Optimal design of shells of uniform stability by variational and hybrid PSO - spline methods

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Abstract

The problem of optimal design of rotationally symmetrical shells of uniform stability was discussed in the current paper. There were used two different methods of solving the optimization problem: variational and numerical one i.e. combining the description of the shape created of meridional curve by means of spline in FEM model along with an algorithm of PSO. The results obtained from both approaches were compared. The second approach allows solving significantly more complex shells, e.g. equipped with circular hole in the bottom, made out of composites, etc. The results obtained confirm, that second approach gives results similar to those obtained on the analytical way.

Keywords: optimization, evolutionary methods, shape design, shells, stability

1. Introduction

Thin-walled shells under external pressures can be subjected to loss of stability and then corresponding constraints should be taken in to account in their structural optimization.

Instability of shells has very often a local form and buckling does not depend essentially on the boundary conditions. This is particularly true in a case of non-uniform stress distribution and a non uniform geometry of the shell (variable curvature, variable thickness).

Then, instability can be determined by the stress state and the geometry of a shell at individual points and the buckling is initiated in the weakest point (zone) of a structure, called the "dangerous point".

For a shell with a double positive curvature Shirshov (Ref. [4]) transformed the problem of global stability to a simpler problem of local stability of such a structure. Using the linear theory of shell stability, applying the equations given by Wlassow and assuming sinusoidal deflection mode, Shirshov obtained rather simple formula for the critical loading parameter q namely:

$$q_{1,2} = 2\sqrt{DEH}\frac{K_\varphi + K_\theta z_{1,2}^2}{\overline{N}_\theta + 2\overline{S}z_{1,2} + \overline{N}_\varphi z_{1,2}^2}, \qquad q_{kr} = min(q_1, q_2) \quad (1)$$

where $z_{1,2}$ are roots of the equation

$$z^2 + \frac{K_\theta \overline{N}_\varphi - K_\varphi \overline{N}_\theta}{K_\theta \overline{S}} z - \frac{K_\varphi}{K_\theta} = 0 \quad (2)$$

$$z = tg\,\phi \quad (3)$$

and ϕ is a certain free parameter with respect to which the loading parameter q should be minimized. Moreover, in the equations above following symbols were introduced: K_θ and K_φ denoting respectively circumferential and meridional curvature, $\overline{N}_\theta$, $\overline{N}_\varphi$ i $\overline{S}$ denoting the membrane resultant stresses related to the loading multiplier q, D – shell stiffness, E – Young modulus, H – thickness of the wall. The critical value of loadings is determined by one of (1), whichever leads to a smaller value. The mentioned problems were intensively discussed in the Ref. [1].

2. Shells of uniform stability

The considered shell features following geometrical parameters: $R_0/L_0 = 0.25$, $H_0/R_0 = 0.005$, where R_0 stands for the radius of an reference cylindrical shell, L_0 means the half of its length and H_0 - thickness of its wall.

The subject of our investigation is the shell described above loaded with external pressure exposed to the loss of stability. The discussed structure is described by double non-negative Gauss curvature.

Thin-walled shells under complex loadings (hydrostatic pressure, compressive force, twisting moment) can be subjected to loss of stability and then corresponding constraints should be taken into account in their structural optimization. Instability of shells has very often a local form and buckling does not depend essentially on the boundary conditions. This is particularly true in a case of non-uniform stress distribution and in a case of a "non-uniform" geometry of a shell (variable curvature, variable thickness).

The similar problem without influence of a hole in a bottom was analysed in the Ref. [3].

With given parabolic shape of the middle surface we look for such distribution of a shell wall thickness, which leads to the maximal critical loading parameter

$$q_{kr} \Rightarrow max \quad (4)$$

Such an optimization problem is stated under a few constraints. Let's introduce the reference cylindrical shell of constant wall thickness H_0, constant radius R_0 and length of $2L_0$. It is assumed that an optimal shell has the same volume of material (weight) as a cylindrical reference shell,

$$2\pi L_0 R_0 H_0 = 2\pi \int_0^{L_0} H R_\theta \, dx \quad (5)$$

and the internal capacity of both containers is also the same,

$$2\pi L_0 R_0^2 = 2\pi \int_0^{L_0} R^2 dx \quad (6)$$

Moreover, the minimal value of the coordinate R (the radius of the bottom of a shell) is constrained by a lower bound. Our investigation is restricted to a doubly convex shell.

Given function, describing the shape of the middle surface takes the form of parabola:

$$R(x) = \overline{R}(1 + \overline{\alpha}x^2) \tag{7}$$

where in general case $\overline{R}$ is an unknown radius of the shell in the beginning of the coordinate system, while $\overline{\alpha}$ is a free coefficient subjected to the optimization.

