Analysis of non-planar shear wall structures with stiffening beams

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Abstract

The paper presents the analysis of non-planar asymmetric shear wall structures with stiffening beams. The stiff deep beams incorporated at the various levels of coupled shear walls improve the stiffness of the structural system of tall building. The analysis is based on a variant of the continuous connection method for three-dimensional shear wall structures having stepwise changes in cross-section. In the continuous approach the connecting beams are replaced by equivalent continuous connections. The compatibility equations have been written at the midpoints of connecting and stiffening beams. The differential equation systems for shear wall structure segments of the constant cross-section are uncoupled by orthogonal eigenvectors. The results of the proposed method have been compared with those obtained using the *SAP2000* structural analysis program and a good match has been observed.

Keywords: computational mechanics, shear wall structures, stiffening beams, continuous connection method, tall buildings

1. Introduction

In the design of tall buildings it is essential that the structure is sufficiently stiff to resist the horizontal loads caused by wind and seismic motion. The shear wall structures have been recognized as one of the most efficient structural systems for such a purpose. The methods, which are available for the analysis of shear wall buildings, can be broadly categorized into the following main groups: a finite element method, a frame analogy method and a continuous connection method [11]. The finite element method is the most powerful and versatile method of the analysis of complex structures but nevertheless, there is a scope for the development of other techniques that may have the advantage of greater efficiency for specific forms of structural systems, such as tall structures containing coupled shear walls and cores. Some difficulties concerning a great number of unknowns and ill conditioning of a problem for slender structures, which appear in a discrete model, may be avoided in a simple way using the continuous model. In the continuous approach, the horizontal connecting beams are substituted by continuous connections. The continuous connection method (CCM) is regarded as the simplest and most efficient method for the design analysis of coupled shear walls.

In practice, however, the depth of connecting beams is limited and coupling effect provided by the lintel beams on structural walls may not be sufficient. Therefore, it is sometimes necessary to insert the deep beams somewhere along the height of the walls. A suitable position for the stiffening beams can be conveniently found at the top of building or at intermediate levels reserved for building services or safety purposes [4]. The stiffened planar coupled shear walls have been analysed in many papers [1, 3, 4, 5, 6, 9, 10]. Coull and Low [7] presented the analysis of non-planar coupled shear walls with additional stiff connecting beam at roof level, based on Vlasov's theory of thin-walled beams and continuous medium technique. Emsen *et al.* [8] studied non-planar coupled shear walls with one band of connecting beams and with any number of stiffening beams.

The aim of this paper is to present the analysis of non-planar shear wall structures with any number of connecting and stiffening beams, using the variant of the continuous connection method (CCM) for structures of variable cross-section [13].

2. Analysis

Equation formulations for a three-dimensional continuous model of the shear wall structure with the constant cross-section have been given, among others, in [2, 12].

The analysis is based on the following main assumptions:

1. The floor slabs are taken as diaphragms with infinite in-plane stiffness.
2. The out-of-plane stiffness of the floor slabs can be modelled by connecting beams of appropriate stiffness spanning between shear walls.
3. Vlasov's theory for thin walled beams of an open section is taken to be valid for the individual walls.

A structure, which changes its cross-section along the height, can be divided into n_h segments, each one of the constant cross-section. For k-th segment, the differential equations can be stated as follows [13]:

$$\mathbf{B}\,\mathbf{n}_\mathbf{N}''(z) = \mathbf{A}\,\mathbf{n}_\mathbf{N}(z) + \mathbf{f}(z) \tag{1}$$

$$\mathbf{v}_\mathbf{G}'''(z) = \mathbf{V}_\mathbf{T}\mathbf{t}_\mathbf{K}(z) - \mathbf{V}_\mathbf{N}\mathbf{n}_\mathbf{N}(z) - \mathbf{V}_\mathbf{R}\mathbf{n}_\mathbf{R}(z) \tag{2}$$

where, the following notation applies: $z \in (h_{k-1}, h_k>$, h_k is the height of upper boundary of k-th segment of the constant cross-section; n_h is the number of segments of the constant cross-section ($k = 1, \dots n_h$); n_w is the number of continuous connections; n_e is the number of shear walls; n_v is the number of vertical loads; $\mathbf{B}$ is $n_w \times n_w$ diagonal matrix containing flexibilities C of continuous connections idealizing the coupling beams, calculated from the following relation:

$$C = (l^3/12EJ_b + 1.2l\ /GA_b)h \tag{3}$$

where, l, J_b, A_b constitute the connecting beam span length, the moment of inertia and the cross-sectional area, respectively, and h is the storey height; $\mathbf{n}_\mathbf{N}(z)$ is the vector containing unknown functions of the shear force intensity in continuous connections which substitute connecting beams; $\mathbf{f}(z)$ is a vector formed on the basis of given lateral loads; $\mathbf{n}_\mathbf{R}(z)$ is the vector of the functions of vertical loads; $\mathbf{t}_\mathbf{K}(z)$ is the vector of the functions of shear forces and a torque due to the action of lateral loads, $\mathbf{t}_\mathbf{K}(z) = col\ [t_X(z),\ t_Y(z),\ m_S(z)]$; and $\mathbf{v}_\mathbf{G}(z)$ is the vector containing the functions of horizontal displacements

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of the structure measured in the global coordinate system *OXYZ*: $\mathbf{v_G}(z) = col\,[v_X(z), v_Y(z), \varphi(z)]$.

