Investigation on the discharge process in a flat-bottomed bin: the experimental measurements and theoretical predictions

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Abstract

This paper presents comparison between experimental measurements of wall pressures made in the flat bottomed silo model and theoretical calculations. The numerical and simplified analytical analysis of deformation and stress states in a silo model during discharge was considered. In the analytical solution, we discuss a method for solving a set of differential equations governing the flow in the model, while numerical analysis is considered by implementing the relations based on discrete element method. The experiments were performed in a bin model made of plexi and filled with the pea grains, while the adequacy of the theoretical and measured values was considered.

Keywords: bin, pressure, outflow rates, experimental measurements, DEM and continuum-based predicitons

1. Introduction

An accurate prediction of the granular flow in bins and silos always play a major role in their reliable design, maintenance and control of granular material handling. In general, the outflow rates, velocity patterns, wall pressures are contributed to each other. This contribution leads to a sophisticated theoretical background, while the postulated assumptions on flow roles and time-dependent states are accounted for the of the granular material behaviour as a whole.

Different approaches [1]-[4] show that researchers consider the material within the bin by using the simplified continuum models. Generally, a treatment of the granular material as linear elastic, satisfying Hooke's law, or perfectly plastic, satisfying Coulomb yield condition, with the associated or non-associated flow rule serve the base of classical continuum-based techniques. The most popular assumption is that the granular material within the silo is in the plastic state of stress. This assumption is so regularly made that it is sometimes not even stated, but the use of failure properties, such as the angle of internal friction, indicates that the assumption is introduced. The fact that this plastic assumption imposes considerable restrictions on the stress history and deformation of the material has always been ignored. Some attempts have been already made to verify this assumption in [1].

The capability to take into account the contribution of individual grain properties on the macroscopic material behaviour is not able in a frame of continuum mechanics. Consequently, discrete approach in terms of the Discrete Element Method (DEM), proposed by Cundal and Strack [6], was efficient numerical tool accounting both micro and macro levels in a time-dependent fashion.

In DEM, the granular material was assumed to be composed of a set of particles treated as the contacting visco-elastic non-cohesive frictional solid bodies. Each individual particle in the system is allowed to undergo translational and rotational motions due to gravity acceleration, forces and the torques originated from contacts between the particles or particle with walls.

The physical state of material is defined through the state of particles. When the number of particles increases their individual behavior smears producing the behavior of the whole material. Implementation of DEM in analysis of hopper filling and discharge was given in [5], [7].

In the current investigation, we present analysis of the experimental measurements and theoretical predictions on wall stresses, outflow rates, velocity patterns in the flat-bottomed bin model filled with pea grains. Main theoretical predictions are based on numerical simulation of the discharge process by DEM. The theoretical predictions in terms of the continuum mechanics approaches are also considered

2. Experiments – wall pressures measurements

The experiments were performed in the laboratory. A bin model made of Plexiglas was put on a stage. Wall pressures were measured at two locations: Sensor 1 was installed at 55 mm above the bottom on the right wall, and Sensor 2 at the height of 165 mm on the left wall. There were two 45 mm diameter tensometric sensors (type: CL17) made of aluminum alloy and representing uniaxial strain gauges measurements. The strain gauges were connected to a data measuring system consisting of AD converter card ADLINK 9112. Measurements were controlled by a data acquisition program, which recorded and analyzed the data in real time. The amount of grains filled the model close to the third sensor that was not fully covered with the grains so that is why the measurements were registered only by two sensors.

