Modeling of the beams with taper cross section using absolute nodal coordinate formulation

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Abstract

This paper presents methods of analyzing the spatial, continuum ANCF beam element with taper cross section. The change of the cross section size alongside was achieved by appropriate integration of the element matrices. Two possible solutions of this problem were analyzed. Numerical examples shows that both models give appropriate results. It was also demonstrated that second ANCF model, in which shape functions include the thicknesses of the cross section, gives better performance then the first model. Negative influence of the locking phenomena on the results was noticed.

Keywords: beams, finite element methods, large deformations, multibody dynamics, numerical analysis

1. Introduction

In the last several years, a new approach for static and dynamic analysis of the flexible multibody systems, called absolute nodal coordinate formulation (ANCF) [6] is intensively developed. This non-incremental formulation was proposed for efficient analysis of the highly flexible structures composed of beam and plate elements. A characteristic feature of the ANCF elements is the lack of rotational parameters in nodal coordinates. Instead of them to describe rotations slopes are used.

Many authors investigated wide range of different types of the beam applications, but it is difficult to find examples in which beam cross section change smoothly its size alongside. Therefore method for modeling spatial beam elements with linearly taper cross section is presented in this paper. Desired effect was achieved by appropriate integration of the element matrices. This method was applied to classical spatial, fully parametrized ANCF beam element [7] which is based on continuum mechanics approach. This element has twenty four nodal parameters and its displacements along beam axis are of the third order.

Presented method was used in several static and dynamic large displacement examples. Also modal analysis of the simply supported beam was performed. Results of the numerical tests were compared with results obtained with commercial, general purpose FEA package [4]. Good agreement was achieved between ANCF and FEA results.

2. Formulation

Matrix with shape functions for original ANCF beam element is given in literature [7] in the form, which do not contain information about the size of the cross section. One can write this matrix as follows:

$$\boldsymbol{S} = [s_1\boldsymbol{I}\ s_2\boldsymbol{I}\ s_3\boldsymbol{I}\ s_4\boldsymbol{I}\ s_5\boldsymbol{I}\ s_6\boldsymbol{I}\ s_7\boldsymbol{I}\ s_8\boldsymbol{I}] \tag{1}$$

where $\boldsymbol{I}$ is the 3 by 3 identity matrix and the polynomials:

$$\begin{array}{ll} s_1 = 1-3\xi_1^2+2\xi_1^3, & s_2 = l(\xi_1-2\xi_1^2+\xi_1^3), \\ s_3 = l(\eta_1-\xi_1\eta_1), & s_4 = l(\zeta_1-\xi_1\zeta_1), \\ s_5 = 3\xi_1^2-2\xi_1^3, & s_6 = l(-\xi_1^2+\xi_1^3), \\ s_7 = l\xi_1\eta_1, & s_8 = l\xi_1\zeta_1 \end{array} \tag{2}$$

where $\xi_1 = x/l$, $\eta_1 = y/l$ and $\zeta_1 = z/l$ are natural coordinates of the element and l is element length.

Since information about thickness of the element is not presented in Eqn (2), one can express the linear change of the cross section size by variable integration limits:

$$\int_V h(\xi_1,\eta_1,\zeta_1)dV = l^3\int_0^1\int_{-a_l}^{a_l}\int_{-b_l}^{b_l} h(\xi_1,\eta_1,\zeta_1)d\xi_1 d\eta_1 d\zeta_1 \tag{3}$$

where h is given function of the natural coordinates, $a_l = a_l(\xi_1)$ and $b_l = b_l(\xi_1)$ are the following integration limits:

$$\begin{aligned} a_l(\xi_1) &= \frac{a(\xi_1)}{2l} = \frac{(1-\xi_1)a_0+\xi_1 a_1}{2l} \\ b_l(\xi_1) &= \frac{b(\xi_1)}{2l} = \frac{(1-\xi_1)b_0+\xi_1 b_1}{2l} \end{aligned} \tag{4}$$

where $a(\xi_1)$ and $b(\xi_1)$ are variable element thicknesses along axises y and z, a_0 and b_0 as well as a_1 and b_1 are constant thicknesses of the cross sections at the beginning and the end of the element.

The problem of the taper cross section can be formulated differently, by changing the definition of the element natural coordinates. When $\xi_2 = x/l$, $\eta_2 = y/a$ and $\zeta_2 = z/b$ are used as natural coordinates the shape functions contain information about the thickness of the cross section:

$$\begin{array}{ll} s_1 = 1-3\xi_2^2+2\xi_2^3, & s_2 = l(\xi_2-2\xi_2^2+\xi_2^3), \\ s_3 = a_0(\eta_2-\xi_2\eta_2), & s_4 = b_0(\zeta_2-\xi_2\zeta_2), \\ s_5 = 3\xi_2^2-2\xi_2^3, & s_6 = l(-\xi_2^2+\xi_2^3), \\ s_7 = a_1\xi_2\eta_2, & s_8 = b_1\xi_2\zeta_2 \end{array} \tag{5}$$

and integral given by Eqn (3) takes the form:

$$\int_V h(\xi_2,\eta_2,\zeta_2)dV = l\int_0^1 a(\xi_2)b(\xi_2)\int_{-0.5}^{0.5}\int_{-0.5}^{0.5} h(\xi_2,\eta_2,\zeta_2)d\xi_2 d\eta_2 d\zeta_2 \tag{6}$$

where $a(\xi_2)$ and $b(\xi_2)$ are linearly varying thicknesses of the element.

In the subsequent part of this paper the way of modeling elements with taper cross section given by Eqn (3) is called the first (I) model, while that given by Eqn (6) is called the second (II) model.