The matrices and vectors appearing in the Equations (1), (2) are described by the following formulae [13]:

$$\mathbf{A} = \mathbf{S}_\mathbf{E}^\mathrm{T}\mathbf{K}_\mathrm{S}\mathbf{S}_\mathbf{E} - \mathbf{C}_\mathbf{N}^\mathrm{T}\mathbf{L}\mathbf{V}_\mathbf{N}$$

$$\mathbf{f}(z) = \mathbf{F_R}\mathbf{n_R}(z) + \mathbf{F_T}\mathbf{t_K}(z)$$

$$\mathbf{F_R} = \mathbf{S}_\mathbf{E}^\mathrm{T}\mathbf{K}_\mathrm{S}\mathbf{S}_\mathbf{R} - \mathbf{C}_\mathbf{N}^\mathrm{T}\mathbf{L}\mathbf{V}_\mathbf{R} \qquad \mathbf{F_T} = \mathbf{C}_\mathbf{N}^\mathrm{T}\mathbf{L}\mathbf{V}_\mathbf{T} \tag{4}$$

$$\mathbf{V_T} = (\mathbf{L}^\mathrm{T}\mathbf{K_Z}\mathbf{L})^{-1} \qquad \mathbf{V_N} = \mathbf{V_T}\mathbf{L}^\mathrm{T}\mathbf{C_N} \qquad \mathbf{V_R} = \mathbf{V_T}\mathbf{L}^\mathrm{T}\mathbf{C_R}$$

where:

- $\mathbf{S_E}$ — $n_e \times n_w$ Boolean matrix, related to the interaction between shear walls and continuous connections
- $\mathbf{K}_\mathrm{S}$ — $n_e \times n_e$ diagonal matrix, $\mathbf{K}_\mathrm{S} = diag\,(1/EA_i)$,
- $\mathbf{K_Z}$ — $3n_e \times 3n_e$ matrix containing transverse stiffness of shear walls in local coordinate systems, i.e. systems of principal axes of the shear walls, $\mathbf{K_Z} = diag\,(-EJ_{y1}\ldots, -EJ_{x1}\ldots, -EJ_{\omega 1}\ldots)$
- $\mathbf{C_N}$ — $3n_e \times n_w$ matrix containing the coordinates of the points of contra-flexure in connections in the local coordinate systems $\mathbf{C_N} = (\mathbf{C_{Nx}}, \mathbf{C_{Ny}}, \mathbf{C_{N\omega}})^T$
- $\mathbf{L}$ — $3n_e \times 3$ matrix of coordinates transformation from the global coordinate system *OXYZ* to the local systems of axes
- $\mathbf{S_R}$ — $n_e \times n_v$ Boolean matrix related to the action of vertical loads on shear walls
- $\mathbf{C_R}$ — $3n_e \times n_v$ matrix containing the coordinates of the points of vertical loads application.

The boundary conditions for Equations (1) and (2) at the bottom and at the top of the shear wall structure can be stated as follows:

$$\mathbf{n}_{\mathbf{N}(1)}(0) = -\mathbf{B}_{(1)}^{-1}\mathbf{S}_E^T\mathbf{z_0}, \qquad \mathbf{n}'_{\mathbf{N}(n_h)}(H) = 0 \tag{5}$$

$$\mathbf{v}_{\mathbf{G}(1)}(0) = 0, \qquad \mathbf{v}'_{\mathbf{G}(1)}(0) = 0, \qquad \mathbf{v}''_{\mathbf{G}(n_h)}(H) = 0 \tag{6}$$

where, $\mathbf{z_0}$ is the vector containing given settlements of shear walls and H is the structure height.

The boundary conditions for the unknown functions $\mathbf{n_N}$(z) at the boundaries of *k-th* and (*k+1*)*-th* segments have been derived on the basis of compatibility consideration at the mid-points of the cut connecting beams in the following form:

$$\mathbf{n}_{\mathbf{N}(k)}(h_k) = \mathbf{B}_{(k)}^{-1}\mathbf{B}_{(k+1)}\mathbf{n}_{\mathbf{N}(k+1)}(h_k) \tag{7}$$

The refined boundary conditions for the derivatives of $\mathbf{n_N}$(z) functions at the plane of contiguity, at which an abrupt change in cross-section occurs, have been established on the basis of the normal forces in shear walls equilibrium consideration in the following form:

$$\begin{aligned}\mathbf{n}'_{\mathbf{N}(k)}(h_k) &= \mathbf{B}_{(k)}^{-1}\mathbf{B}_{(k+1)}\mathbf{n}'_{\mathbf{N}(k+1)}(h_k) \\ &+ \mathbf{B}_{(k)}^{-1}(\mathbf{C}_{\mathbf{N}(k)}^T\mathbf{L}_{(k)}\mathbf{v}''_{\mathbf{G}(k)}(h_k) - \mathbf{C}_{\mathbf{N}(k+1)}^T\mathbf{L}_{(k+1)}\mathbf{v}''_{\mathbf{G}(k+1)}(h_k)) \\ &+ \mathbf{B}_{(k)}^{-1}\mathbf{S}_{\mathbf{E}(k)}^T(\mathbf{K}_{\mathbf{S}(k+1)} - \mathbf{K}_{\mathbf{S}(k)})\,\mathbf{n}_{\mathbf{E}(k)}(h_k)\end{aligned} \tag{8}$$

where, $\mathbf{n_E}(z)$ is the vector containing the normal forces in shear walls.

Here, it should be emphasized that the midpoints of the connecting beams in different segments should lie on the same vertical line. The derivation of boundary conditions (7), (8) is given in [13].

