Concrete at high strain rates, a material formulation based on delayed damage

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Abstract

Concrete damage strength at higher strain rates are formulated with an enhanced delayed material formulation which is based on gradient continuum damage. Therefore classical damage formulation is extended by inertia effects which will lead to a retardation of damage at higher strain rates and the typical increase of the dynamic strength exciting the quasi-static strength. Furthermore a time depending viscous part was added to reproduce the behaviour at lower strain rates. The capabilities of this configuration will be demonstrated by numerical investigated Split Hopkinson Pressure Bar experiments within the tensile and compressive domain.

Keywords: concrete, damage, dynamics, impact, viscoelasticity

1. Introduction

High strain rates have a large influence on the strength of concrete. A number of experimental investigations have been performed to study this effect, which was observed for the tensile [3] and compressive [4] strength. Figure 1 is summarizing a fraction of the experimental results of various authors. Therein the dynamic strength is defined as the maximum peak stress at constant strain rate, the static one as the maximum stress at zero strain rate and there ratio as the dynamic strength increase factor (DIF). Since low strain rate effects are dominated be various effects like cross aggregate cracking, moisture content etc., the damage at high strain rates appears to be only dominated by inertia effects [2]. The presented method attempts to consider both effect and will be numerically investigated.

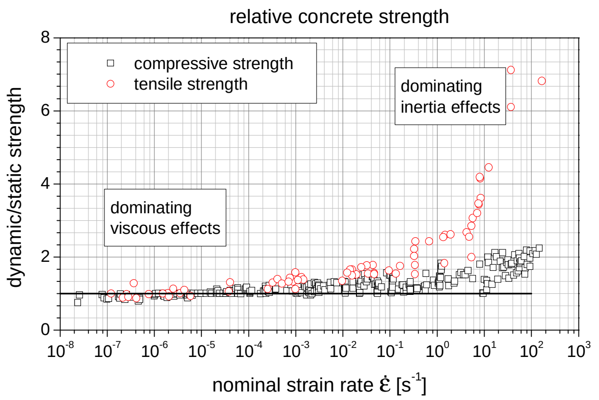

Figure 1: Sength increase factors (DIF), extracted from [4]

2. Isotropic strain based gradient damage law

A scalar extension D to the elastic tensor **E** is chosen to consider the occurrence of small cracks within brittle materials like concrete. The basic constitutive law, with $\boldsymbol{\sigma}$ as the stress and $\boldsymbol{\varepsilon}$ as the strain tensor, can be rewritten with

$$\boldsymbol{\sigma} = (1-D)\cdot\mathbf{E}\cdot\boldsymbol{\varepsilon}\,. \tag{1}$$

The damage evolution of concrete can be considered as Weibull distribution of micro defects with a variable equivalent nonlocal damage strain $\bar{\kappa}$ and the material constants e_d, e_0, g_d.

$$D(\bar{\kappa}) = \begin{cases} 0, & \bar{\kappa} < e_0 \\ 1 - e^{-\left(\frac{\bar{\kappa}-e_0}{e_d}\right)^{g_d}}, & \bar{\kappa} \geq e_0 \end{cases} \tag{2}$$

Furthermore the equivalent damage strain $\bar{\kappa}$ is connected to the triaxial strain states with the damage limit function

$$F(\boldsymbol{\varepsilon},\kappa) = a_1 J_\varepsilon + \kappa\left[a_2\sqrt{J_\varepsilon} + a_3\varepsilon_1 + a_4 I_\varepsilon\right] - \kappa^2 = 0 \tag{3}$$

including the first Iε and second Jε invariant of the strain tensor, the largest principal strain ε1 and further material constants a1…a4. It conforms the Kuhn-Tucker conditions: $F \leq 0,\quad \dot{D} \geq 0,\quad \dot{D}F = 0$.

Since material softening leads to strong localisation, the mesh dependence of those was eliminated by introducing non-local and gradient formulations for the damage variables. The gradient approach is used with a nonlocal equivalent damage strain $\bar{\kappa}$ as a function of the location **x**, which is connected to the local value κ with the Laplace operator Δ and the material dependent characteristic length R by

$$\bar{\kappa}(\mathbf{x}) - \frac{R^2}{2}\Delta\bar{\kappa}(\mathbf{x}) = \kappa(\mathbf{x})\,. \tag{4}$$

3. Inertially delayed damage extension

The inertially delayed damage bases on the idea that micro cracks cannot propagate arbitrarily fast as a movement of internal crack faces relative to their immediate surrounding is involved, i.e. a movement of masses on a microscopic scale, which leads to an delay of crack propagation. The measure for damage is given by $\bar{\kappa}$ and by extending Eqn. (4) with an inertial term and the additional parameter m_κ, this leads to:

$$m_\kappa\ddot{\bar{\kappa}}(\mathbf{x}) + \bar{\kappa}(\mathbf{x}) - \frac{R^2}{2}\Delta\bar{\kappa}(\mathbf{x}) = \kappa(\mathbf{x}) \tag{5}$$

Thus this approach to some extent decouples strain and damage in time and delays damage increments with respect to strain increments.