Table 1: Comparison of the frequencies

Mode	BEAM188 1el	ANCF I 1el	ANCF II 1el	BEAM188 10el	ANCF I 10el	ANCF II 10el
1st bending	0.4166	0.7425	0.5556	0.4106	0.4796	0.4773
2nd bending	1.5631	32.412	32.411	1.8323	2.1654	2.1559
torsion	2.5800	6.6619	4.3672	2.5347	2.7872	2.7631
3rd bending	-	-	-	4.0699	4.9098	4.8877
4th bending	-	-	-	7.1591	8.8814	8.8414
longitudinal	8.1299	8.2118	8.2119	8.1148	8.1149	8.1150

3. Implementation

The algorithm is entirely implemented in commercial numerical computing environment [3]. In the examples of dynamic analysis, to solve the ordinary differential equations, Newmark method [1] with constant integration step size was used.

For numerical integration of integrals given by Eqn (3) and (6) Gauss-Legendre quadrature [1] was used. Like for the original ANCF beam element the number of used quadrature points was carried out with four integration points in the x direction and two integration points in the y and z directions.

In the case of integration of Eqn (3) for every integration point in beam axis direction ξ_1 the internal integrals are evaluated with fixed limits a_l and b_l which are computed from Eqn (4). Similarly in the case of Eqn (6) for every ξ_2 the current values of the element thicknesses a and b are computed. It is important to point out that every beam element in the body have got variable thickness.

4. Numerical examples

To verify presented formulation several analysis of simple structures are provided. The obtained results were compared with the results of the simulations performed with commercial FEA package [4] with the use of the BEAM188 element.

In the first example modal analysis of simply supported beam is analyzed. The beam is 1m long and has a square taper cross section with side length of 10mm at the beginning and 30mm at the end. Results of this analysis is showed in Table 1. BEAM188 element was used in its cubic formulation.

In the case when single element was used in the analysis, significant differences can be noticed between two ANCF models. Second model shows better agreement with FEA results. The great difference in the second bending mode between ANCF and FEA beams is caused by the shear locking in continuum ANCF element [5]. When ten elements were considered, both ways of modeling taper cross section show good agreement with FEA. Because second model provides more accurate results in the analysis with single element one can expect that second model will have a better performance.

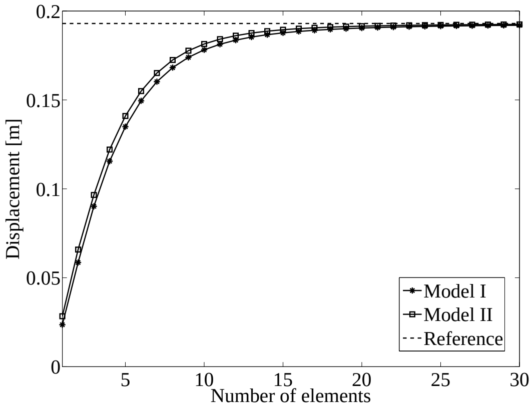

Figure 1: Covergence test

To investigate convergence of both ANCF models static bending of the beam was analyzed. Beam with taper cross section, used in the previous example, was clamped at one end and loaded with bending moment at free end. Poisson ratio was set equal to zero to avoid the influence of the Poisson locking phenomena. In Fig. 1 vertical displacements of the beam end are presented as the function of the number of elements. As it was expected second model has better convergence then the first one but differences are not very significant. Both ANCF models converge to the reference solution given by FEA with thirty elements.

Dynamic analysis was also performed. The displacements of the physical pendulum under gravity forces was considered. Results was compared with FEA pendulum and good agreement with both ANCF models were achieved.

5. Conclusions

The continuum ANCF beam element allows to model beams with linearly varying size of the cross section. This may be performed by changing the integration schema for the matrices in the equations of motion. Two possible solutions of this problem were presented. This paper demonstrates that second ANCF model, in which shape functions include the thicknesses of the cross section, gives better performance then the first model.

As it can be seen in Fig. 1 good agreement with reference solution is achieved when approximately twenty ANCF elements are used. In this example of the static analysis the third order BEAM188 finite element achieves sufficient accuracy with only four elements. The weak performance of the ANCF element most likely results from the appearance of the locking phenomena, which is reported in literature [2]. However the proposed method of modeling allows to include methods used to eliminate locking.

References

[1] Bathe, K.J. *Finite Element Procedures*. Prentice Hall, New Jersay, 1996.

[2] Gerstmayr, J. and Shabana, A.A. Efficient integration of the elastic forces and thin three-dimensional beam elements in the absolute nodal coordinate formulation. In *Multibody Dynamics 2005 ECCOMAS Thematic Conference*, Madrit, Spain, 2005.

[3] *MATLAB User's Guide*, Release R2010b. The MathWorks, Inc., 2010.

[4] *ANSYS Mechanical APDL Documentation*, Release 12.1. SAS IP, Inc., 2009.

[5] Schwab, A.L. and Meijaard, J.P., Comparison of three-dimensional flexible beam elements for dynamic analysis: Classical finite element formulation and absolute nodal coordinate formulation. *Journal of Computational and Nonlinear Dynamics*, 5, pp. 011010, 2010.

[6] Shabana, A.A., Definition of the slopes and the finite element absolute nodal coordinate formulation. *Multibody System Dynamics*, 1, pp. 339–348, 1997.

[7] Shabana, A.A. and Yakoub, R.Y., Three dimensional absolute nodal coordinate formulation for beam elements: Part I and II. *Journal of Mechanical Design*, 123, pp. 606–621, 2001.