Stochastic finite element method study on temperature sensitivity of the steel telecommunications towers

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Abstract

This paper addresses an important question in structural analysis how to efficiently model the uncertain temperature influence on the internal forces in particular structural elements of the steel telecommunication towers. The computational methodology proposed here is based on the traditional Finite Element Method enriched with the generalized stochastic perturbation technique and the computational implementation is performed using the FEM commercial system ROBOT jointly with the symbolic algebra computer system MAPLE. Contrary to the previous straightforward differentiation techniques, now the local response function method is employed to compute any order probabilistic moments of the structural state functions. The response function is assumed in the polynomial form, whose coefficients are computed from the several solutions of the deterministic problem around the mean value of the random temperature. This method is illustrated with the example of telecommunications tower with a height equal to 52 meters modelled as the 3D truss structure under Gaussian random temperature load, which leads to determination of probabilistic moments of the internal forces into this system.

Keywords: stochastic finite element method, steel structures, telecommunication towers, reliability analysis

1. Introduction

The analysis of structures with random parameters plays an important role in structural design, optimization and reliability modelling. There are several well established both mathematical and numerical models enabling for inclusion of randomness in design parameters into the structural statics and dynamics problems solutions [1,6]. Starting from the analytical approaches based on the response spectrum analysis, through the crude Monte-Carlo stochastic simulation until the computational spectral methods based on the Karhunen-Loeve or polynomial chaos expansions of the input random fields. On the other hand, there are some lower order stochastic perturbation methods, however they have fundamental limitations on the input random dispersion level, so that their application to the real engineering problems seems to be rather limited. This new method is based on the Taylor expansion of any desired order with random coefficients of all uncertain parameters and state functions around their expected values. The second new idea here is an application of the response function method in conjunction with this generalized stochastic perturbation technique. We suppose that the structural response may be represented via some polynomial form of the input random parameter, whose order strongly depends on the final approximation error. The coefficients for this response polynomium are computed from the several solutions of the original problem obtained for this parameter values taken around its mean value. The polynomial form of the response function leads to analytical determination of its partial derivatives with respect to the input random parameter, which can be finally employed for a symbolic determination of the probabilistic moments; it essentially differs from the previous straightforward solution to the equations of the increasing order.

The entire procedure is applied to analyze the steel telecommunication tower in 3D stress state with random temperature load applied uniformly on all the structural members. It shows that the generalized stochastic perturbation method converges relatively fast (eight and tenth order approaches return almost the same results) and enables for final determination of the reliability indices.

2. The governing equations of the problem

We consider the boundary value problem of the linear elastostatics consisting of the following equations in the presence of random temperature T:

- equilibrium equations

$$D_{jk}\sigma_k(T)+\hat{f}_j(T)=0,\quad x\in\Omega,\quad j,k\equiv x,y,z, \tag{1}$$

- constitutive equations

$$\sigma_k(T)=C_{kj}\varepsilon_j(T),\quad x\in\Omega,\quad j,k=x,y,z, \tag{2}$$

- geometrical relations

$$\varepsilon_k(T)=D_{kj}u_j(T),\quad x\in\Omega,\quad j,k=x,y,z, \tag{3}$$

- Dirichlet boundary conditions

$$u_j=\hat{u}_j,\quad x\in\partial\Omega_u, \tag{4}$$

- von Neumann boundary conditions

$$N_{jk}\sigma_k(T)=\hat{t}_j(T),\quad x\in\partial\Omega_u. \tag{5}$$

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The basic idea of the stochastic perturbation approach is to expand all input variables and all the state functions of the given problem via Taylor series about their spatial expectations using some small parameter ε>0, so that the following expression is employed for a random temperature T and the state variable q:

$$q = q^0 + \sum_{n=1}^{\infty} \frac{1}{n!} \varepsilon^n \frac{\partial^n q}{\partial T^n} (\Delta T)^n \tag{6}$$

where

$$\varepsilon \Delta T = \varepsilon \left(T - T^0\right) \tag{7}$$

$$\varepsilon^2 (\Delta T)^2 = \varepsilon^2 \left(T - T^0\right)^2 \tag{8}$$

denote the first and the second variation of T about T^0. A symbol $(.)^0$ represents the considered function value taken at the expectation of random input; all partial derivatives are evaluated at the expectations. Then, the expected value of any state function $f(T)$ may be derived from the definition using the expansion (6) in the following way [3,4]:

