Joint reactions in rigid or flexible body mechanisms with redundant constraints

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Abstract

It is known that when redundant constraints exist in a rigid body mechanism, some or all joint reaction forces cannot be uniquely determined. Methods of finding joints for which reactions are unique are available. However, to find the unique reaction solution for all joints it is necessary to abandon the assumption that all bodies are rigid. In most cases, after taking into account flexibility of selected bodies, redundant constraints are no longer observed. The presented study shows that it is not obvious which parts of the investigated mechanism should be modelled as flexible bodies to obtain the unique reaction solution. The most important point is that not always taking into account flexibility of selected mechanism bodies guarantees that calculated reactions are the unique, "real" ones. The results obtained for a partially flexible model, which is not overconstrained, can be misleading or completely erroneous.

Keywords: multibody dynamics, flexible bodies, redundant constraints, joint reactions

1. Introduction

The problem of joint reactions indeterminacy is – in engineering simulations – most often caused by redundant constraints which are defined as constraints that can be removed without changing the kinematics of the system. By their nature, the redundant constraints cannot be uniquely selected.

It can be proven that in the case of an overconstrained rigid body mechanism, despite the fact that all constraint reactions cannot be uniquely determined, selected single constraint reactions or selected groups of reactions can be specified uniquely [1]. In our previously published works [1, 2] an algebraic criterion, allowing for detection of these joints for which reaction forces can be uniquely determined, was formulated. This criterion was followed by three numerical methods of finding such joints. It can be shown that these methods can also be used to check whether the simulated motion is unique (in terms of positions, velocities and accelerations of bodies) in the case of a redundantly constrained rigid body mechanism with Coulomb friction in joints [2].

The methods presented in [1, 2] can be helpful when analyzing selected joint reactions. In order to find a unique set of all joint reactions in an overconstrained system, it is necessary to reject the assumption that all bodies are rigid. Flexible bodies introduce additional degrees of freedom to the mechanism, which usually makes the constraint equations independent. Quite often only selected bodies are modelled as the flexible ones, whereas the other remain rigid.

In this paper we show that taking into account flexibility of selected mechanism bodies does not guarantee that unique joint reactions can be found. For the sake of simplicity, a case study of an overconstrained spatial mechanism is presented.

2. Joint Reactions in Rigid or Flexible Body Mechanisms

A spatial parallelogram mechanism (Fig. 1), consisting of the basis 0 and seven moving bodies 1–7, is discussed in this paper. The bodies are connected by 12 spherical joints. The mechanism has seven degrees of freedom. The platform 7 can perform a translational motion (with constant orientation), moreover, links 1–6 can rotate along their longitudinal axes. The theoretical mobility calculated using the Grübler-Kutzbach equation is 6, not 7, thus redundant constraints are present in the mechanism.

Since the studied mechanism is overconstrained, some of the joint reaction forces cannot be uniquely determined using a rigid body model. The problem of this mechanism joint reactions solvability was investigated in [1]. It was found that the reactions in joints *K*, *L*, *M* and *N* can be determined uniquely. The reactions in the remaining joints cannot be determined uniquely.

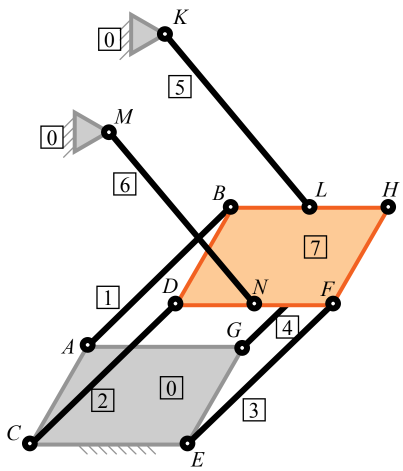

Figure 1: A spatial parallelogram mechanism

Firstly, a simulation in which all moving bodies were modelled as flexible was performed. The reactions calculated during this simulation can be regarded as the "true" ones, since the multibody system was modelled in its full complexity, and no arbitrary decisions on where to introduce flexibility were made. The exemplary results are shown in Fig. 2.

Next, different variants of the mechanism were studied. In the first one all links were rigid, in the other variants selected links were modelled as flexible bodies. Various placements of flexible links in the mechanism structure were analysed. Motion simulation for each variant was performed to observe the joint reaction forces. The platform load and initial conditions were the same for all simulations.

