Shape optimization of machine parts made of PRMMCs using approximate low cycle fatigue models

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Abstract

Shape optimization of notched machine parts made of PRMMCs with respect to low cycle fatigue life is discussed in this paper, treated as max-min approach, converted latter to the simple max problem. Many approximate fatigue models named as Neuber type correction rules and their generalized models, are used to compute the actual strain-stress field in the notch zone are the main components of the fatigue analysis module. The response of PRMMCs is based on an analytical homogenization methods (Eshelby's equivalence principle and Mori-Tanaka method). Average strain and stress (s-s) pseudoelastic fields are mapped to Ramberg-Osgood s-s relation for PRMMCs. Number of cycles in crack initiation phase is computed using an analytical model which has been proposed by Zhang and Chen. Increasing low cycle fatigue time is observed in comparison to the traditional shapes of machine parts.

Keywords:optimization,fatigue, computationals methods, PRMMCS

1. Introduction

In particulate reinforced metal matrix composites (PRMMCs) non-metallic particles or whiskers are incorporated in metallic alloys to improve their mechanical properties. The fatigue behaviours of PRMMCs are very important factors for these engineering applications. We generally have the following points that characterize the low cycle fatigue (LCF) life behaviour of PRMMCs [1]: (1) The microstructure of almost all commercially available PRMMCs have relatively uniform particle distribution. (2) The reinforcement particle volume fraction strongly influences crack initiation and crack propagation time. (3) The LCF life of the PRMMCs follows a Coffin-Manson relationship as well. (4) The LCF failure mechanism is essentially a matter of crack growth. (5). When machine parts are subjected to cyclic loading, cracks and therefore failures generally initiate at notches. Although it can be assumed that the bulk of a PRMMCs component remains elastic, the stresses in the metallic matrix in the vicinity of these concentrations may exceed the yield stress of the material. The possibility exists to modify fatigue properties of PRMMCs component via of the notch shape optimization process.

2. Problem formulation

In this paper we treat the optimization problem as follows: for a given boundary conditions, external loading and material properties:

find $\mathbf{D} \in R^n$ to maximize

$$\{\min N_i(\mathbf{D}) \colon \mathbf{g}(\mathbf{D}) \le \mathbf{0},\ \mathbf{D}_{lower} \le \mathbf{D} \le \mathbf{D}_{upper}\} \qquad (1)$$

and $\Gamma(\mathbf{D}) \in \Gamma^*$,

where $\mathbf{D}$ is n-dimensional vector of design variables, min N_i ($\mathbf{D}$) is the objective function, $\mathbf{g}(\mathbf{D})$ are the constraint functions, Γ(D) is a boundary shape to be modified, $\mathbf{D}_{lower}$ and $\mathbf{D}_{upper}$ are the lower and upper bounds on vector D, Γ^* is a given variation domain of Γ (Fig. 1), and (i) is the number of critical BEM nodes (these nodes are equivalent to a location of critical points (planes) in Local Strain Approach). around $\Gamma \equiv AB$, where number of cycles N_i are evaluated. This is the familiar max-min approach with discontinuous objective function. Using the bound (beta) formulation, the problem is converted to the simple max problem [3].

3. Approximate LCF models

Approximate fatigue models named as Neuber type correction rules (Neuber, Molski and Glinka, known also as ESED, Ye et al., Sethuranian and Gupta methods [2,3] and their generalized models, both uniaxial or multiaxial variants), coupled with Mori-Tanaka homogenisation method, are used to compute the actual strain-stress field in the notch zone.

4. LCF crack initiation model for PRMMCs

Number of cycles in crack initiation phase is computed using an analytical model which has been proposed by Zhang and Chen [4]. This proposal is an extension of Ding et al. [1] LCF life prediction model limited only to the fatigue crack propagation process. This formula is given in the explicit form:

$$N_i = \frac{\pi E_c \gamma - 4\left\{K'\left[\dfrac{\Delta\varepsilon_p}{2(1-V_f)}\right]^{n'}\right\}^2 a^*(1-\nu_c^2)}{2\pi E_c\, f t K'\left[\dfrac{\Delta\varepsilon_p}{2(1-V_f)}\right]^{n'} \dfrac{\Delta\varepsilon_p}{(1-V_f)}} \qquad (2)$$

where N_i is the crack initiation life, V_f is the volume fraction of the reinforcement, E_c is the Young modulus of PRRMCs, γ is the surface energy density, K' is the cyclic strength coefficient,

n' is the cyclic strain hardening exponent, $\Delta\varepsilon_p$ is the plastic strain range in the cycle, a^* is the radius of newly created penny-like crack, ft is the indicator of the energy that is stored from cycle to cycle, and ν_c is the Poisson ratio of PRMMCs. For details see the paper of Zhang and Chen [4].

