Shape update strategies in finite element modelling of finite wear

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Abstract

A general class of frictional contact problems with wear is considered in which both the deformation due to contact interactions and shape changes due to wear are finite (i.e. not assumed to be infinitesimal). A theoretical framework is provided on the continuum level, allowing consistent time and spatial discretization. Moreover, with some assumptions on the class of problems at hand, several shape update schemes are proposed to improve the performance and the accuracy of the finite element solution.

Keywords: contact mechanics, numerical analysis, sensitivity, finite element methods, large deformations

1. Introduction

Wear is a process of material removal from the surfaces of contacting bodies which results from damage processes developing in the surface layers. This work is concerned with modelling of finite shape changes resulting from wear. Several shape update schemes are discussed from the point of view of their accuracy and computational efficiency, and their performance is illustrated using 2D and 3D finite element simulations. The details of the formulation can be found in [2].

2. Three configurations and two time scales

The continuum formulation is based on the concept of three configurations. The contacting bodies are assumed to be hyperelastic. For each body, in addition to the initial configuration $\hat{\Omega}$ and the current configuration ω, an intermediate time-dependent configuration Ω is introduced, which corresponds to unstressed and undeformed body of the shape changed due to wear. Accordingly, the kinematics involves the shape evolution mapping $\boldsymbol{\Psi}$ and the deformation mapping $\boldsymbol{\varphi}$

$$\boldsymbol{X} = \boldsymbol{\Psi}(\hat{\boldsymbol{X}}, t), \quad \boldsymbol{x} = \boldsymbol{\varphi}(\boldsymbol{X}, t), \quad t \in [0, T], \tag{1}$$

where $\hat{\boldsymbol{X}} \in \hat{\Omega}$, $\boldsymbol{X} \in \Omega$, $\boldsymbol{x} \in \omega$, cf. Fig. 1.

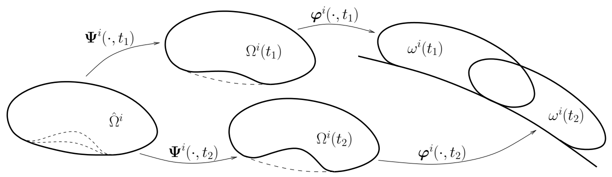

Figure 1: The three configurations at two time instants.

Two time scales are also introduced to distinguish the fast time scale τ of the actual contact interactions from the slow time scale t of the wear process. The shape and deformation mappings (1) are rewritten accordingly,

$$\begin{aligned} \boldsymbol{X} &= \boldsymbol{\Psi}(\hat{\boldsymbol{X}}, t), & t &\in [0, T], \\ \boldsymbol{x} &= \boldsymbol{\varphi}_t(\boldsymbol{X}, \tau), & \tau &\in [t, t + \Delta\tau], \end{aligned} \tag{2}$$

where $\boldsymbol{X} \in \Omega_t$, $\Omega_t = \boldsymbol{\Psi}(\hat{\Omega}, t)$, and $\Delta\tau$ is a characteristic or representative time of the deformation problem, for instance, one cycle of a cyclic loading program. Those time scales are then separated to partially decouple the contact deformation problem and the shape evolution problem. We assume that wear is negligible at the fast time scale, therefore a classical contact problem is solved at the fast time scale, and the wear rate $\dot{W}_t(\tau)$ is calculated as a postprocessing quantity. This wear rate is then averaged over a representative time interval $\Delta\tau$,

$$\dot{\overline{W}}(t) = \frac{1}{\Delta\tau} \int_t^{t+\Delta\tau} \dot{W}_t(\tau) d\tau, \tag{3}$$

giving the input data for the problem of shape evolution due to wear at the slow time scale, i.e.

$$\dot{\boldsymbol{\Psi}}_t \cdot \boldsymbol{N}_t = \begin{cases} -\dot{\overline{W}}(t) & \text{on contact part of boundary } (\Gamma_t^c), \\ 0 & \text{elsewhere on boundary } (\Gamma_t \setminus \Gamma_t^c). \end{cases} \tag{4}$$

Here, $\boldsymbol{N}_t$ is the outer normal to the intermediate undeformed configuration Ω.

3. Shape update schemes

Equation (4) specifies only the evolution of the boundary alone, while in the finite element framework the evolution of all mesh nodes must be provided to prevent mesh degeneration. Two methods are adopted and discussed in this work. The fist method relies on solving an *auxiliary elasticity problem*, in which displacement boundary conditions are applied in normal direction to the surface in undeformed configuration, with the magnitude corresponding to previously calculated average wear profile (3).