3. Theoretical predictions of wall pressures

Analytical modelling may be defined in terms of perfectly elastic and the elastic-plastic solution providing the relations for wall stress [2]-[4]. In this case, the material discharge may be predicted in terms of the imposed vertical (z axis) displacement. In particular, it is assumed that the upper surface is free, but the bottom material surface undergoes increasing displacement u_1 starting from the initial zero value. Therefore the following boundary conditions may be assumed

$$\sigma_z(z)\big|_{z=\frac{1}{n}} = 0 \;\;, \; u(z)\big|_{z=1} = -u_1 \tag{1}$$

At the walls the friction stress is acting $\tau_n = \sigma_x(z)\tan\mu$ ($\sigma_x(z)$ is horizontal stress at z distance from the material surface, μ is the coefficient of wall friction) and it include in the equilibrium equations, so there are no boundary conditions stated for. For brevity, the solution in terms of the horizontal stress acting on the wall may represented by relation of the following form:

$$\sigma_x\big|_{z=1} = \frac{\psi - \nu_1}{\psi + 2\nu_1}\sigma_z\Bigg|_{z=1} \tag{2}$$

where ψ, ν_1 are the constant evaluating the elastic/plastic material states for active/passive deformation zones [2]-[4].

When wall stress is analysed by means of the discrete element method, a time-dependent material state during discharge is considered. The evaluation of the inter-particle as well as particle-wall contact forces is determined by the contact mechanics models in terms of Hertz and Mindlin-Deresiewicz's theories, while the time-dependent state of granular material is defined by second Newton's law for translation and rotational motions. Thus, the stresses acting on the wall denoted by superscript w may be found from the following expression

$$\boldsymbol{\sigma}_{ij}^{w} = \frac{1}{A}\sum_{c_w \in A} \mathbf{F}_{ij}^{c_w}\left(h_{ij}^{w}\right), \tag{3}$$

$$h_{ij}^{w} = R_i - \left|\mathbf{x}_s^{w} - \mathbf{x}_i(t)\right|, \tag{4}$$

$$\left\{\mathbf{x}_1(t), \ldots \mathbf{x}_N(t)\right\} \in \left\{ \mathbf{x}_i(t) \;\middle|\; \left\{ \begin{array}{l} \iint \frac{\sum \mathbf{F}_i(t)}{m_i} \mathrm{d}t\mathrm{d}t + \mathbf{C}_{i,1}t + \mathrm{C}_{i,2} \\ \iint \frac{\sum \mathbf{M}_i(t)}{I_i} \mathrm{d}t\mathrm{d}t + \mathbf{C}_{i,3}t + \mathrm{C}_{i,4} \end{array} \right. \right\}, \tag{5}$$

where $\mathbf{F}_{ij}^{c_w}\left(h_{ij}^{w}\right)$ is the contact force vector at contact c_w, depending on overlap between the particle i and wall, A is the representative area of the wall (or the sensor area) on which the contact forces are homogenized, $\mathbf{x}_s^w$ is the vector for the location of the representative area or the sensor, $\sum \mathbf{F}_i(t)$ and $\sum \mathbf{M}_i(t)$ is the resultant force vector and torque acting on *i-th* particle, respectively, $\mathbf{C}_{i,1,\ldots 4}$ are the unknown constant vector for *i-th* particle, m_i, I_i are the mass and moment of inertia for *i-th* particle, t is the time being considered.

As a role, the solution of (5) cannot be defined explicitly. To this end, the numerical approach given in [8] is used in the current analysis.

4. An adequacy to the experiment

Comparison of the numerically predicted wall stress and those measured experimentally during the discharge process is illustrated in Figure 1. As can be seen in the plotted graph, the numerical prediction of wall stress is quite adequate to the measured wall pressure.

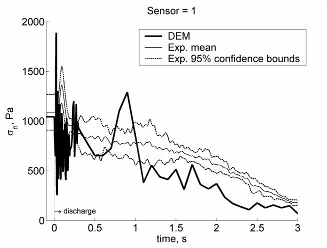

Figure 1: Development of horizontal stress of the wall in discharge.

5. Concluding remarks

The obtained results of the wall pressures demonstrate the quite satisfactory adequacy of numerical prediction and experiments. Comparison of the other discharge parameters will be also discussed.

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