Sensitivity analysis and optimal design of shear deformable sandwich panels

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Abstract

The aim of the study is to find optimal dimensional and material parameters of panels including their support parameters under the constraints following from the conditions of the ultimate and serviceability limit states for prescribed load. Multi-span sandwich panels with a soft core and flat steel facing subjected to static load and thermal action are considered. Introduction of elastic supports allows us to reduce the unfavorable influence of temperature on the state of stress. The sensitivity operators are derived using the adjoint variable method. These operators account for the jump in slope of slabs at the line of concentrated active load or reaction, thus extending the sensitivity operators known in literature. The jump of slope results from shear deformable core.

Keywords: optimization, sensitivity, structural mechanics, plates, composites

1. Introduction

The common aim of designers and technologists is to find solutions which combine high quality, safety and low price. This can be reached when the characteristic properties of different materials in composite or sandwich structures are used reasonably. In the construction industry three-layered panels (sandwich panels) with a soft core and very thin steel flat or profiled facings play special role. They offer high load-bearing capacity coupled with small weight and good thermal insulation. The structural behavior of these panels is strongly affected by the shear flexibility of the core. Moreover, several failure mechanisms are faced, which are not observed in the response of solid slabs. Hence, in the design particular attention must be paid to wrinkling failure of the facings and the local deformation at the support. Important role is often played by the temperature action.

The optimal design of sandwich structures is frequently taken up in literature. There is great demand of the market to design slabs which provide maximal range of applications at minimal cost. Such optimization problems were undertaken by a number of authors [4, 8, 10]. In the paper [1] core junctions were proposed to improve shear panel capacity. Simultaneously the geometric shape of the boundary of the adjoined core materials was improved to significantly diminish local stress concentrations at the core. Some recent works tackle the problem of different failure modes and their influence on the state of stresses. The comparison of the behavior of sandwich panels with various combinations of materials for achieving minimum mass is presented in [7]; meanwhile in [9] failure maps are created. They illustrate the dependence of failure mechanisms on the structural parameters and load/support conditions.

Several theories of shear deformable beams and panels have been proposed in the literature [11]. The panels widely used in civil engineering are characterized by sufficiently large length/width ratio, supporting and loading conditions so that satisfactory precision is guaranteed by Timoshenko beam theory generalized to sandwich sections. It belongs to the class of the First Order Shear Deformation Theory (FSDT).

The response of sandwich panels with soft core is quantitatively and qualitatively different from the response of a typical panel. It is illustrated in Fig. 1 by the example of a beam loaded by a concentrated force at the tip of the beam. Important is that at the points of concentrated load and reaction forces there are jumps in the slope of deflection line. This effect has been taken into account in sensitivity analysis with respect to parameters of joints [2] and further developed in [6]. In this paper a new problem is studied, when the jump appears at the place of the support.

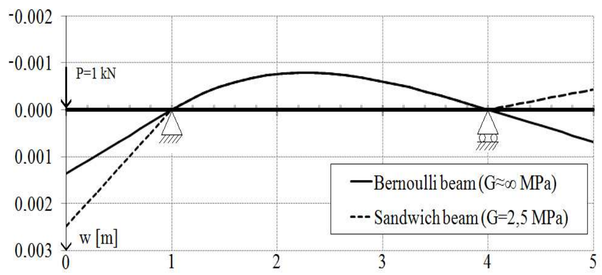

Figure 1: Displacement lines for a sandwich and Bernoulli beam.

In the present paper the problems of the sensitivity analysis and optimal design are taken up. Interaction of external load and temperature is taken into account. It is well known that such interaction leads to conflict in optimization. Therefore special attention is paid to support conditions, because by optimization of support conditions better compromise can be found [3]. The sensitivity operators accounting for variable support conditions and geometric and material parameters are derived in explicit form using the continuous formulation and the adjoint variable method. The response functional is defined in a general form allowing optimization of stress, strains or displacements. Necessary conditions for optimal placement and stiffness of the support are derived. Special cases of the response functional are discussed. In optimal design several behavioral constraints are assumed.

2. Formulation of the problem

It is assumed that the materials are isotropic, homogenous and linear. Because the Young modulus of the core is about 70 thousand less than Young modulus of the facings, the normal stresses in the core are negligible, hence shear stresses τ in the core are constant.