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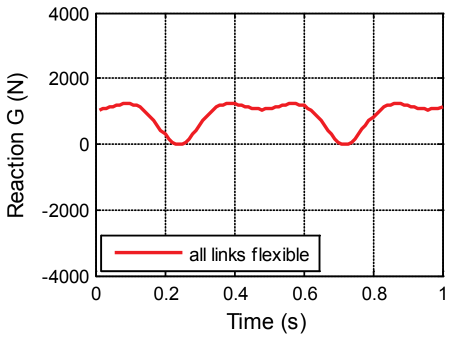

Figure 2: Reaction force in joint G (fully flexible model)

In the case of rigid body approach, redundant constraints were eliminated from the mathematical model of the mechanism. Since redundant constraint equations cannot be selected uniquely, several possible variants of elimination were studied. It was found that only selected reaction forces (in joints *K*, *L*, *M* and *N*) were not affected by the choice of eliminated redundant constraints. Thus, quantitative results are in accordance with qualitative information about reactions solvability.

In the next series of simulations the upper links, i.e. *KL* and/or *MN*, were modelled as flexible bodies. It was found that, despite introducing some elasticity effects, redundant constraints were still present in the partially flexible mechanism. As in the case of the rigid body model, redundant constraints were eliminated, and several possible variants of elimination were examined. The simulations showed that taking into account flexibility of the upper links changes practically nothing when joint reactions are concerned. Negligibly small differences between the joint reactions calculated using the rigid and the partially flexible model are observed, since the models are similar but not equivalent to each other. Again, reactions in joints *K*, *L*, *M* and *N* are not affected by the choice of eliminated constraints, whereas the other reactions depend on this choice.

During the third series of simulations one arbitrarily selected lower link was modelled as a flexible body; all four possibilities were tested. This time no redundant constraints were detected, thus – at least formally – the mechanism was not overconstrained anymore. Detecting no redundant constraints may be a bit misleading when joint reactions uniqueness is considered. Since three lower links are rigid, the fourth, flexible link cannot change its length, and thus is unable to carry axial load. Hence, in this case introduction of flexibility may be regarded as being equivalent to (non unique!) redundant constraints elimination. The simulations showed that reactions in joints *K*, *L*, *M* and *N* are not affected by the choice of the link treated as a flexible body (moreover, these reactions coincide with those obtained using the rigid body model), whereas the other joint reactions depend on this choice. The exemplary results are presented in Fig. 3a.

Two out of four lower links were modelled as flexible bodies during the next series of simulations; all possible combinations were analysed. This time the flexible bodies were able to carry axial loads. As in the previous cases, it was found that arbitrary choice of flexible links placement in the mechanism structure affects the obtained joint reaction forces. Only reactions in joints *K*, *L*, *M* and *N* were not affected by this choice and were the same as in the previous simulations. Exemplary results are presented in Fig. 3b. The simulations were repeated for model with two flexible lower links and one or two flexible upper links. It was found that for the obtained reactions it was practically unimportant whether the upper links are rigid or flexible, since only negligible differences were observed.

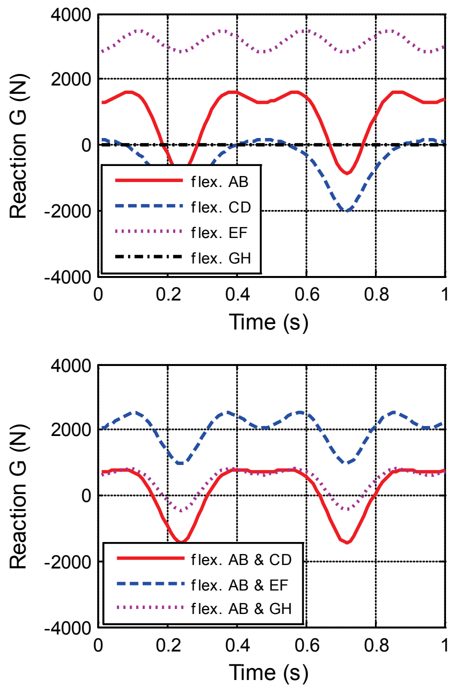

Figure 3: Joint reaction forces in a partially flexible mechanism

3. Conclusions

When redundant constraints exist in a multibody system, it is necessary to abandon the assumption that all bodies are rigid in order to find a unique set of all joint reaction forces. The presented study shows that it is not obvious which parts of the investigated mechanism should be modelled as flexible bodies to obtain the unique reaction solution. The most important point is that not always taking into account flexibility of selected mechanism bodies guarantees that calculated reactions are the unique, "real" ones. The results obtained for a partially flexible model, which is formally not overconstrained, can be misleading or completely erroneous. In some situations the system remains overconstrained, despite introduction of flexibility of some bodies. In other cases of partially flexible systems, formally there are no redundant constraints, however, the obtained reaction solution depends on the choice of parts modelled as flexible bodies. The simulations also show that, if all reactions acting on a selected part can be uniquely determined using a rigid body model, there is no point in taking into account flexibility of this part when solving for joint reactions.

References

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