$$E[q(T)] = \int_{-\infty}^{+\infty} \left(q^0 + \sum_{n=1}^{\infty} \frac{1}{n!} \varepsilon^n \frac{\partial^n q}{\partial T^n} (\Delta T)^n \right) p(T) dT \tag{9}$$

where $E[q(T)]$ denotes the expected value of the function $q(T)$ and where $p(T)$ denotes the probability density function of external temperature. Quite similarly one can find the higher order probabilistic moments of the structural response. It is important to mention that the integration provided in above equation is never carried out with infinite upper and lower bounds. Usually it is done for some real values reflecting the physical meaning of the given random input variable. Analogous issue is an expansion in the same relation, which most frequently is provided up to the tenth order only and is a subject of the additional numerical verification. Finally, we apply the perturbation parameter ε=1, although inclusion of the analytic expansion with respect to this parameter is also natural as far as the computer algebra system assists in all the computational procedures. We apply the formula (4) with the symmetry assumption and using the integral definitions of all probabilistic moments of the input random variable to calculate the expectation as

$$E[q(T)] = q^0(T) + \tfrac{1}{2}\varepsilon^2 \frac{\partial^2 q}{\partial T^2} \mu_2(T) + \tfrac{1}{4!}\varepsilon^4 \frac{\partial^4 q}{\partial T^4} \mu_4(T) + \tfrac{1}{6!}\varepsilon^6 \frac{\partial^6 q}{\partial T^6} \mu_6(T) + ... \tag{10}$$

A classical definition of the variance leads to the coefficient of variation illustrating a random dispersion of the temperature; there holds

$$\alpha[q(T)] = \frac{\sqrt{\int_{-\infty}^{+\infty} (q(T) - E[q(T)])^2 \; p(T) dT}}{\int_{-\infty}^{+\infty} \left(q^0 + \sum_{n=1}^{\infty} \frac{1}{n!} \varepsilon^n \frac{\partial^n q}{\partial T^n} (\Delta T)^n \right) p(T) dT} \tag{11}$$

Higher order moments and characteristics, like the third and the fourth one are determined to recognize the type of probabilistic distribution of the structural state functions. One calculates the skewness and kurtosis from the following relations, where also stochastic perturbation together with integral definitions of the moments enable for the discrete formulas. We compute in turn the skewness coefficients as the ratio of the third central probabilistic moment and the third power of the standard deviation in the following way:

$$S = \frac{\int_{-\infty}^{+\infty} (q(T) - E[q(T)])^3 \; p(T) dT}{\left[\sqrt{\int_{-\infty}^{+\infty} (q(T) - E[q(T)])^2 \; p(T) dT} \right]^3} \tag{12}$$

and kurtosis as

$$K = \frac{\int_{-\infty}^{+\infty} (q(T) - E[q(T)])^4 \; p(T) dT}{\left[\sqrt{\int_{-\infty}^{+\infty} (q(T) - E[q(T)])^2 \; p(T) dT} \right]^4} - 3 \tag{13}$$

As far as the perturbation-based approach is implemented, a reliable and efficient determination of higher order probabilistic moments may demand relatively larger expansions than the expectations but we apply only lowest order terms to recognize the main tendencies. It should be mentioned that at this stage the proposed procedure is independent from a choice of the initial probability distribution function (one can employ the lognormal one, for instance [5]), however a satisfactory probabilistic convergence of the final results may demand various lengths of the expansions for random variables with different distributions.

3. Computational implementation

As it is well-known, the elastostatic analysis in the Generalized perturbation-based Stochastic Finite Element Method implemented with the use of the response function approach proceed using the following deterministic algebraic equations series:

$$\mathbf{K}^{\alpha}(T)\mathbf{q}^{\alpha}(T) = \mathbf{R}^{\alpha}, \tag{14}$$

where α=1,…,n indexes all iterations necessary for the response function approximation, $\mathbf{K}$ and $\mathbf{R}$ denote stiffness matrix and the vector of nodal forces. Let us remind that this is applied contrary to the classical Direct Differentiation Method, where only a single zeroth order equation, also of the deterministic nature, is solved

$$\mathbf{K}^{0}(T)\mathbf{q}^{0}(T) = \mathbf{Q}^{0}, \tag{15}$$

and this process is continued for higher order equations having the recursive form given below

$$\sum_{n=1}^{n} \binom{n}{k} \frac{\partial^{n-k}\mathbf{K}(T)}{\partial T^{n-k}} \frac{\partial^{k}\mathbf{q}(T)}{\partial T^{k}} = \frac{\partial^n \mathbf{Q}}{\partial T^n} \tag{16}$$