In the second approach, an *approximate shape evolution mapping* $\tilde{\boldsymbol{\Psi}}(\hat{\boldsymbol{X}}, \boldsymbol{\phi}(t))$ is constructed, which is expressed in terms of time-dependent shape parameters $\boldsymbol{\phi}$. In general, the mapping can satisfy Eq. (4) in a weak sense only, e.g. according to the follow-

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ing minimization problem with respect to $\dot{\boldsymbol{\phi}}$,

$$\min_{\dot{\boldsymbol{\phi}}} \int_{\Gamma_t} \left(\boldsymbol{N}_t \cdot \frac{\partial \tilde{\boldsymbol{\Psi}}}{\partial \boldsymbol{\phi}} \dot{\boldsymbol{\phi}} + \dot{\overline{W}}_t \right)^2 dS. \qquad (5)$$

In the time-discretized setting, at each increment of the slow time t, a load cycle of pure contact problem is simulated, providing the averaged wear velocity on the contact surface, and then the updated shape of the intermediate configuration Ω is calculated. Two time-integration schemes are considered for that purpose. The first-order scheme is a simple forward-Euler time integration scheme which utilizes only the value of averaged wear rate (3). In the second-order scheme, (implicit) derivatives of the averaged wear rate with respect to shape parameters are necessary, which are obtained from sensitivity analysis [4]. The latter scheme is more complex, however, it is more accurate, so larger time steps can be used.

4. Finite element analysis

The finite element formulation employs contact smoothing and the augmented Lagrangian technique [3] to enforce the contact constraints. Sensitivities of averaged wear rate with respect to shape parameters are obtained using the direct differentiation method (DDM). *AceGEN/AceFEM* system [1] is used to derive the low-level finite element code and to solve the direct and sensitivity contact problems.

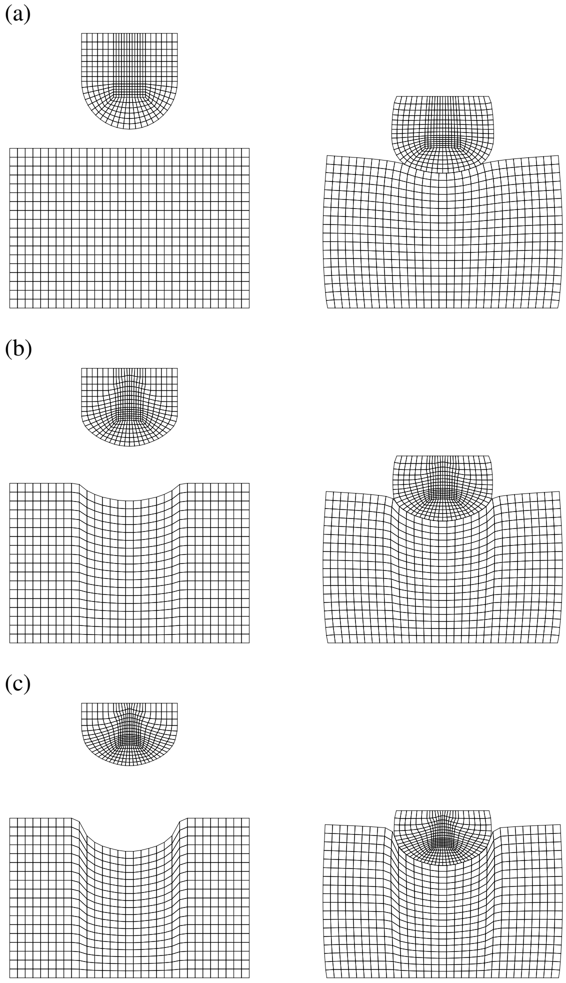

Figure 2: Steady-state pin-on-flat problem (both surfaces wear-off due to out-of-plane motion): undeformed and deformed finite element mesh at three time instants: (a) $t = 0$, (b) $t = T/2$ and (c) $t = T$ (Lengiewicz and Stupkiewicz [2]).

In the numerical examples, the performance and accuracy of the two time integration schemes and of various definitions of mapping $\tilde{\boldsymbol{\Psi}}$ are studied. For instance, deformation and shape evolution due to wear in a steady-state pin-on-flat problem are illustrated in Figs. 2 and 3. Oscillations visible in Fig. 3(b) result from instability of the first-order integration scheme.

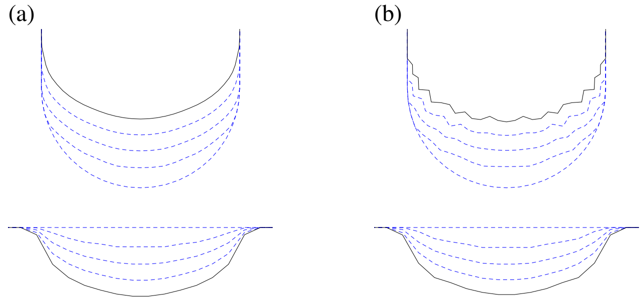

Figure 3: Shape evolution due to wear obtained in (a) 128 and (b) 8 time increments (first-order scheme). The initial and four subsequent shapes are shown, the final configuration is plotted with a solid line (Lengiewicz and Stupkiewicz [2]).

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