The boundary conditions for the functions of horizontal displacements $\mathbf{v_G}$(z) at the boundaries of *k-th* and (*k+1*)*-th* segments can be stated as follows:

From the geometric compatibility consideration we have:

$$\mathbf{v}_{\mathbf{G}(k)}(h_k) = \mathbf{v}_{\mathbf{G}(k+1)}(h_k), \quad \mathbf{v}'_{\mathbf{G}(k)}(h_k) = \mathbf{v}'_{\mathbf{G}(k+1)}(h_k) \tag{9}$$

Taking into account that centres of gravity of walls in different segments should lie on the same vertical straight line, from equilibrium consideration the following condition can be stated:

$$\mathbf{m}_{\mathbf{E}(k)}(h_k) = \mathbf{m}_{\mathbf{E}(k+1)}(h_k) \tag{10}$$

where, $\mathbf{m_E}(z)$ is the vector of bending moments and bimoments in the shear walls, expressed by the relation:

$$\mathbf{m_E}(z) = \mathbf{K_Z}\,\mathbf{L}\,\mathbf{v}''_\mathbf{G}(z) \tag{11}$$

Substituting Equation (10) in Equation (9) and then pre-multiplying by $\mathbf{V}_{\mathbf{T}(k)}\mathbf{L}^T_{(k)}$, the following condition has been obtained:

$$\mathbf{v}''_{\mathbf{G}(k)}(h_k) = \mathbf{S}_{\mathbf{V}(k+1,k)}\,\mathbf{v}''_{\mathbf{G}(k+1)}(h_k) \tag{12}$$

where:

$$\mathbf{S}_{\mathbf{V}(k+1,k)} = \mathbf{V}_{\mathbf{T}(k)}\mathbf{L}_{(k)}^T\mathbf{K}_{\mathbf{Z}(k+1)}\mathbf{L}_{(k+1)}$$

In the analysis of stiffened shear wall structures, storeys with stiffening beams are considered as the individual segments of the constant cross-section.

3. Solution method

In the proposed method, the algorithm of solving the differential equation system, used for structures of constant cross-section [12], has been extended so as to enable taking the structures of the variable cross-section into account.

In order to uncouple differential equation systems (1), auxiliary functions $\mathbf{g}_{(k)}(z)$ satisfying these relations have been introduced:

$$\mathbf{n}_{\mathbf{N}(k)}(z) = \mathbf{B}_{(k)}^{-1/2}\mathbf{Y}_{(k)}\,\mathbf{g}_{(k)}(z) \tag{13}$$

where, $\mathbf{Y}_{(k)}$ is matrix the columns of which are eigenvectors of the symmetrical matrix

$$\mathbf{P}_{(k)} = \mathbf{B}_{(k)}^{-1/2}\mathbf{A}_{(k)}\mathbf{B}_{(k)}^{-1/2}$$

Consequently, for the *k-th* segment n_w second-order differential equations have been obtained in the following form:

$$z \in (h_{k-1}, h_k > \qquad \begin{aligned} g''_{i(k)}(z) - \lambda_{i(k)}\,g_{i(k)}(z) &= F_{Bi(k)}(z) \\ F_{Bi(k)}(z) &= \mathbf{Y}_{i(k)}^T\,\mathbf{B}_{(k)}^{-1/2}\,\mathbf{f}_{(k)}(z)\end{aligned} \tag{14}$$

where, $\lambda_{i(k)}$ is the *i-th* eigenvalue of matrix $\mathbf{P}_{(k)}$, and $\mathbf{Y}_{i(k)}$ is the eigenvector corresponding to the *i-th* eigenvalue.

In the analysis, a polynomial form of functions $\mathbf{f}_{(k)}(z)$ has been used:

$$\mathbf{f}_{(k)}(z) = \mathbf{F}_{(k)} \mathbf{W}_S(z), \quad F_{Bi(k)}(z) = F_{Bi(k)} \mathbf{W_S}(z), \\ \mathbf{W_S}(z) = col(z^0, \dots, z^{(s-1)}) \tag{15}$$

A procedure for least squares fit by orthogonal polynomials for the approximation of the functions of load has been applied.

The eigenvalues and eigenvectors of symmetric matrix $\mathbf{P}_{(k)}$ are computed by a set of procedures realizing the Householder's tridiagonalization and the **QL** algorithm, which have been inserted in [14] and later written in Pascal. The matrix **A** is positive semi-definite, thus matrix **P** can also have zero eigenvalues.

The solutions of uncoupled differential equations (14) corresponding to zero eigenvalues have the following form:

$$g_{i(k)}(z) = F_{Bi(k)} col(z^2/2,\ z^3/6, \dots, z^{(s+1)}/(s(s+1))) \\ + C_{1i(k)}\, z + C_{2i(k)} \tag{16}$$

The form of solutions corresponding to the non-zero eigenvalues λ_i is as follows:

$$g_{i(k)}(z) = C_{1i(k)} e^{\sqrt{\lambda_{i(k)}}\, z} + C_{2i(k)} e^{-\sqrt{\lambda_{i(k)}}\, z} + \mathbf{r}_{\mathbf{S}i(k)} \mathbf{W_S}(z) \tag{17}$$

where, $C_{1i(k)}, C_{2i(k)}$ are the integration constants and $\mathbf{r}_{\mathbf{S}i(k)}$ are particular solution coefficients calculated by the indeterminate coefficient method.

