for thin plates.

The numerical results show that the strain based element **SBRPE** is free from shear locking since it converges to the Kirchhoff solution for thin plates, contrarily for the corresponding displacement based element **R4** and it have quite rapid rate of convergence to reference solutions for both thick and thin plates. The influence of the transverse shear is much more important for clamped plates than for the simply supported plates

4. Conclusion

The Reissner/Mindlin element based on the strain approach is simple, straightforward with only 12 degrees of freedom and satisfies equilibrium equations. Numerical results obtained, using this element, agree well with those from other investigations and theoretical results for both thin and thick plates. It has been confirmed that the present element (**SBPRE**) is free from any transverse shear locking. The performances of this element have been demonstrated, and the advantages of using the strain approach have again been confirmed.

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