Numerical study of tangential contact behaviour of spherical particle

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Abstract

Numerical study of the tangential contact behaviour of the elastic frictional spherical particle is presented and the influence of numerical implementation procedures was studied. Tangential contact is assumed to follow elastic spring model. The elasticity properties are characterised by the tangential stiffness which presents in general case the time-history dependent parameter. A modified prediction of the tangential force-displacement behaviour by reaching Coulomb limit is proposed. The oblique sphere-plane interaction is considered for illustration purposes. It was found that straightforward application of routine algorithmic approach may lead to artificial jumps in stick-slip transition points. On the contrary, the suggested prediction provides physically adjustable results. Moreover, evaluation of the tangential direction may also significantly contribute to numerical solutions. Investigation of particle's tangential contacts is continued by considering multi-particle systems.

Keywords: discrete element method, tangential force calculation, switching between stick and sliding

1. Introduction

The Discrete Element Method (DEM) became recently conventional numerical technique extensively applied for simulation of particulate solids. The motion of each individual particle is described by solving equations of motion. Therewith, evaluation of contact forces is significant procedure of the DEM computations, where the force is considered to be divided into two perpendicular normal and tangential components. Generally, the normal contact is well studied while the tangential behaviour, in opposite, is treated controversially. It is commonly agreed, that direct derivation of the tangential force-displacement relationship is not possible.

The review of friction models may be found in [1] while DEM related aspects were comprehensively discussed by Di Renzo and Di Maio [2] and continued by Kruggel-Emden et al. [3,4]. Unfortunately, discussion on the issue of detailed calculation of tangential forces is rather limited [5].

The focus of this work is investigation of the computational details of the tangential contact behaviour. Performance of modified prediction the tangential force and displacement behaviour is illustrated by considering the oblique sphere-plane impact. Occurrence of artificial jumps during the stick-slip transition will be demonstrated.

2. Particles interaction models

The non-cohesive elastic frictional spherical particle and boundary plane are considered in a frame of conventional DEM methodology. Damping and rolling friction is neglected. The normal interaction is defined in terms of normal contact force $\mathbf{F}_n$ and displacement (particles overlap) $\mathbf{h}_n$. It is calculated according to Hertz contact law and remains independent on tangential interaction. Elasticity properties are characterised by normal stiffness k_n. Additionally, arbitrary time variation of normal force $\mathbf{F}_n(t)$ is allowed.

The tangential behaviour is defined in terms of contact force $\mathbf{F}_t$ and displacement $\boldsymbol{\delta}$. Elasticity properties are characterised by tangential time-history dependent stiffness $k_t(\mathbf{F}_n)$.

Tangential force $\mathbf{F}_t$ is bounded by the Coulomb limit. For calculation purposes the following equation can be taken

$$\mathbf{F}_t = \mu \|\mathbf{F}_n\| \mathbf{t} \quad (1)$$

where μ is coefficient of sliding friction while $\mathbf{t}$ is direction vector. Numerically evaluated values of $\mathbf{t}$ are sensitive to contact data, especially to direction changes. Hereby, using of forces, velocities or displacements to evaluate tangential direction yield to no unique results.

3. Numerical calculation of tangential particle behaviour

Interaction of particles is time-dependent process. It is tracked by applying incremental integration procedure. Assuming particles state at time instant t_n to be known, integration deals with prediction of the new state in time $t_{n+1} = t_n + \Delta t$ after increment Δt.

Conventionally employed integration methods are aimed to deal with smooth functions and concerns basically kinematical variables such as particles position and velocities. Our focus is prediction of secondary tangential components $\mathbf{F}_t$ and $\boldsymbol{\delta}$ at time instant t_{n+1}. Generally, this force-displacement relationship could be expressed as follows

$$\mathbf{F}(t_{n+1}) = \mathbf{F}(t_n) - \int_{t_n}^{t_{n+1}} k_t(\tau) \frac{d\boldsymbol{\delta}(\tau)}{d\tau} d\tau \quad (2)$$

To simplify integration the constant tangential stiffness k_t is assumed during increment. Consequently, Eq. (2) is usually replaced by

$$\mathbf{F}(t_{n+1}) = \mathbf{F}(t_n) + k_t \left(\boldsymbol{\delta}(t_{n+1}) - \boldsymbol{\delta}(t_n)\right) \quad (3)$$

Since during sliding tangential force $\mathbf{F}_t(t_{n+1})$ is defined according to Coulomb limit (1), Eq. (3) could be resolved with respect to final displacement and modified force compatible expression is suggested

$$\boldsymbol{\delta}(t_{n+1}) = \frac{-\mu\left\|\mathbf{F}_n\left(t_{n+1}\right)\right\|\dfrac{\mathbf{F}_t\left(t_{n+1}\right)}{\left\|\mathbf{F}_t\left(t_{n+1}\right)\right\|} + k_t\left(\xi\right)\boldsymbol{\delta}(t_n) + \mathbf{F}_t\left(t_n\right)}{k_t\left(\xi\right)} \tag{4}$$