Introducing these solutions of uncoupled differential equations into the relation (13) and later considering boundary conditions, given by Equations (5), (7) and (8) we will obtain the system of $2\, n_h n_w$ linear equations for the determination of all the integration constants in the form:

$$\mathbf{R}_W\, \mathbf{c} = \mathbf{p}_S \tag{18}$$

where, $\mathbf{R}_W$ is an unsymmetric matrix and $\mathbf{p}_S$ is a vector dependent on the load. The vector **c** successively for each segment contains: integration constants C_1 corresponding to zero and non-zero eigenvalues and next integration constants C_2 corresponding to the zero and non-zero eigenvalues, respectively. The solutions are computed by the procedures based on the **LU** factorization, where **L** is lower triangular and **U** is upper-triangular, taken from [14].

After the determination of the integration constants **c**, the functions of shear force intensity in continuous connections for each segment are computed in a given number of points. Then they are replaced by appropriate polynomial functions using the interpolation. This allows the user to calculate easily final solutions for any ordinates of the structure height.

The next step of computations is the determination of functions of horizontal displacements $\mathbf{v_G}(z)$ and their derivatives necessary to calculate the internal forces and stresses from Eqn (2) with boundary conditions (6),(9),(12).

The refined boundary conditions for the derivatives of shear force intensity functions $\mathbf{n_N}'(z)$, given by Equation (8), will have to be satisfied in an iterative manner. To obtain the first approximation we shall assume that the last two terms of Equation (8) are equal to zero. From this analysis the values of derivatives of displacement functions $\mathbf{v_G}''_{(k)}(h_k)$, $\mathbf{v_G}''_{(k+1)}(h_k)$ and normal forces $\mathbf{n}_{\mathbf{E}(k)}(h_k)$ at the boundaries of segments can be found and then, according to Equation (8), the improved value of the right-hand side $\mathbf{p_S}$ of the Equation (18) is obtained. The analysis then carries on repeatedly until the solution $\mathbf{v_G}(H)$ is found to be sufficiently convergent. The solutions converged to four significant figures in about 5-10 iterations. In spite of the number of iterations required, the calculation is very fast.

On the basis of this algorithm the software in Object Pascal of Delphi 5 environment has been implemented and included in the system for the analysis of shear wall tall buildings [12].

4. Numerical examples

Several examples are presented in order to demonstrate the versatility and accuracy of the proposed technique.

4.1 Example 1: A plane shear wall with three stiffening beams and four rows of openings

The stiffened multi-bay coupled shear wall (Fig.1), previously investigated by Bikce *at al* [1], using the CCM and *SAP2000* structural analysis program, was first analysed. Numerical input values for the 25-storey structure are as follows: the total height H = 75 m, for successive segments, starting from the base: thickness, 0.30 m, 0.25 m, 0.20 m; storey height, 3.1m, 2.9m, 3.0m; the stiffening beam height, 2.85m, 2.65m, 2.75m; modulus of elasticity E = 2.876 10^7 kN/m^2 and the connecting beam heights, from left to right, respectively, 0.25 m, 0.20 m, 0.35 m, 0.30 m. The rigid foundation (Type 1) has been considered. Table 1 presents values of axial wall forces at the base and horizontal displacement at the top of structure. The first column includes results, obtained by Bikce *at al* using the CCM, the second column results of *SAP2000*, given in [1] and the last one results obtained by the present analysis, assuming the Poisson's ratio value ν = 0.2.

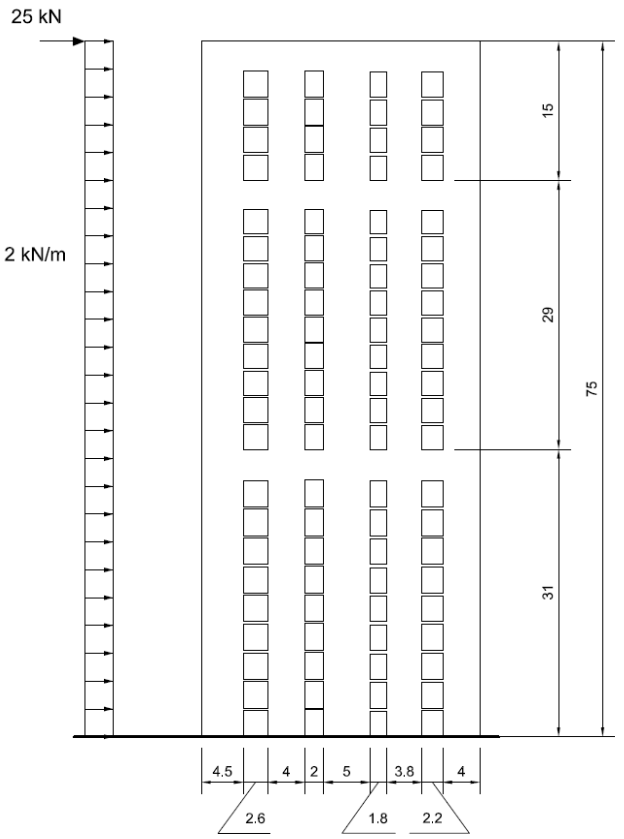

Figure 1. Example 1: Plane shear wall with three stiffening beams and four rows of openings [1]

Table 1. Example 1: Stiffened multi-bay coupled shear wall - normal forces at the base and top displacement

Forces and displacement	Method of approach		
	Bikce *at al* [1] CCM	*SAP2000* [1]	Present study CCM
Ne1(0) [kN]	172.4	174.1	172.6
Ne2(0) [kN]	70.9	71.8	71.3
Ne3(0) [kN]	37.1	35.8	36.5
Ne4(0) [kN]	-109.7	-108.7	-104.1
Ne5(0) [kN]	-172.2	-172.9	-176.3
v_X (H) [mm]	2.22	2.21	2.16

The satisfactory agreement between the results obtained by the CCM and *SAP2000* may be observed. In the presented example, the stiffening of coupled shear walls reduces the maximum displacement at the top by 67% and maximum stress at the base by 29%.