5. Numerical example

The example treated is a plate containing a central hole and subjected to uniaxial in-plane cyclic loading (y – direction). The characteristic dimension of the hole with respect to the width of the plate is ¼ (Fig.1). The maximum and minimum stresses are S_{max}=200 MPa and S_{min}=20 MPa.

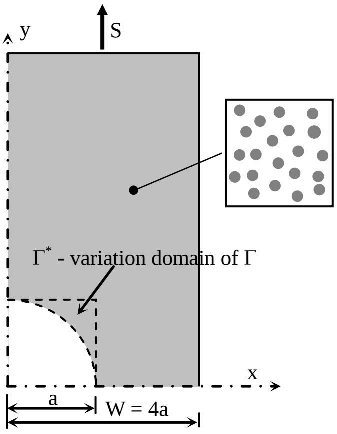

Figure 1: Plate made of PRMMC with central hole

To predict fatigue life the equation (2) is used. The problem is to find the shape Γ of the hole for which min N(Γ) reaches maximum. The shape of the hole is defined by special concept of one segmented Bezier interpolants (modification of the standard Bezier's curve) [3]. The stress field is evaluated using the Fictitious Stress (Load) Method (indirect variant of the BEM) [3]. The Neuber's rule coupled with Mori-Tanaka scheme is used to compute the actual strain and stresses in PRMMC. The optimization procedure uses the SLP method with move limits. The reference shape of the hole is the circular hole contour. The material data reported in the literature are listed in Table 1. Figs 3-4 show the reference and optimal contours for V_f = 0.15 and the distribution of number of cycles during the crack initiation phase corresponding to the reference and optimal solution for V_f equal 0.15 and 0.2.

Table 1: Material parameters for AA6061-Al_2O_3 T6 composite

V_f	n'	E_c (Gpa)	K' (MPa)	Ft (10^{-9} m)	$\sigma_{y0.2\ c}$ (MPa) exp
0.15	0.163	93.3	1066	3.74	387
0.20	0.110	97.2	792	5.91	400

6. Summary and conclusions

It has been shown in this paper how by proper shape modification of notches or machine part boundaries, made of PRMMCs, we can significantly increase the number of cycles corresponding to crack initiation. Average strain and stress (s-s) pseudoelastic fields (Mori-Tanaka approach) are mapped to Ramberg-Osgood s-s relation for PRMMCs (Neuber method in this Abstract). The design procedure is the combination of the CAGD mathematical methods for the shape definition of notch boundary, and the BEM is used for analysis of the pseudoelastic response state. All mentioned above components of the overall optimization algorithm are assisted by the Sequential Linear Programming method with move limits. Numerical example displays a significant increase in a number of cycles corresponding to crack initiation phase in comparison to a traditional (regular) shape of the machine part.

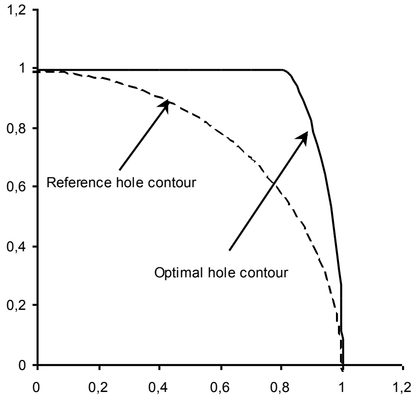

Figure 2: Reference and optimal contour of the hole (V_f=0.15)

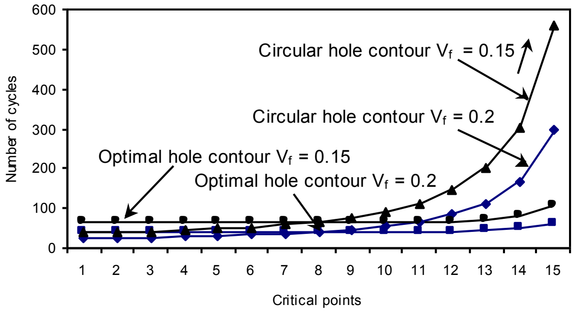

Figure 3: Distribution of number of cycles at critical points

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