4. Viscoelastic extension

Since higher strain rates are dominated by inertia effects in the microscopic consideration the lower parts are affected by slower time dependent straining phenomena like creep, swelling and relaxation. Rewriting Eqn. (1) considering the time dependency of the elastic behaviour leads to:

$$\boldsymbol{\sigma}(t) = (1-D)\cdot\mathbf{E}\cdot\boldsymbol{\varepsilon}(t)\,. \qquad (6)$$

With combining the well known Maxwell material model to the standard linear solid using the parallel extension of a elastic spring element G_1 to the serial Maxwell combination of the viscous damper η_2 and the viscoelastic spring G_2, the governing equation for the deviatoric part of eqn. (6) can be rewritten as

$$\boldsymbol{\tau} + p_0\dot{\boldsymbol{\tau}} = q_1\boldsymbol{\varepsilon} + q_0\dot{\boldsymbol{\varepsilon}} \qquad (7)$$

with $p_0 = \dfrac{G_2}{\eta_2}$, $q_1 = 2(G_1+G_2)$, $q_0 = \dfrac{2G_2G_1}{\eta_2}$.

Considering the shear modulus $G = E/(2+2\nu)$ with ν as the Poisson's ratio, the damage of the extended spring is integrated with $G_1 = G(1-D)$.

5. Application for uniaxial tensile and compressive loading

5.1. Split Hopkinson pressure bar (SHPB) numerical setup

A 50 mm cylindrical concrete specimen was placed between two aluminium bars and subjected to a short loading impulse. The aluminium bars were shortened for numerical speed up. The acoustical impedances are characterized separately for each material and the loading function were assumed trapezoidal distributed over time with the amplitude *P(t)*, the flange duration $t_{f.}$ and the pulse duration t_i respectively. The load has been smoothed by an overlaid superelliptic function to avoid discontinuities. The strains were measured analogous to the experimental setup at the middle of the two bars and different material formulation and strain rates were observed.

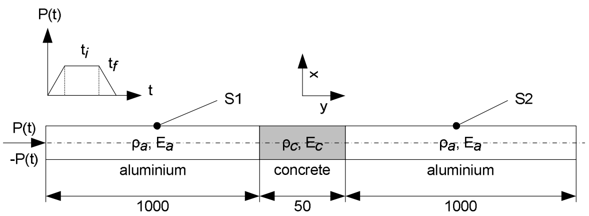

Figure 2: Split Hopkinson pressure bar test with simplified geometrical setup

5.2. Numerical results

Figure (3) is comparing a subset of the resultant transmitted wave shapes for different material formulations as it can measure from SHPB experiments in the same way. It can be shown, that the viscoelastic formulation and the delayed damage approach lead to different stress transmittance through the specimen. This is caused by the delayed straining and strain rearrangement within the specimen which is additionally overlaid by several wave reflections at the specimen interfaces. The stress-strain relation at different strain rates and amplitudes can be determined by the mean values of the introduced ε_i, reflected ε_r and transmitted ε_t strain distributions with

$$\sigma_m = E\frac{A_{spec}}{A_{bar}}\varepsilon_t \quad \varepsilon_m = -\frac{2C_0}{L_{spec}}\int_0^t \varepsilon_r \quad \dot{\varepsilon}_s = \frac{-2C_0}{L}\varepsilon_r\,, \qquad (8)$$

where C_0 is the materials sound speed, A the bars and specimens cross sectional area and E the elastic modulus. Thus deriving material strength for each strain rate can be easily done with regarding the averaging effect of this experimental method.

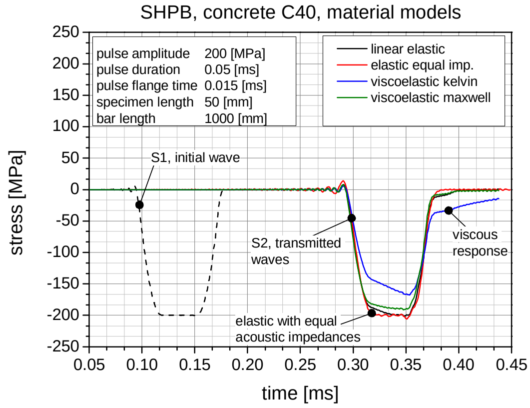

Figure 3: Initial and transmitted wave shapes for different material formulations at constant load.

6. Conclusion

The approaches of inertially delayed damage in combination with the viscoelastic formulation may describe strain rate effects of quasi-brittle materials like concrete, while ensuring regularization of the localization behaviour. The damage formulation bases on the assumption that micro cracking cannot spread out arbitrarily fast which leads to dynamic strength increase factors which will confirm to experimental observations at higher strain rate. Otherwise the additional viscous formulation is affecting the wave propagation and leads to delay effects at lower strain rates which can be causative for the increasing DIF factor at lower rates.

References

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