The response functions between the displacement vector components and random temperature are assumed in the following polynomial form [3,4]:

$$q_{\beta}(T) = A_1^{(\beta)} T^{n-1} + A_2^{(\beta)} T^{n-2} + ... + A_n^{(\beta)} T^0, \;\; \beta = 1,..., N \tag{17}$$

where N is the total number of degrees of freedom. These response functions are found using the few traditional FEM tests with varying value of the randomized parameter and, at the same time, some classical approximation procedures implemented in any computer algebra system. Finally, we calculate the partial derivatives necessary for probabilistic moments evaluation from the recursive formula:

$$\frac{\partial^k q_\beta}{\partial b^k} = \prod_{i=1}^{k} (n-i) A_1^{(\beta)} b^{n-k} + \ldots + A_{n-k}^{(\beta)}. \tag{18}$$

It should be underlined that randomness in the environmental temperature is much more difficult in such an analysis because practically temperature as the physical parameter may exhibit enormously large random fluctuations; they are for sure significantly larger than for traditional structural parameters. Therefore, a usage of the stochastic perturbation-based technique must proceed with as many orders as it is justified by the probabilistic convergence analysis. On the other hand, such an algorithm enables for any commercial or the academic FEM codes application [2].

4. Numerical experiments

Computer analysis was performed using 3D model of the telecommunication tower with 52,0 meters height and this tower was subjected to the following loadings: self-weight, constant and technology loads all adopted according to the Eurocodes. This model consists of 72 two-noded beam elements with linear shape functions and 138 two-noded elastic 3D truss elements joined in 77 nodal points (see Fig. 1), which was a subject of the eigenfrequency analysis with random parameters before [5].

Random parameter in this analysis is the temperature load of all the structural elements varying in the range [-80°C, 50°C]. The temperature is treated as the Gaussian random variable with the expected value $E[T]$ equal to -15°C, while its standard deviation (through the coefficient of variation) is treated as the additional design parameter in this study. The FEM analysis is performed using the engineering system ROBOT and is utilized using the generalized stochastic perturbation technique thanks to the computer algebra system MAPLE, v. 13. First four probabilistic moments and the additional coefficients are computed for this tower as the functions of the coefficient of variation of the temperature varying in the range $\alpha(T)=(0.0,0.3)$.

Temperature deformation of the bars (Δl) in this system have been taken recalculated in each temperature using the well-known formula stating that the elongation of each member

$$\Delta l(T) = \alpha_T \cdot T \cdot l_0, \tag{19}$$

where α_T is thermal deformation of the steel and l_0 stands for the initial length of the considered structural element. Since the reference temperature is 0, then the explicit environmental temperature is included into this equation.

Therefore, taking this as the initial Gaussian random variable of the problem with the given first two probabilistic moments, one may easily calculate the probabilistic characteristics of the deformation of all bars. These are:

- the expected values

$$E[\Delta l(T)] = \alpha_T \cdot l_0 \cdot E[T], \tag{20}$$

- the variances

$$Var(\Delta l(T)) = \alpha_T^2 \cdot l_0^2 \cdot Var(T), \tag{21}$$

- k^{th} order central probabilistic moment

$$\mu_k(\Delta l(T)) = \alpha_T^k \cdot l_0^k \cdot \mu_k(T), \tag{22}$$

- the coefficient of variation, asymmetry and kurtosis

$$\alpha(\Delta l(T)) = \alpha(T),\ S(\Delta l(T)) = S(T),\ K(\Delta l(T)) = K(T). \tag{23}$$

A cycle of thirteen computational problems has been carried out to determine the extremum values of internal forces and displacements into the tower. The structure given schematically in Fig. 1 is loaded with the dead load only (excluding the wind effects) to show transparently the effects of the environmental temperature, whose uniform value is applied on all structural members; some deformed configurations are attached in Fig. 2.