4.2 Example2: Stiffened non-planar coupled shear walls

To verify the correctness of the technique used, the non-planar coupled shear walls with additional stiff connecting beam at the roof level, studied by Coull and Low [7] (model 4), have been analysed. In [7] the validity of solutions, based on Vlasov's theory of thin walled beams and continuous medium technique, was checked by comparison of the theoretical results with those from tests on 18 storey models, which were constructed from 12 mm thick Perspex sheet. In Fig.2 the cross-section of the analysed model is presented.

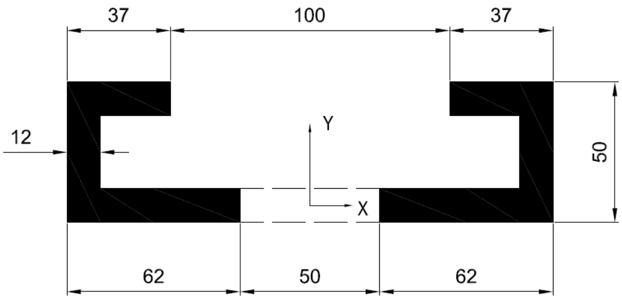

Figure 2. Example 2: The non-planar coupled shear wall Perspex model [7]

The storey height and connecting beam depth were maintained constant at 25 mm and 6 mm, respectively. The results for the stiff top beam refer to a very stiff condition in which the stiffening beam consisted of 24.4x10x2.64 mm steel channels bolted to either side of the web [7]. The uniformly distributed lateral loading q_x = 1 N/mm was applied in the plane of the web (in X direction). Longitudinal strains were measured by electrical resistance gauges located at 16 points at the height z = 59.5 mm.

In Fig.3 the obtained stress distribution in stiffened model due to loading in plane of web is presented. The reasonable agreement is achieved between obtained and experimental stress distribution, given in [7], both with and without a stiff top beam. In this case the stiff top beam has the effect on reducing the top displacements v_x by 36% and normal stresses by 14%.

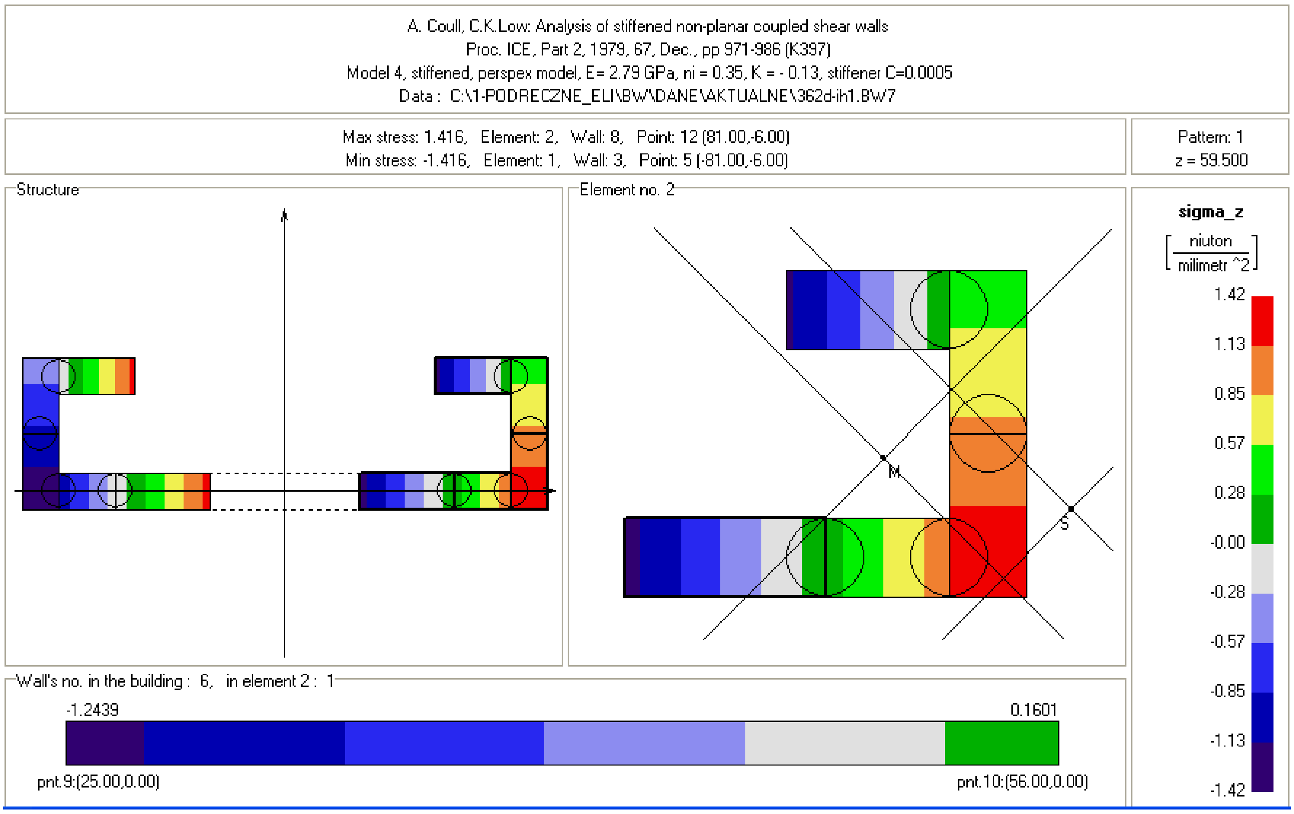

Figure 3. Example 2: The stress distribution in the stiffened non-planar coupled shear wall model at the height z = 59.5 mm

