Multi-constrained topology optimization using constant criterion surface algorithm

Mirosław Mrzygłód
Institute of Rail Vehicles, Cracow University of Technology
al. Jana Pawła II 37, 31-864 Kraków, Poland
e-mail: mrzyglod@mech.pk.edu.pl

Abstract

This paper sets out to describe a multi-constrained approach to least-weight topology optimization of structures. In the optimization, a constant criterion surface algorithm and multi-constrains procedure is used. The multi-constraints procedure consists of constraints normalization and equivalent design space assembling. The work is illustrated by an example of an L-shaped domain optimization with horizontal line support and complex loads. The example takes into consideration fatigue and static stress constraints. The constraints applied separately as well as a parallel showed significant differences in structures topology layouts. The application of a fatigue constraint gave a more conservative results when compared to static stress limitation. The multi-constrained approach allowed effectively lowering the mass of the structure while satisfying both constraints.

Keywords: multi-constrained topology optimization, fatigue constraint, constant criterion surface algorithm

1. Introduction

Topology optimization methods like the homogenization method, SIMP or ESO/BESO can give an answer to the demand for solving large and multi-load problems. However, for multi-constrained design problems, there are only a few propositions in the literature [1]. In this paper, investigation on the multi-constrained structure optimization with the usage of a constant criterion surface (CCS) algorithm is presented.

2. Constant criterion surface algorithm with multi-constraints

The topology optimization algorithm is based on Mattheck's idea of shaping structures in a form of the surface of constant stresses. In the CCS algorithm, the possibilities of adding various constrain criteria were added.

The algorithm consists of a standard procedure of the removal of FE elements with low values of constraint criterion parameters g. The removal procedure is controlled by a ΔFR parameter of volume percentage reduction. To find the ΔFR a minimum criterion bound level Δg_{MIN} is calculated at every iteration.

In a standard way, the algorithm starts from decrease of the material. However, when the structure crosses the bound value of the state parameter (e.g. equivalent stress), it can switches to adding material. The process of material expansion is continued until the parameter g returns to admissible values. By enlarging and decreasing the structure, the algorithm goes to the best topology of the structure. As it was observed, for the definite value of constraint criterion $\bar{g}$, the obtained solutions are characterized by similar layouts.

Let us assume the optimization problem formulated as follows:

$$\min_{\eta} f(\eta) \tag{1}$$

the constraints are: $g(x) \le \bar{g}$;

$\eta(x) = \eta_{\min}$ or 1, $\forall x \in \Omega$;

$$\frac{\int_{\Omega} \eta \, dx}{V_0} \le FR \tag{2}$$

where: $x = [x_1, x_2, \ldots x_N]$ is a vector of finite elements; $\eta = [\eta_1, \eta_2, \ldots \eta_N]$ is a vector of design variables defined as $\eta(x) = E_i / E_0$, E_i and E_0 are respectively, intermediate and real material Young's modules; g is the constraint criterion parameter; $\bar{g}$ is the upper bound of constraint; V_o is the starting volume of the structure limited by design space Ω; $f(\eta)$ is the objective function (the volume of the structure). The N design variables represent a pseudo-density (stiffness parameter) of each finite element of the structures that vary between $\eta_{\min}$ and 1. $\eta_{\min}$ is lower bound of pseudo-density introduced to prevent singularity of the equilibrium problem.

For multi-constrained topology optimization problems following normalized constraints are introduced:

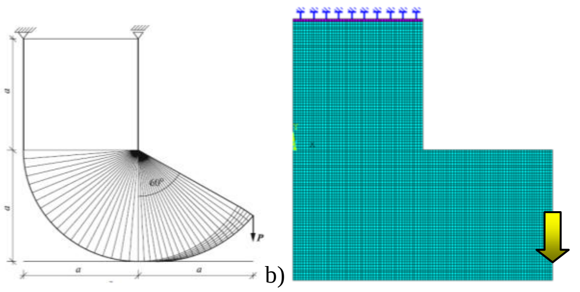
$$g_{nj}(\eta) \le \frac{g_j}{\bar{g}_j}, \; j = [1, 2, \ldots, K] \tag{3}$$

In addition, for every iteration equivalent design space Ω_{eq} is constructed. The equivalent design space for all constraints is calculated according to 'compare and save max' rules. It means that only maximum values among compared g_{nj} vectors are transferred to the resulting equivalent design space.

The normalized constraints of final equivalent design space are used by the constant criterion surface algorithm of topology optimization.