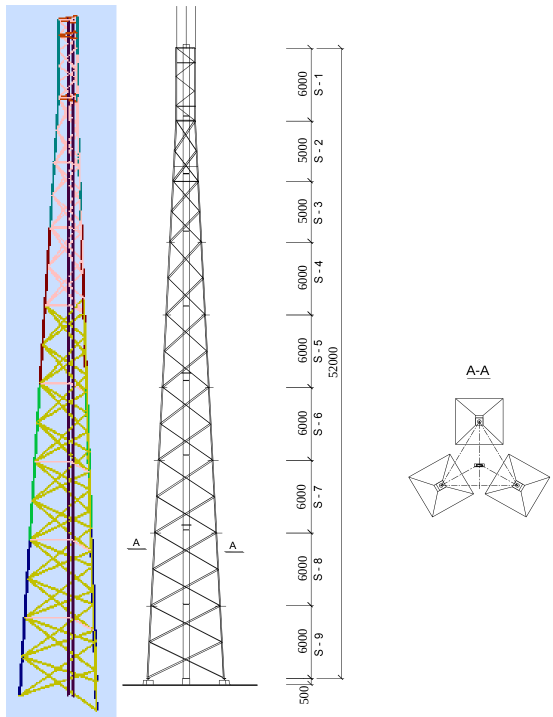

Fig. 1. FEM model of the telecommunication tower

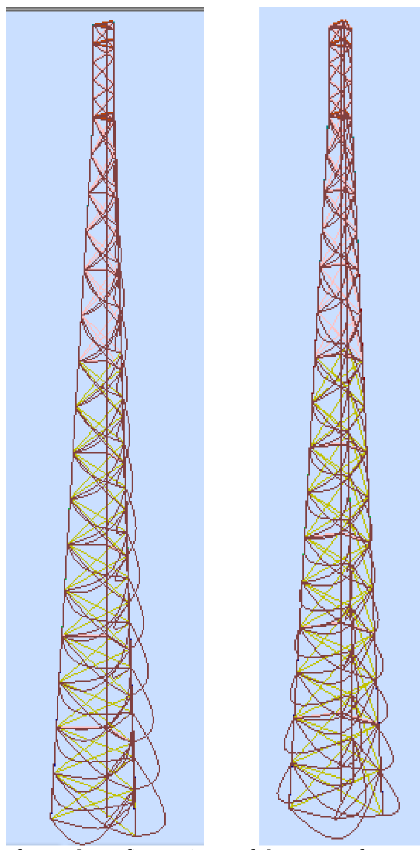

Fig. 2. Deformed configurations of the tower for T=-80, 50°C

Further analysis of the probabilistic moments is performed with respect to two most loaded bars – from segments 5 (around the half of this tower) and 9 (the lowest segment). From the structural point of view all the members are made of round cross-sections, Ø95 mm and Ø80 mm and we postpone at all local bending effects in this truss resulting from the bolted and welded joints.

The exemplary normal forces variations for those two bars are collected in Table no 1, where positive values are relevant to the compression state, whereas negative – to the tension members. It is quite clear from this table that the compression state in the lowest segment of the tower increases together with the external temperature the few times in the given temperature range (the highest temperature the larger compression). It should be underlined that some selective symbolic analysis is necessary for all large scale probabilistic computations considering the few parameters or functions to be to determined and drawn for each degree of freedom, so that full, pure computational reliability analysis without any pre-selection may be very time and space consuming, whereas final visualization – somewhat misleading.

Table 1. Normal forces variations versus temperature changes

Temperature	F_x in section 9 [kN]	F_x in section 5 [kN]
-50	27,93	1,38
-40	31,07	5,30
-30	34,20	9,22
-20	37,34	13,15
-10	40,47	17,07
0	43,60	20,99
10	46,76	24,92
20	49,87	28,84
30	53,00	32,76
40	56,14	36,69
50	59,27	40,61

Next, we notice the response functions relevant to the verified bars in two segments with respect to the input temperature within the entire temperature range. As one may expect this interrelation is totally almost linear, without any numerical discrepancies, local oscillations, especially at the edges of the computational domain. It guarantees very good probabilistic convergence of the basic characteristics for those forces at least. The discrete results coming from the FEM computations are marked in Figs. 3 and 4 with the black dots, while the MAPLE symbolic processing result is a continuous red line.