4.3 Example 3: Non-planar asymmetrical coupled shear walls with four stiffening beams

The multi-stiffened non-planar asymmetrical coupled shear walls, previously investigated in [8], have been analysed (Fig.4). The total height of the shear wall is 72 m and the storey height is 3 m. The height of the connecting beams is 0.4 m. Three stiffening beams of 3 m height are placed at the levels of the ninth, the thirteenth and seventeenth storeys and the last one two metres high at the top. The thickness of the walls in the left and right external parts is 0.2 m and 0.3 m in the central part. The elasticity and shear moduli, given in [8], are $E = 2.85$ GPa and $G = 1.056$ GPa, respectively. The external loads P_x =500 kN, P_Y = 400 kN and M_S = -1500 kNm act at the top of the structure. In Fig.5, the lateral displacement v_X at points on Z axis, found by the present analysis using the continuous connection method and given in [8], obtained by the *SAP2000* structural analysis program using the frame method, are compared, for the unstiffened and stiffened cases, and a good agreement has been observed.

Figure 6 shows the graphs of lateral displacements and rotations of stiffened structure and normal forces in shear walls.

Figure 7 shows the horizontal displacements at the top of the stiffened structure. In this example, the stiffening of coupled shear walls reduces the displacements v_x at the top by 41%, displacements v_Y and rotations φ by 9% and the maximum stresses at the base by 12%.

The further analysis showed, that when two additional bands of connecting beams were introduced between the external walls in the central part, torsional stiffness of the shear wall structure was considerably increased. The results obtained for connecting beams of the height 0.4 m indicated 24% reduction in displacements v_x, a 77% reduction in rotations and a 25% reduction in maximum stress.

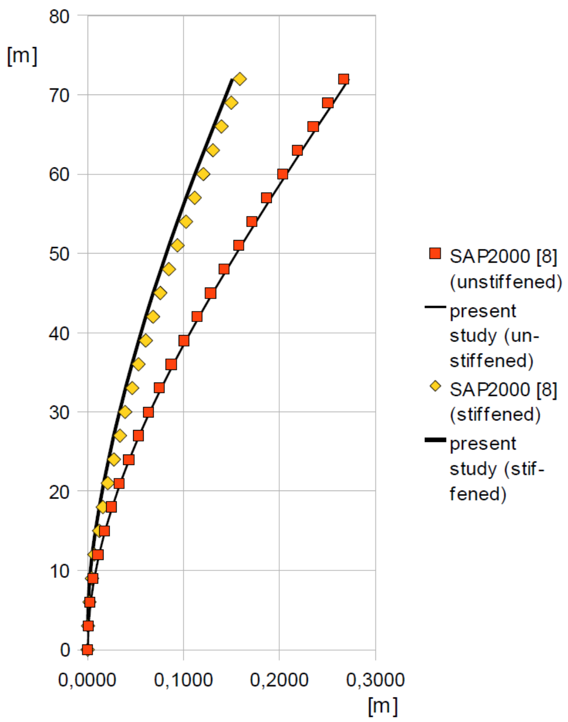

Figure 5. Example 3: Comparison of the lateral displacements in x direction for the unstiffened and stiffened cases

Figure 8 shows normal stress distribution at the base in stiffened shear wall structure, obtained by the presented method.