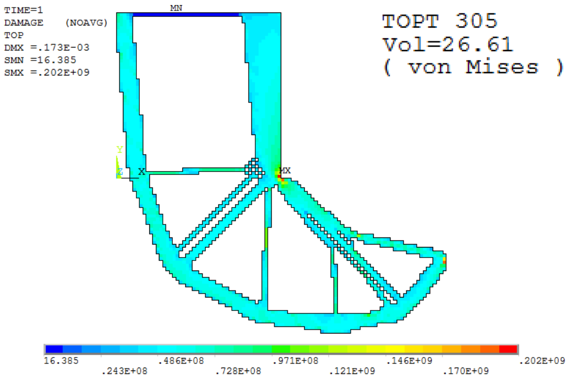
3. Example of optimization

The problem of L-shaped domain optimization with horizontal line support and complex loads was selected as an optimization example. The example is based on the Lewiński-Rozvany analytical benchmarks for topological optimization III [2] (Fig. 1a). The FE model of the example structure with boundary conditions is shown in Fig. 1b, $N = 7500$. A pulsating (R = 0) load was applied for the chosen structure. To test the convergence of the algorithm the two solution paths of the algorithm were used for von Mises constraint. In the Fig. 2a the result for starting from full design space is presented. A very close final result was obtained for a start from minimum design space (reversing) (see Fig. 2b).

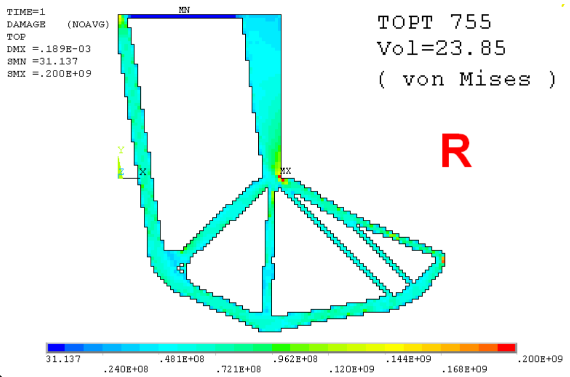


a) b)

Figure 1: Analytical solution [2] (a) and FE model (b) with displacement boundary conditions and pulsating load (R = 0)



a)



b)

Figure 2: The topology optimization results for von Mises constraint $\overline{g_1}$ = 205 MPa; start from max. (a) and min. design spaces (reversing) (b)

For an assumed material model (f_{-1} = 190 MPa, t_{-1} = 114 MPa, σ_f = 205 MPa) the topology optimization results for a single constraint (von Mises or Dang Van) were presented in Fig. 2a and 2b.

The Dang Van criterion is as follows [3]:

$$\max_{A} \left[\tau(t) + \kappa_1 \sigma_H(t)\right] \leq \lambda \tag{4}$$

where: *A* is the area of studied object,

$$\tau(t) = \frac{\sigma_1(t) - \sigma_3(t)}{2}; \quad \sigma_H(t) = \frac{1}{3}\left(\sigma_1(t) + \sigma_2(t) + \sigma_3(t)\right);$$

$\lambda = t_{-1}$; $\kappa_l = 3\, t_{-1}/ f_{-1} - 3/2$ (f_{-1}; t_{-1} are reversed bending and reversed torsion fatigue limits).

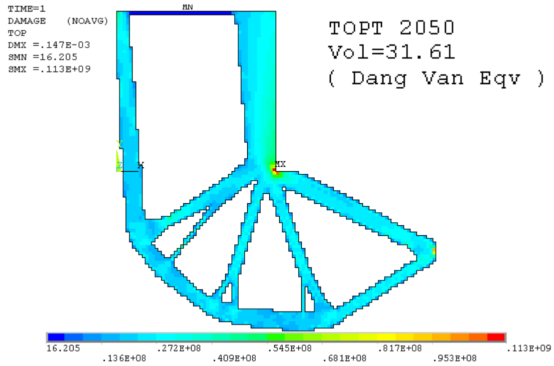


Figure 3: The topology optimization result for Dang Van constraint $\overline{g_2}$ = 114 MPa (c)

For a multi-constrained test, two types of constraints were used, the Dang Van high-cycle fatigue and von Mises equivalent stress criteria (K=2). The topology optimization result for multi-constraints (Dang Van and von Mises) was presented in Fig. 4.

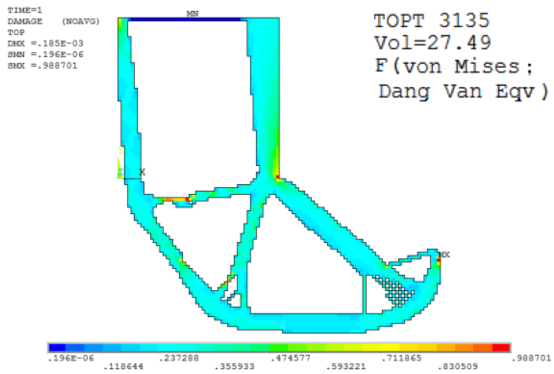


Figure 4: The topology optimization result for multi-constraints (Dang Van and von Mises), $\overline{g_n}$ = 1

4. Conclusions

In this article, the constant criterion surface algorithm with multi-constraints was presented. As illustrated in the example, the application of normalized constraints and equivalent design space to topology optimization gives a possibility to solve the optimization problem with fatigue and static stress constraints. In optimization investigation different layouts were obtained for single and multi-constrains optimization. The multi-constrained methodology can be easily applied to complex industrial-size problems.

References

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