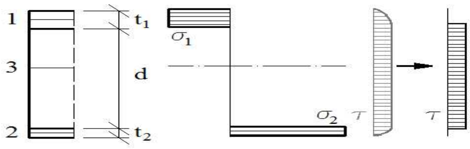

Figure 2: Distribution of normal stresses (σ_1, σ_2) and shear stress (τ) in a sandwich panel with thin flat facings.

Let us consider a response functional (1) as a measure of structural behavior of the sandwich structure. We assume F as a Gateaux differentiable function of displacements w, stresses $\mathbf{Q}$ and strains $\mathbf{q}$. The functional $G(\boldsymbol{\xi})$ can play the role of the objective function or a constraint,

$$G(\boldsymbol{\xi}) = \int_0^L F(w, \mathbf{Q}, \mathbf{q}) \cdot dx \,. \tag{1}$$

The design vector $\boldsymbol{\xi}$ has the components: position and stiffness of the support x_s and k_s, thickness of the facings t_{Fi}, total thickness of the panel D, shear modulus of the core G_C, and theYoung modulus of the facings E_{Fi} .

$$\boldsymbol{\xi} = \left[\, x_s, k_s, t_{F1}, t_{F2}, D, G_C, E_{F1}, E_{F2} \,\right]. \tag{2}$$

Variation of (1) takes the form

$$\delta G(\boldsymbol{\xi}) = \int_0^L \left(\frac{\partial F}{\partial w} \delta w + \frac{\partial F}{\partial \mathbf{Q}} \delta \mathbf{Q} + \frac{\partial F}{\partial \mathbf{q}} \delta \mathbf{q} \right) dx \,, \tag{3}$$

where δw, $\delta\mathbf{Q}$ and $\delta\mathbf{q}$ represent implicit functions of the variations of $\boldsymbol{\xi}$.

Following [4] we assume that the structure is subjected to initial distortions, both in form of the initial strain field $\mathbf{q}^i$ which is kinematically inadmissible and initial stress field $\mathbf{Q}^i$ which is statically inadmissible. This causes the elastic strain $\mathbf{q}^e$ and stress $\mathbf{Q}^e$ (Fig. 3), which are interrelated by the Hook's law (4^3):

$$\mathbf{Q} = \mathbf{Q}^i + \mathbf{Q}^e, \quad \mathbf{q} = \mathbf{q}^i + \mathbf{q}^e, \quad \mathbf{Q}^e = \mathbf{K}\,\mathbf{q}^e \,. \tag{4}$$

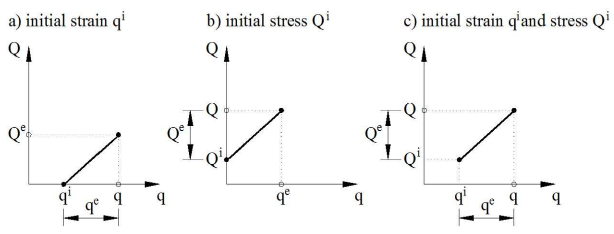

Figure 3: Conception of the initial stress and strains, see [4].

The stiffness matrix $\mathbf{K}$, stress $\mathbf{Q}$ and strain $\mathbf{q}$ have the forms

$$\mathbf{K} = \begin{bmatrix} B_s & 0 \\ 0 & S \end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix} M \\ V \end{bmatrix}, \quad \mathbf{q} = \begin{bmatrix} -w''_M \\ w'_V \end{bmatrix} \tag{5}$$

where B_S and S denote bending stiffness and shear stiffness, respectively.

Introducing the adjoint structure and using the generalized reciprocity theorem accounting for loads and distortions [2] we arrive at the sensitivity gradient as the explicit function of all variations of the design vector (2).

The paper is also concerned in problems of optimal design of slabs with regard to the design vector (2) and accounting for a number of box and behavioral constraints.

3. Concluding remarks

Interesting is that due to the jump of slope, the sensitivity gradient with respect to the position of the support x_S is discontinuous at $x = x_S$. Hence, the necessary condition for the optimal support position has no more the form of equality known from the literature, but takes the new form of the set of inequalities. The derived formulae for sensitivity operators were applied to some special cases and checked using the central difference method. Good agreement was observed. The applicability and usefulness of the response functional in the form of the total interior elastic energy induced by the load and distortions proposed in [4] was also studied.

It appeared that in case of interaction of load and temperature induced distortions the introduction of elastic supports can remarkably improve the structural response. It reduces the stresses and hence diminishes the danger of wrinkling and local deformation (indentation) at the place of the support. As a result the elastically supported slab can safely carry out larger temperature variation and/or external loads.

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