A structural optimization viewpoint on growth phenomena

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Abstract

Models of growth and adaptation are similar to models of structural optimization. Hence, variational sensitivity analysis based on an improved continuous formulation of continuum mechanics plays an important role in structural optimization. The parallelism of variations in physical and material space are highlighted incorporating the changes of mass, of properties and of shapes.

Keywords: computational mechanics, structural optimization, sensitivity analysis, configurational mechanics, growth

1. Introduction

Models of growth and adaptation are similar to models of structural optimization in that both involve *design* variables in addition to the usual mechanical and thermomechanical state variables. In growth problems such design variables are treated by evolution laws, while in structural optimization they are fixed by criteria of optimality. A structural optimization viewpoint on biological evolution is simply to base the determination of growth and adaptation variables on optimality principles instead of evolution. This comes close to the biological concept of homeostasis: to adapt to a changing environment by keeping properties constant (optimal), see [3] for details.

As a consequence, variational sensitivity analysis based on an improved continuous formulation of continuum mechanics plays an important role in structural optimization in order to derive the gradient information needed to formulate optimality criteria, see [1]. The parallelism of variations in physical and material space are highlighted incorporating the change of mass, the change of properties and the change of shapes.

The concept consists of three parts, i.e. a complete *separation of all fundamental quantities*, a complete *reformulation of continuum mechanics* based on this novel viewpoint and a *consistent design linearization technique* for these basic entities within sensitivity analysis.

2. Separation of fundamental quantities

An initial step is devoted to the identification of the fundamental functions and properties in continuum mechanics which are needed to describe changes of mass, of properties and of shapes independently from each other. This idea enforces a *pull back transformation* of the considered domain of the growing body similarly to the *intrinsic concept* introduced in [2] or the *Arbitrary Lagrangian-Eulerian* (ALE) concept.

It has been shown in [1], see references therein for hints to original works of the author, that the complete separation and independent presentation of design and deformation is achieved by two mappings based on local coordinates as outlined in figure 1. The local mappings $\boldsymbol{\kappa}$ and $\boldsymbol{\mu}$ are combined to yield the deformation mapping $\boldsymbol{\varphi} = \boldsymbol{\mu} \circ \boldsymbol{\kappa}^{-1}$. Therefore, the material deformation gradient $\mathbf{F}$ is decomposed via $\mathbf{F} = \mathbf{M}\mathbf{K}^{-1}$, where $\mathbf{K}$ and $\mathbf{M}$ are the gradients of $\boldsymbol{\kappa}$ and $\boldsymbol{\mu}$ with respect to the local coordinates $\boldsymbol{\Theta}$, respectively. The benefits are that the placement mappings are completely independent from each other and that they are only linked via the equilibrium condition. Thus, an easy presentation and understanding of sensitivity analysis is outlined in [1] and this viewpoint is directly linked to configurational mechanics.

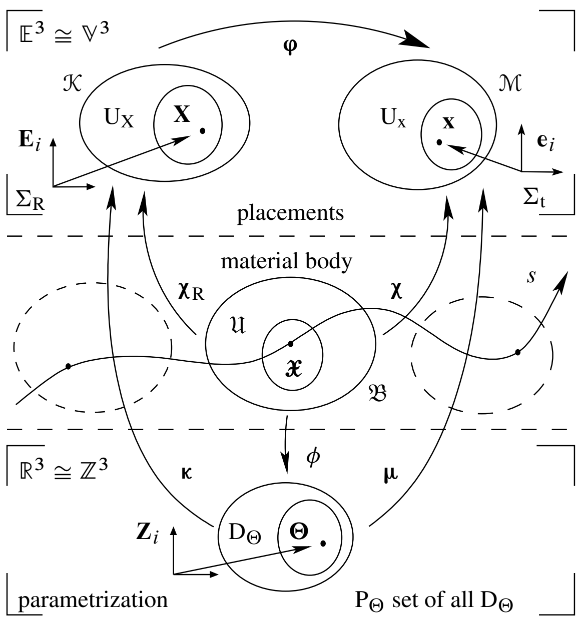

Figure 1: Local definition of placements

This theory has been successfully applied to variational shape sensitivity analysis. Herein, the *growth process* between different designs is not described in detail due to the fact that engineering modifications are always applied to the initial and undeformed structure whose deformation is subsequently completely recomputed from scratch. This situation is depicted with the lines (A) and (B) in figure 2.

This ideal situation of structural optimization is no longer valid in case of growth. Here, an interaction between design s and time t is valid to yield the dashed curve $s = \hat{s}(t)$ in figure 2. Consequently, the complexity of design sensitivity analysis is enlarged even more due to the underlying time dependency of the design. Thus, the question arises which is the most fundamental separation of continuum mechanical quantities in case of growth in order to apply similar ideas as outlined for shape optimization.