Where $\xi \in [t_{n+1},\ t_{n+2}]$. Here, a force-based evaluation of the direction vector $\mathbf{t} = \mathbf{F}_t / \|\mathbf{F}_t\|$ [5] is applied. The tangential stiffness k_t can be approximated by rectangular rule $k_t(\xi) = k_t(t_{n+1})$. Conventionally, see for example [3], when Coulomb limit is reached displacement is considered as elongation of tangential spring calculate by the formula

$$\left\|\boldsymbol{\delta}\left(t_{n+1}\right)\right\| = \mu\left\|\mathbf{F}_n\left(t_{n+1}\right)\right\| / k_t\left(t_{n+1}\right) \tag{5}$$

It is evident that $\boldsymbol{\delta}$ norms according to (4) and (5) are different.

4. Numerical results and discussion

The oblique impact of the spherical particle bouncing on the plane was investigated by applying elaborated approximation (4). By specification of particle's parameters and initial conditions various interactions may be observed. A case of low attack angle $\alpha = 1.5^{\circ}$ will be shown below.

Characteristic simulation results in terms of time histories of the tangential displacement $\boldsymbol{\delta}$ and tangential force $\mathbf{F}_t$ are illustrated in Fig. 1 and Fig. 2, respectively. The Illustration was limited by showing four contacts. The characteristic time instants indicating switch of particle contacts regimes are denoted by points ranging between *a* and *m*. Three particle-plane interaction regimes, namely slip, elastic stick with loading and reloading and reversed stick occur during contact before rebound. Zero values indicate contact-free motion.

Let us focus on first contact. The segment between *a* and *b* points illustrate sliding behaviour, between points *b* and *c* illustrate stick behaviour, between *c* and *d* illustrate again sliding behaviour.

Variation of displacement (Fig. 1) is characterised by stick-slip transition in point *b* and rebound at point *c*. The higher values and continuity in sticking-sliding transition point were detected by applying the modified history dependent technique (curve 1).

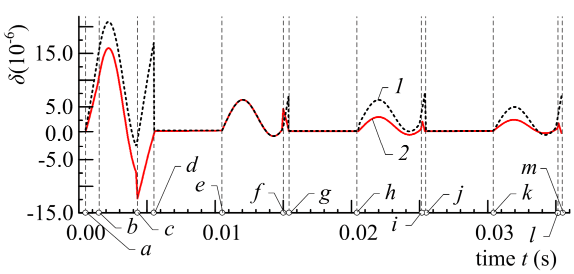

Figure 1: Variation of tangential displacement in time:
1 −modified approach according to (4);
2 − conventional approach according to (5)

The artificial jump in transition point (curve 2) is obtained by applying the conventional methodology. It could be noticed that the obtained jump magnitude is not sensitive to integration step length. The other contacts exhibit the same tendencies in transition points.

Variation of the tangential force obtained on the basis of (4) (Curve 1, Figure 2) demonstrates continuous switch in transition points and seems to be physically correct. Additionally, other computation possibility to evaluate tangential direction was studied i.e. $\mathbf{t} = \boldsymbol{\delta} / \|\boldsymbol{\delta}\|$. The non-physical jump occurs in transition points *f* and *i* (Curve 2, Figure 2) in tangential force diagram when tangential direction was taken on the basis of tangential spring elongation.

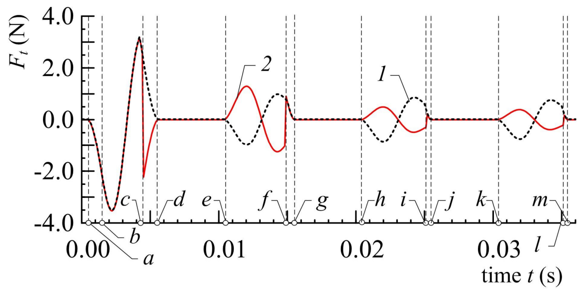

Figure 2: Variation of the tangential force in time for variously calculated direction vectors $\mathbf{t}$: 1 - $\mathbf{t} = \mathbf{F}_t / \|\mathbf{F}_t\|$; 2 - $\mathbf{t} = \boldsymbol{\delta} / \|\boldsymbol{\delta}\|$.

5. Conclusions

The tangential contact behaviour of spherical particle was considered and influence of numerical implementation procedures was studied. It is found that straightforward application of the routine algorithmic approach may yield artificial jumps in the turn off points. In contrary, the suggested prediction of dual force-displacement (shear slip) provides physically adjustable results. Independently of the above, considering of tangential direction is important.

It could be stated, however, that the numerical instabilities are time history-dependent and sensitive to the particle properties. Their contribution to behaviour of multi-particle systems will be demonstrated separately.

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