3. The hybrid PSO - spline algorithm

One of the most popular modern stochastic search algorithm is Particle Swarm Optimization (PSO). The algorithm is inspired by social living forms: bee swarms, bird flocks and fish schools from the world of the nature. Individuals forming a swarm influence each other and are simultaneously affected by the environment. A modified algorithm of PSO is adapted to cope with constrained nonlinear optimization tasks. A detailed description can be found in Ref. [2].

Particles are the points in a task domain, defined by position vectors. The update equations of moving swarm are expressed below:

$$\boldsymbol{v}_{k+1}^i \leftarrow w_{1k}^i \boldsymbol{v}_k^i + w_{2k}^i [\, c_1 r_{1k}^i (\boldsymbol{p}_k^{ii} - \boldsymbol{x}_k^i) + \\ + c_2 r_{2k}^i (\boldsymbol{p}_k^{\boldsymbol{g}} - \boldsymbol{x}_k^i) + c_3 r_{3k}^i (\boldsymbol{p}_k^{\boldsymbol{n}i} - \boldsymbol{x}_k^i)] \tag{8}$$

$$\boldsymbol{x}_{k+1}^i \leftarrow \boldsymbol{x}_k^i + \boldsymbol{v}_{k+1}^i \tag{9}$$

The following symbols are applied: k - iteration step index, i - particle index, c_1, c_2, c_3 - fixed coefficients named as acceleration constants or learning factors, r_1, r_2, r_3 - uniformly distributed random numbers in range [0,1], w_1 - inertia weight, w_2 - binary switching coefficient, $\boldsymbol{x}$ - position vector, $\boldsymbol{v}$ - velocity vector, $\boldsymbol{p^i}$ - the best own particle position found so far, $\boldsymbol{p^n}$- the best particle neighbours leader position found so far, $\boldsymbol{p^g}$ - the best swarm leader position found so far. The dimension of the position and velocity vectors equals the number of design variables in the optimization task.

The initial velocity is randomly generated, so the particle can move in any direction of the task domain at the beginning. The fitness function evaluates the particle position by calling the FEM analysis task. The work of the particle swarm algorithm is managed by just a few parameters. Nevertheless, choosing the best value for these is crucial for obtaining a rapid solution and a correct result.

Uniform polynomial type B-spline curve was applied to the description of the meridian of the shell. The conditions of the C^2 smoothness type are satisfied in the joints of the splines. This way leads us to the approximated function of the exact solution by means of polynomial of the relatively low-degree (third).

4. Model of the shell in Ansys

An attempt of verification of results acquired by the method of variational optimization in Ansys was taken. The parametric model of the shell was build in APDL (ANSYS Parametric Design Language). Composition of formulas describing the critical loading parameter and supplementary formulas was written in the way, which enables defining structures of various geometrical properties. Written formulas make easy adopting the model for other loading cases, including e.g. axial compressive force.

It is possibility to confirm of the results (i.e. geometry) of the optimal shell solved with variational optimization versus with hybrid PSO – spline method.

Figure 1 presents the obtained results in the form of contour maps of intensity of circumferential membrane force.

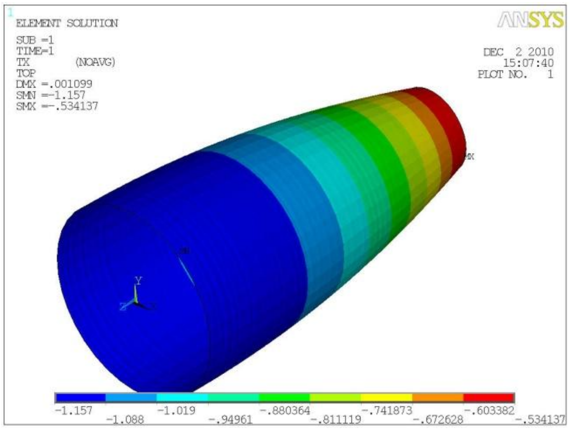

Figure 1: Plot of intensity of circumferential membrane force

5. Closing remarks

The parabolic shell under external pressure, exposed to the loss of stability was considered. An APDL model of such structure was build, verified and discussed.

One can acquire partially differential equations from the solution of mentioned problem with the variational optimization method. These equations were further developed to power series in the vicinity of a zero point and hence they were integrated numerically with fourth row Runnge - Kutta method. The result of integration were next discretized; they state the solution of isoperimetric problem in the form of tabulated points.

The results obtained by two different methods of solving the optimization problem: variational and numerical one are compared in the paper.

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