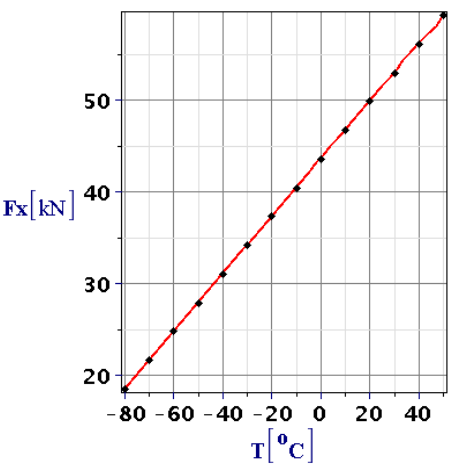

Fig. 3. The response function of the normal force, segment no 9

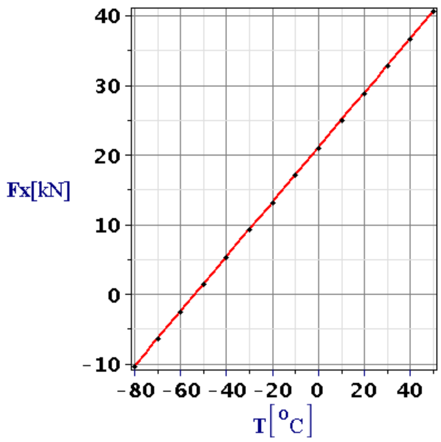

Fig. 4. The response function of the normal force, segment no 5

Using those functions we compute the expected values of the given forces for the mean value of the external temperature equal to -15°C (Figs. 5 and 6 with respect to segment no 9 and no 5). Up to 14th order expansions are compared to show an optimal length of Taylor expansion necessary in this particular case. Considering the fact that the overall differences are noticed in the total numerical error range and are not quite clear from those graphs, we have attached the additional Table no 2 equivalent to the input coefficient of variation α=0,3. So that neglecting those marginal fluctuations one can conclude that the second order is quite sufficient in the view of an accurate determination of these particular variables. Nevertheless, they somewhat decrease together with an increase of the input coefficient of variation marked almost everywhere on the horizontal axis. The differences in-between particular orders solutions are practically invisible in Figs. 5 and 6 starting from the 6th order, while consistently with initial conclusions, the second order method is efficient enough for $\alpha \le 0.10$.

Table 2. The expected values of normal forces computer for various temperatures

Perturbation method order	Expected values in the segment 9th	Expected values in the segment 5th
2	38,90525262	15,11018244
4	38,90589553	15,11044127
6	38,90580545	15,11041100
8	38,90581287	15,11041321
10	38,90581252	15,11041312