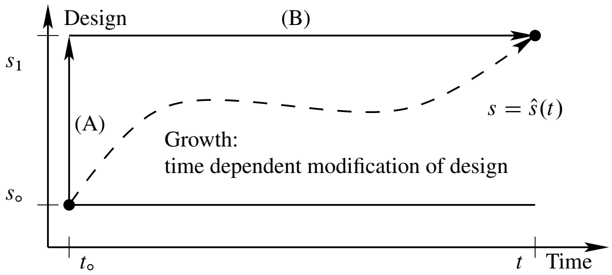

Figure 2: Interaction of design and time parameters

The fundamental idea of this contribution is explained below. The mass density $\varrho = m/V$ as quotient of mass m per occupied volume V is an already composed quantity. Here, *molar mass* M_m, *molar volume* V_m and *number of entities* $N = N_A \cdot n$, where N_a is the Avogadro constant and n denotes the *amount of substance*, are more fundamental. The traditional values are given by $m = M_m \cdot n$ and $V = V_m \cdot n$. Finally, the density reads $\varrho = m/V = M_m/V_m$. Although well-known in chemistry, these fundamental variables are almost not used in continuum mechanics. This situation is evident for an engineering viewpoint leading to a phenomenological formulation on the macroscale. Therefore, the density ϱ is the only material property which is usually available for engineering applications.

3. Reformulation of continuum mechanics

The classical presentation of continuum mechanics is very much based on the idea of an initially given and fixed reference configuration. The author's viewpoint and concept is to withdraw this standard picture and to replace it with a novel composition based on the above introduced fundamental quantities. The novel form is just a reformulation which can be obtained by suitable but unconventional pull back operations similar to the situation obtained for structural optimization, see figure 1.

This advanced decomposition is furthermore motivated by multiscale considerations. Here, every material point $\boldsymbol{\mathfrak{X}}$ is linked to a representative volume element on the microscale. Thus, macroscopic material parameters are based on the microscale phenomena which naturally include also more detailed physical as well as chemical and biological theories. Thus, *molar mass, molar volume, amount of substance* and *number of entities* are clearly items of the microscale. Furthermore, growth should be initially described on the microscale leading to a modification of the phenomenological representation on the macroscale.

Consequently, an equivalent density of the amount of substance can be defined on the macroscopic level. Thus, the molar mass, the molar volume and the number of entities can be varied independently from each other on the microscale. This viewpoint gives an extra degree of freedom in order to describe different phenomena such as *remodeling*, *growth* and *morphogenesis*.

This paper describes the so far obtained results based on the outlined decomposition, see figure 3. Now, the *intrinsic* (or convected) coordinates $\boldsymbol{\xi}$, which are more or less names of existing material points $\boldsymbol{\mathfrak{X}}$, must be distinguished from the *local* coordinates $\boldsymbol{\Theta}$, which are more abstract variables. Thus, a modification of the amount of substance is described by a variation of mapping $\boldsymbol{\pi}$. The advantages and disadvantages are outlined.

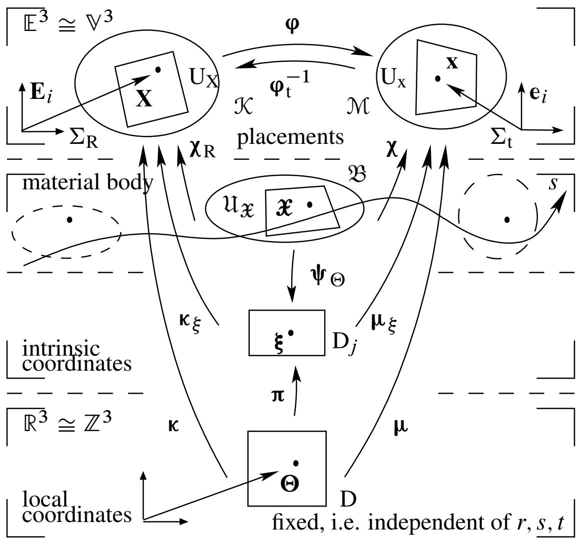

Figure 3: Enhanced decomposition with differences in local and intrinsic coordinates

4. Consistent design linearization technique

The role of consistent linearization is well-known in computational mechanics e.g. to derive the necessary tangents used in Newton's method. Similarly, this technique must be adopted to structural optimization to be used in sensitivity analysis. This is generally well-known but its correct adaptation to growth phenomena modeled by the structural optimization approach is far from being trivial.

The benefit of the complete separation into basic quantities and its consequent reformulation of the most fundamental domain, which is independent from all fundamental quantities, can be seen in sensitivity analysis. Suitable examples explain the ease of the approach after the massive theoretical investment has been performed as outlined above.

Last but not least, the outlined separation must be combined with the ideas formulated in [3] in order to present a complete theoretical framework. This last step is devoted to further research. Finally, the computational approach obtained so far in an ongoing research project is discussed and examples are shown.

References

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