

# Going beyond the classic experimental data error analysis with physically based global method of approximation

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## Abstract

Classic methods for analysis of experimental error concentrate in the first place on finding data statistics which serve for evaluation of experimental error. It is done by calculating various statistical parameters that serve for further inference. Averaging of data series or other smoothing methods may also serve for enhancement of experimental data, providing fields of higher quality. In the paper presented are capabilities of the so called physically based global method approximation (PBGGM) for analysis of experimental data that goes beyond applicability of these classic methods. The PBGM approach is somehow unique in this area as it combines into one method both smoothing and error analysis functionalities, building fits to data that guarantee conformance with the theoretical relations that hold and offering all these capabilities even for a single set of data. Thanks to high quality fits the PBGM offers it is possible to go beyond a mere evaluation of statistical parameters/data credibility into the area of reconstruction of the lost quality and correction of what went wrong, sometimes even before the data was collected, as it will be demonstrated in the case of application of the PBGM method to analysis of real, extremely noisy experimental data that came from neutron diffraction experiments for railroad rails.

*Keywords: meshless methods, error estimation, experimental mechanics, inverse problems*

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## 1. Introduction

In the last two decades the US DOT, Federal Railroad Administration was sponsoring a series of research activities that were oriented on theoretical and experimental analyses of the rail 3D residual stress. Partially as an offspring of these efforts and the needs they created as far as experimental approaches are concerned, developed was a new approach to enhanced analysis of experimental data called as physically based approximation (PBA). In this approach the measurements are treated only as preliminary values, a vague representation of the field that is being restored – similarly to the fuzzy sets approach – and their true values are searched among combinations of admissible functions that obey the requirements that come from the theory of the examined problem or other various relations they should be conformant to (like heuristic requirements, experimental relations, etc.). What is important to stress here, is the fundamental difference with many hybrid experimental-theoretical approaches where the theoretical relations are used to compensate for the lacking experimental relations and are treated as an essential add-on for the whole method to make it possible to obtain the final results. In the PBA approach, the theoretical information is always treated as redundant, used for building a smoothing engine that guarantees the overall conformance with the theoretical requirements. Thanks to this characteristic, the PBA method – and especially the presented here global method formulation PBGM – offers high quality fits to the experimentally derived fields that are suitable not only for further non-statistical error analysis but for other analyses that transcend the usual application of experimental data analysis approaches.

The paper presents results of an advanced, physically based global method (PBGGM) analysis of extremely noisy experimental data obtained during the neutron diffraction (ND) scans of low-stress railroad rail samples. The results

demonstrate robustness of the PBGM method by presenting high quality fits to noisy data which made it possible not only to regain sense and credibility in the otherwise useless data but also to discover certain more fundamental errors that biased the raw data. Without the PBGM methodology those errors would remain unnoticed, as they managed to in the considered case.

## 2. The data

The experimental data was collected at the US DOC, NIST atomic reactor facility in Gaithersburg, MD for several rail samples analysed for residual stress. The experiments were performed according to the experimental scheme called as the transverse/oblique slicing [1], which is basically a destructive technique in which two (or more) thin slices of rail are scanned for in-plane residual stresses in two mutually inclined coordinate systems and then required a 3D stress restoration numerical technique that compensated for the (theoretically) total loss of the slices' normal stress components and its repercussions on the partially-relieved 2D in-plane states that remained in the slices. The 3D technique is not discussed here, only analysis on the 2D level will be in focus here.

An exemplary data for one of the two slices comprising a rail sample is shown in Fig. 1. The data is extremely noisy, which comes to no surprise once one takes into account that the sample was extracted from a rail slab taken out of manufacture which was air-cooled, but neither roller straightened nor subject to actual loading program after installation in the track, thus the stress state in the rail is in very low range of amplitudes, hardly measurable by the neutron diffraction technique.

## 3. Advanced analysis with the PBGM

The formulation of the PBGM is not new and has been already given, e.g., in a paper presented during the CMM 2007

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\*Footnotes may appear on the first page only to indicate research grant, sponsoring agency, etc. These should not be numbered but referred to by symbols, e.g. \*,+. The footnote text may be produced in a small font.

conference [2]. Due to a limited space it will not be recalled here.

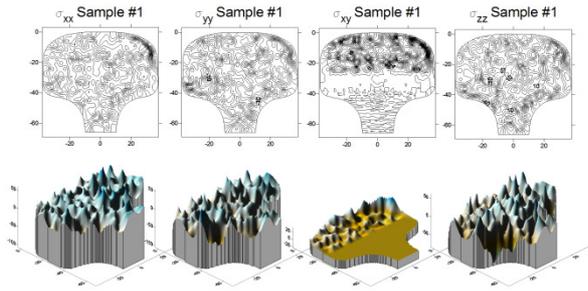


Figure 1. Sample #1, raw data for transverse slice (Slice 1)

The purpose of the research presented in this paper was to perform smoothing and error analysis of the data presented in Fig. 1 as well as for the companion oblique slice, for which the data is not shown here. Unfortunately, after smoothing the raw data for the both slices, it was found that there was a severe discrepancy in the distributions of the horizontal stress components – see Fig. 2 – which commanded an advanced analysis of the whole case. The need to address this problem stemmed from the fact that – as proved in [1] – the horizontal components for the transverse and oblique slices should be exactly the same, whereas – as easily visible in Fig.2 – they were found to be of opposite signs. The fact the other two in-plane components had conformant signs did not make the assumption that a trivial error had been committed very plausible. To make things more complex, one has to be aware that in case of the neutron diffraction technique determined are the strains in at least three (usually four) mutually rotated coordinate systems and the stresses are found as a solution to a linear set of equations.