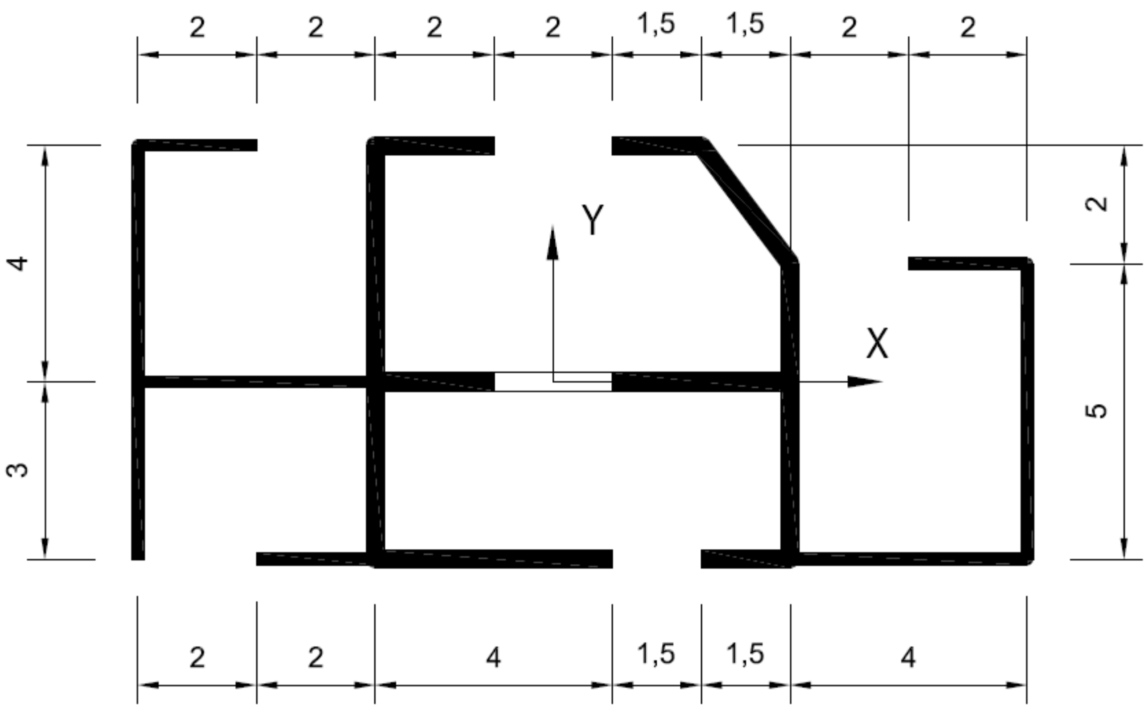

Figure 4. Example 3: Non-planar asymmetrical coupled shear walls with four stiffening beams [8]

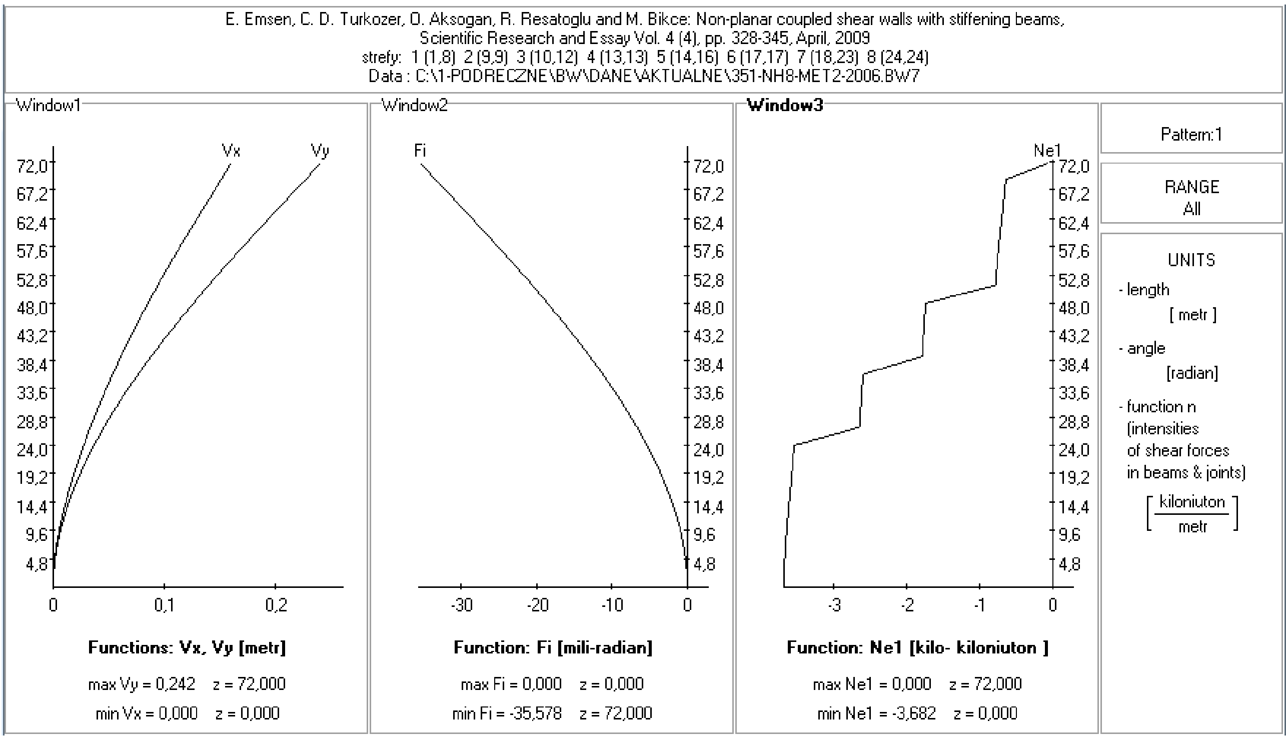

Figure 6. Example 3: Graphs of horizontal displacements and rotations of stiffened structure and normal forces in shear walls