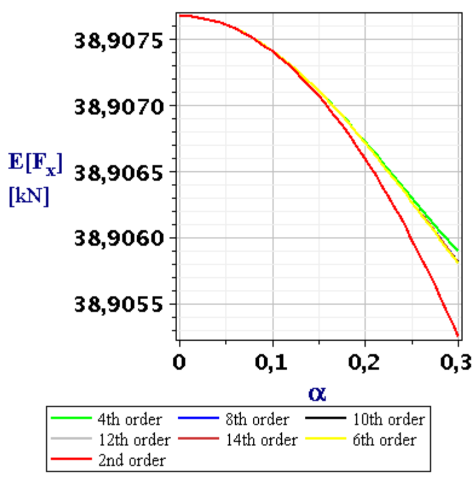

Fig. 5. The expected values of the normal forces, segment no 9

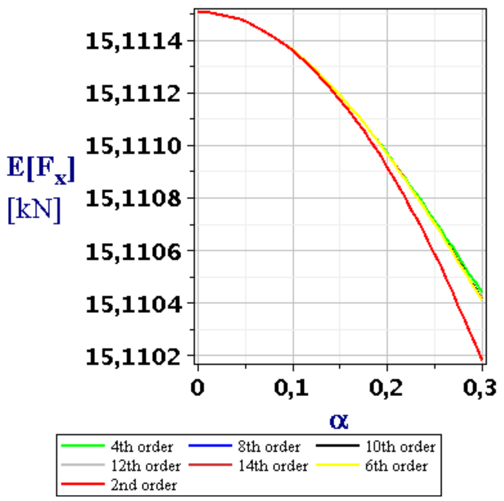

Fig. 6. The expected values of the normal forces, segment no 5

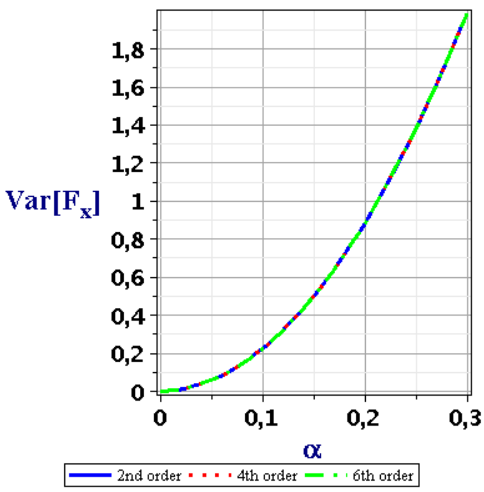

Fig. 7. The variance of the normal forces in the segment no 9

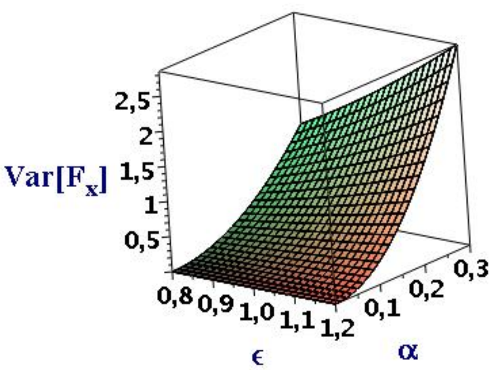

Fig. 8. The variance of the normal forces in the segment no 9 as the function of the perturbation parameter ε

Figures 7 and 8 show the variability of the most loaded element in segment no 9 with respect to both – input coefficient of variation as well as the perturbation parameter. Figure 6 shows that practically there is no difference in-between 2nd, 4th and 6th order approximations of this variance for input coefficient of variation taking the values $\alpha \in [0.0{,}0.3]$. Similarly to previous analyses in this area, an increase together with an additional increase of the input coefficient is nonlinear and represented by the convex curve. It starts from the beginning of the coordinates system, which perfectly reflects the fact that for zero input random dispersion the pure deterministic problem is obtained.

The perturbation parameter ε is of rather smaller importance and even for maximum value of the parameter α the final interrelation with the computed variance is almost linear, as it is illustrated in Fig. 8 above. It is worth to underline that the variability interval for the perturbation parameter is not so small, because it obeys around 20% fluctuations in plus and in minus. A simple consequence of the variances and the expected values computations is the output coefficient of variation given in Fig. 9, which is directly proportional to its input counterpart. Analyzing the particular values it is apparent that this system shows significant probabilistic damping having output coefficients almost ten times smaller than the additional input parameters. There is practically no difference in-between various orders of the stochastic perturbation-based analysis in this case (see Fig. 9).

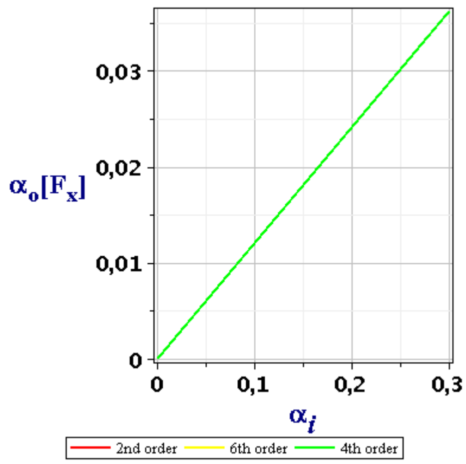

Fig. 9. Coefficient of variation of normal force, segment no 9

Finally, the kurtosis and skewnesses of the normal forces in the segments no 5 and 9 are presented computed according to the second, fourth and sixth order perturbation theories, also as a function of the input coefficient of variation, for $\alpha \in [0.0{,}0.3]$.

