Topology optimization of large-scale trusses using ground structure approach with selective subsets of active bars

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Abstract

The paper presents the new method of finding the optimal topology of large-scale Michell trusses. The method is based on the ground structure approach with selective subsets of active bars. The solution is obtained in iterative way by sequential adding or removing successive bars. Such a strategy significantly reduces as well the memory as the processor time and hence enables effective optimization of very dense ground structures with large numbers of potential bars (the order of hundreds million). The results obtained indicate that the method is very promising due to its high accuracy and robustness. Despite the iterative approach the method presented in the paper uses a linear programming formulation of the optimization problem and assures finding the global optimum, hence it may be considered as the useful tool for finding new, yet unknown results in Michell truss theory.

Keywords: topology optimization, Michell trusses, minimum weight design, interior point method

1. Introduction

The first computerized discrete topology optimization methods for trusses was developed over four decades ago by Dorn et al. [2], who proposed the solution strategy based on the ground structure and the linear programming formulation of the optimization problem. Their method was soon extended to more complicated problems: multiple-load cases, self-weight, joint costs, etc. (see [5,8]).

On one hand, the traditional ground structure approach with fixed nodes is very attractive because the optimization problem may be formulated and solved using the linear programming (LP) methods. Contrary to other approaches (non-linear or non-convex formulation), it assures finding the global optima. On the other hand, the ground structure has to be sufficiently dense to obtain satisfactory results. This leads to large-scale optimization problems which require subtle and powerful linear programming methods. For example the fully connected ground structure defined in a square region 1x1 with the mesh 0.01 has $101^2 = 10\,201$ nodes and $101^2(101^2 - 1)/2 = 52\,025\,100$ potential bars. These numbers are clearly unmanageably large for any direct solution method.

The program developed by the author [9] takes advantage of the fast interior point method [6,11,12] with sparse matrix representation and utilizes the high regularity of the ground structure. Its efficiency and robustness have been confirmed in many numerical tests [9,10]. Nevertheless, it uses the direct solution strategy hence the largest problem, which can be solved in a reasonable time, may contain only a few millions of bars (see [9] for details). It is clear that most of potential bars defined in a ground structure are not necessary for carrying applied loads and they disappear in the final optimum solution (they have zero cross section areas). Thus it is obvious that the large amount of memory and processor time can be preserved if one can eliminate this large number of unnecessary bars. Unfortunately, such a prediction cannot be done in advance because the optimum solution is unknown. Gilbert and Tyas [3] developed the iterative member adding method to overcome this issue. The method proposed in the present paper is close in spirit to the method given in [3] but uses a different member adding strategy.

2. Classical plastic topology optimization of trusses

According to well known duality principles, the plastic design optimization of trusses can be written in two equivalent formulations given bellow. Both of them play important role in the method developed in this paper. Other interesting equivalences in truss topology optimization are given in [1].

2.1. Primal (lower bound) formulation

The primal formulation of the truss plastic design topology optimization problem, for different stress limits for tension σ_T and compression σ_C, and for single load case, can be defined as follows [5]:

$$\begin{aligned} &\min_{\mathbf{T},\mathbf{C}\in R^M} V = \frac{\mathbf{L}^T\mathbf{T}}{\sigma_T} + \frac{\mathbf{L}^T\mathbf{C}}{\sigma_C} \\ s.t.\quad & \mathbf{B}^T(\mathbf{T}-\mathbf{C}) = \mathbf{P} \\ & \mathbf{T}\geq \mathbf{0},\quad \mathbf{C}\geq\mathbf{0} \end{aligned} \tag{1}$$

Here the objective function V is the total volume of the fully-stressed truss, **L** represents the vector of bar lengths, **T** and **C** are the vectors of tension and compression axial forces in bars, **P** is the vector of nodal loads (external loading) and **B** is the geometric matrix including directional cosines of bars. Note that in this formulation the components of both **T** and **C** vectors are the design variables. For truss composed of M members it involves $2M$ design variables with N equality constrains, where N denotes the number of degrees of freedom. Problem (1) is often called the lower bound formulation, because every feasible solution (any truss satisfying the equilibrium equations $(1)_2$) has the volume greater then or equal to the optimum one.