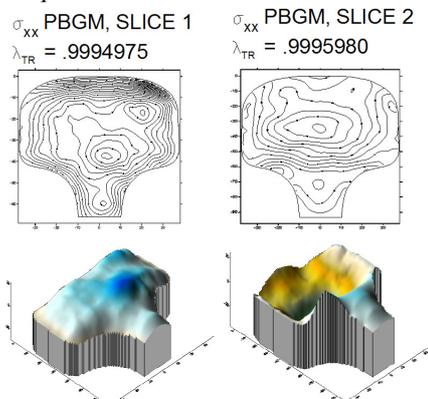


Figure 2. Sample #1, PBGM smoothed data for transverse and oblique slices (Slice 1/Slice2)

Several scenarios were hypothesized and thoroughly tested with the PBGM approach, out of which the hypothesis that the experimenters made an error on a raw data preprocessing stage turned out as very probable. In this scenario it was assumed that there was an error in a cell formula for computing strains as the experimental team used MS Excel spreadsheet application for processing of the experimental data. Indeed, the neutron diffraction technique collects information about the strain tensor, upon application of the classic formula:

$$\epsilon = \Delta d/d = (d - d_0)/d_0 \quad (1)$$

where  $d$  is the actual distance between the crystallographic lattices measured with the ND technique and  $d_0$  is the crystallographic lattices' distance in the stress free body. This

formula is really dangerous if somebody misinterprets data in the spreadsheet by switching  $d$  and  $d_0$  columns, because in that case the formula would lead to a reverted sign of a strain component.

To verify this hypothesis, a symbolic solution to the linear set of equations that binds the stress and strain components was done and it was found that the structure of this set that is expressed as:

$$\begin{aligned} \sigma_{ss} &= \frac{(C_1+C_2) \cdot \epsilon_{ss} - C_2 \cdot \epsilon_{tt} - C_2 \cdot \epsilon_{nn}}{C_1^2 + C_1 \cdot C_2 - 2 \cdot C_2^2} \\ \sigma_{tt} &= \frac{(C_1+C_2) \cdot \epsilon_{tt} - C_2 \cdot \epsilon_{ss} - C_2 \cdot \epsilon_{nn}}{C_1^2 + C_1 \cdot C_2 - 2 \cdot C_2^2} \\ \sigma_{nn} &= \frac{(C_1+C_2) \cdot \epsilon_{nn} - C_2 \cdot \epsilon_{tt} - C_2 \cdot \epsilon_{ss}}{C_1^2 + C_1 \cdot C_2 - 2 \cdot C_2^2} \end{aligned} \quad (2)$$

leads to almost a direct relation between the  $\epsilon_{ss}$  and  $\sigma_{ss}$  components due to dominant diagonal values of  $(C_1+C_2)$  over the out-of-diagonal  $C_2$  values (the  $C_1$  and  $C_2$  are material constants). Thus, taking into account the nature of the MS Excel cell formulas and the structure of the linear set that binds the stress and strain data it was found that this hypothesis is really plausible. The PBGM approximation to the corrected  $\epsilon_{ss}$  data was performed and the outcome of this analysis (for the horizontal stresses only) is shown in Fig. 3. These results seemingly justify the correctness of the hypothesis and the effort undertaken to track down the source of this error.

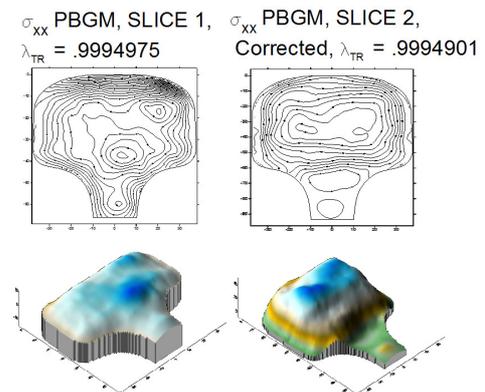


Figure 3. Sample #1, PBGM smoothed data for transverse and oblique slices, the data for Slice 2 was corrected

#### 4. Conclusions

It was demonstrated here that with help of the high quality physically based PBGM data fits it is possible to perform advanced analysis of experimental data error that goes far beyond the standard methodologies. It was proved that this approach lends itself well not only for noise removal but also reveals many hidden facts about the data, too. Thanks to the performance of the PBGM approach it was possible to compensate for a severe errors in the data without repetition of very expensive experiments.

#### References

- [1] Magiera J., Orkisz J., Karmowski W., Reconstruction of residual stresses in railroad rails from measurements made on vertical and oblique slices, *Wear*, 191, pp. 78-89, 1996.
- [2] Magiera J., Enhancement of noisy neutron diffraction data by the meshless physically based global method, *Proceedings of the 17<sup>th</sup> International Conference on Computer Methods in Mechanics, Łódź-Spała, 2007.*