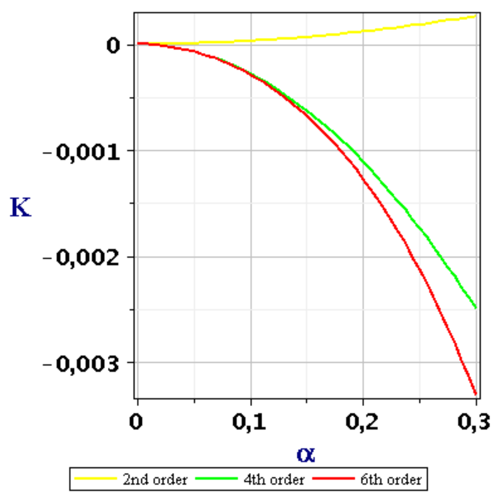

Fig. 10. Kurtosis of the normal forces in the segment no 9

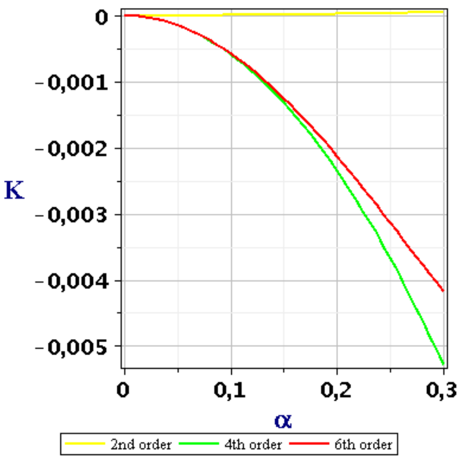

Fig. 11. Kurtosis of the normal forces in the segment no 5

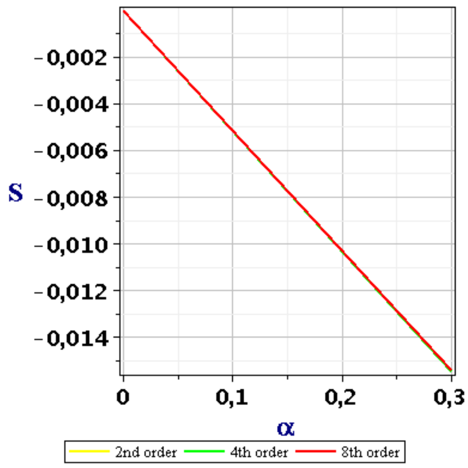

Fig. 12. Skewness of the normal forces in the segment no 9

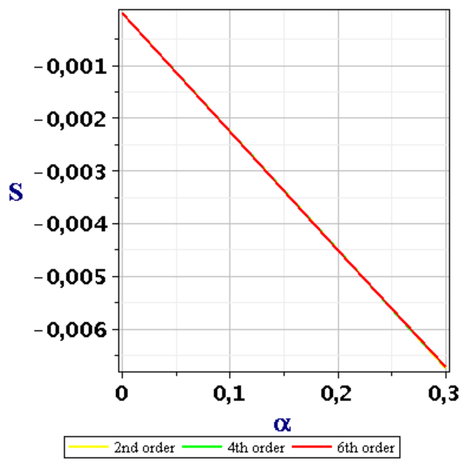

Fig. 13. Skewness of the normal forces in the segment no 5

The very apparent result in Figs. 12 and 13 is that both coefficients oscillate around 0, so that the output distribution may be treated with good accuracy as the Gaussian one. Determination of the skewness is so stable, that second, fourth and sixth order quantities are exactly the same and depends almost linearly upon the input coefficient of variation. This is a basic difference with respect to the Monte-Carlo analysis, where those two parameters are independent. Determination of kurtosis although needs higher order approximations, since the second order results in small positive values, while higher orders fluctuate into the clearly negative ones. Each time all those coefficients (at least their absolute values) increase together with the input coefficient of variation, which needs separate analysis and convergence computational studies.

5. Concluding remarks

Computational analysis provided in this paper documents that the temperature variations typical for our climate may have a significant influence on the overall internal forces into the high and tall spatial trusses like the telecommunication towers studied here. These forces fluctuations may result for higher segments in the sign change for the normal forces, although a randomness in temperature, according to the proposed variability range, has smaller importance in this context. The analyzed elastic system exhibits probabilistic damping since a ratio of output to the input coefficients of variation is decisively smaller than the unity. Nevertheless, the output state variables for the tower appear to be Gaussian provided that the random temperature is also Gaussian, but the method proposed is not restricted to this type of randomness. This conclusion is drawn on the basis of the skewness and kurtosis equal to 0 with relatively small numerical error.

This study may be extended next towards a fire simulation and its results on the reliability and integrity of the telecommunication towers, where temperature variations will intermediately affect also some material properties of the structural steel, so that the output randomness will have apparently nonlinear character. The interesting issues in this context would be also elastoplastic analysis of the tower structure, an application of the aluminium instead of the stainless steel as the basic material as well as forced dynamics of the entire spatial truss in the presence of some random parameters analyzed separately or as a single input vector with the partially correlated components.

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