2.2. Dual (upper bound) formulation

The dual (upper bound) formulation for the same problem is given by [5]

$$\begin{aligned} &\max_{\mathbf{u}\in R^N} W = \mathbf{P}^T\mathbf{u} \\ s.t.\quad & \mathbf{B}\,\mathbf{u} \leq \mathbf{L}/\sigma_T \\ & \mathbf{B}\,\mathbf{u} \geq -\mathbf{L}/\sigma_C \end{aligned} \tag{2}$$

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where **u** represents the vector of virtual nodal displacements (corresponding to Lagrange multipliers) and W is the virtual work of the given forces **P** over the displacements **u**. The latter problem involves only N design variables (for dense ground structure N is many times smaller than M) but subjected to $2M$ inequality constrains. Therefore, without resorting to the specialized LP methods (like active set method, see [1]), this problem is computationally slightly harder than problem (1).

It is worth to note here that the inequality constrains given in (2) correspond to Michell's optimality criteria [7] which states that in an optimal truss the magnitude of the virtual member strains are restricted to:

$$-1/\sigma_C \le \varepsilon_i \le 1/\sigma_T \quad \text{for} \quad i = 1, 2, \ldots, M. \tag{3}$$

The constraints of problem (2) are equivalent to Michell's constraints and hence they are sufficient to obtain an optimal truss.

3. The method of selective subsets of active bars

The main idea of the method developed consists in the iterative improvements of the successive solutions. The initial ground structure includes only the shortest bars connecting the neighbouring nodes (see Fig. 1a). In the next iterations longer and longer bars are considered as the candidates for addition (see Fig. 1 b-d) and at the same time the old unnecessary bars are eliminated from the structure. The Michell's constrains serve as the criterion for adding or removing bars. Due to limited space of this abstract, the new method will be presented in detail in the full paper version of this work.

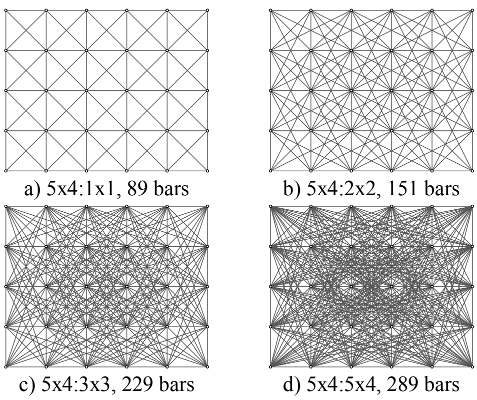

Figure 1: The exemplary successive ground structures

4. Example

Let us consider one of the benchmark problems given in [4] (see Fig. 2, 3). This problem was solved using highly dense mesh 140x280 with 477 190 280 of potential bars. The value of the optimal "numerical" volume is equal to 13.83525 which is only 0.027% greater than the exact volume. The total CPU time required for solving this problem is only 3.5 hours. This proves the extremely high efficiency of the new method.

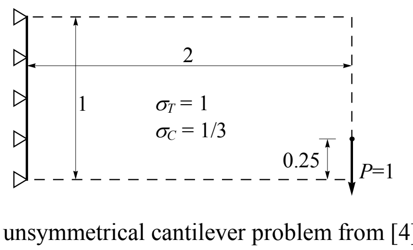

Figure 2: The unsymmetrical cantilever problem from [4]

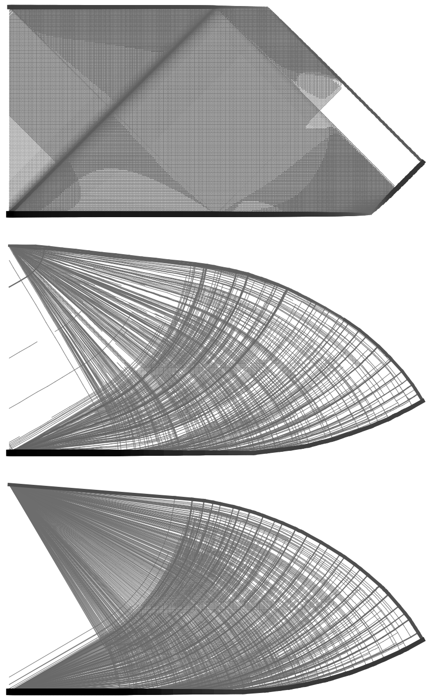

Figure 3: Numerical layouts for iterations: 1, 5 and 11

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