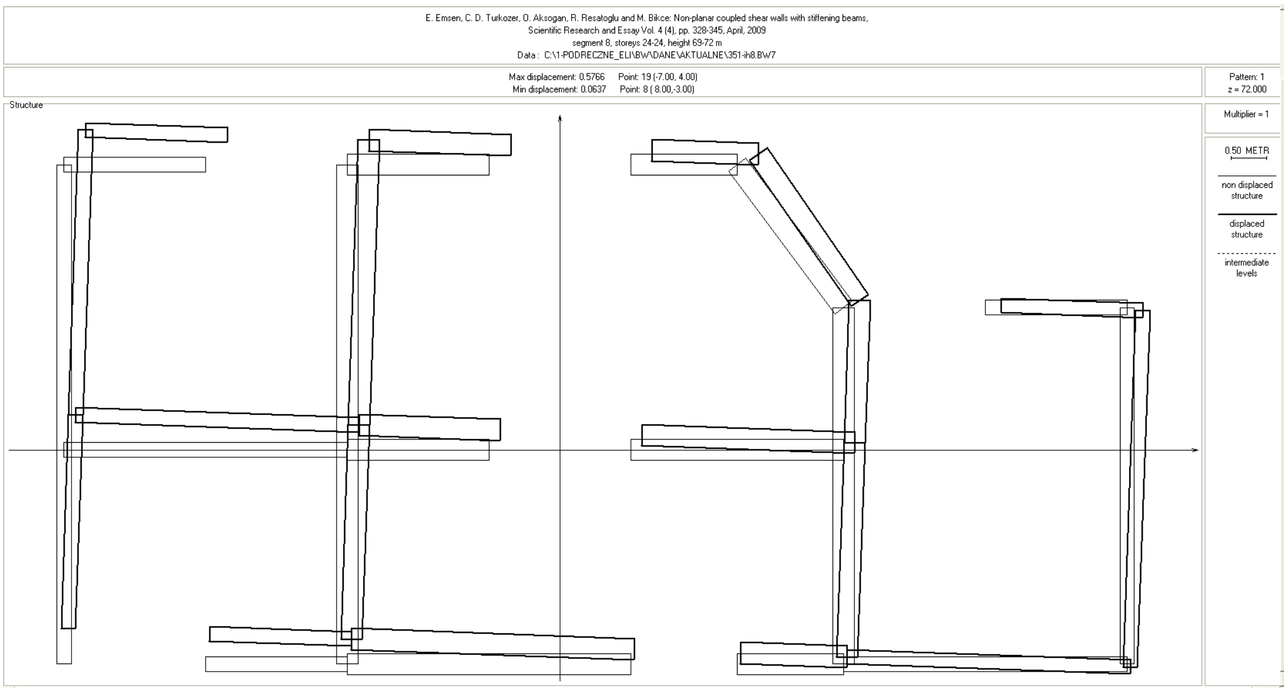

Figure 7. Example 3: Horizontal displacements at the top of stiffened non-planar shear wall structure

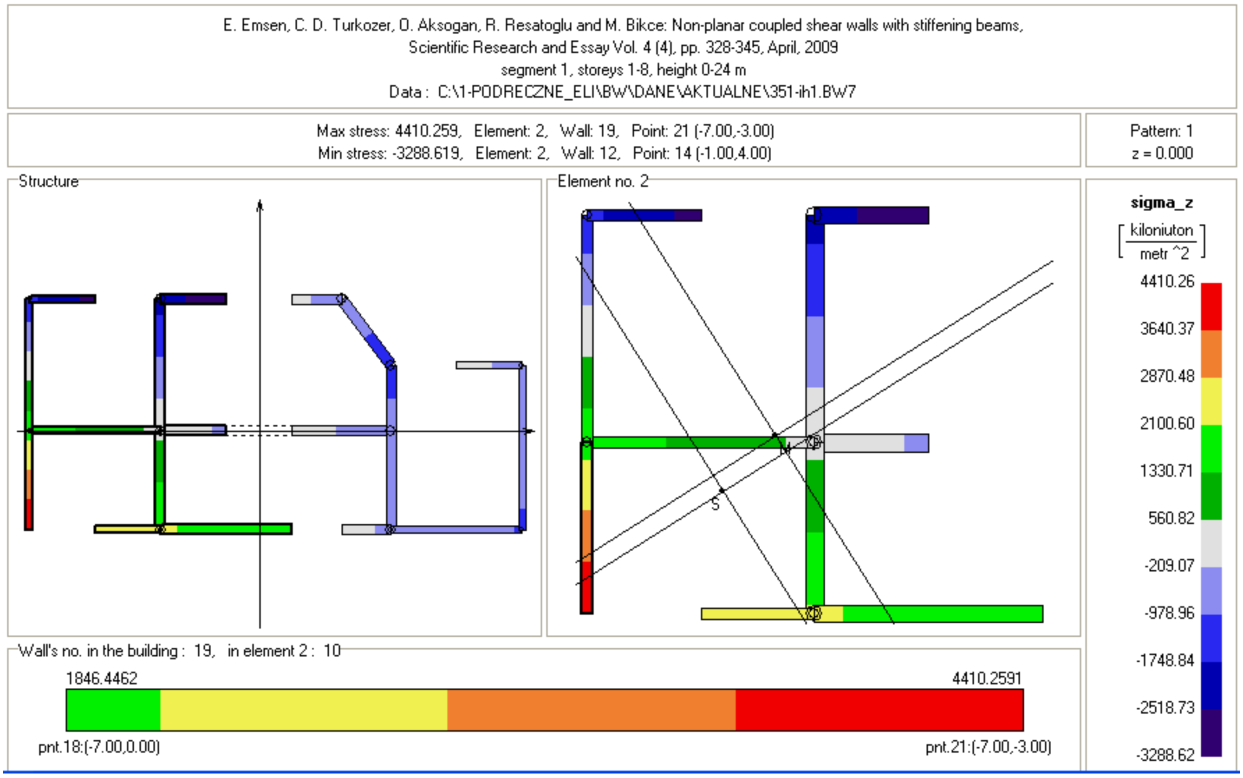

Figure 8. Example 3: Distribution of normal stresses at the base of stiffened shear wall structure

5. Conclusions

The paper presents the analysis of non-planar asymmetric shear wall structures with any number of connecting and stiffening beams, using a variant of the continuous connection method for structures of variable cross section. The refined boundary conditions for derivatives of shear force intensity functions have been included. The results obtained by the presented method have been compared with those obtained experimentally and analytically, given in literature, and a good match has been observed. The numerical examples showed that the insertion of stiffening beams could reduce considerably the lateral deflection of the structure and normal stress in the walls. The proposed method is efficient and can be very useful, particularly, at the preliminary design stage when quick checks with different structural arrangements and dimensions are needed.

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