Criterion for angle prediction for the crack in materials with random structure

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Abstract

Presented paper contains results of fracture analysis of brittle composite materials with a random distribution of grains. The composite structure has been modeled as an isotropic matrix that surrounds circular grains with random diameters and space position. Analyses were preformed for the rectangular finite element models. These models were generated using the authors' computer program *RandomGrain*. Fracture analyses were accomplished with the authors' computer program *CrackPath3* executing the "fine mesh window" technique. Calculations were preformed in 2D space assuming the plane stress state. Current efforts focus on brittle geo-materials such as rocks or concrete.

Keywords: numerical analysis, fracture mechanics, cracks, anisotropy, composites, concrete

1. Generating the random structure of the model

For generating the geometry of the model containing randomly spread inclusions surrounded with matrix material, authors propose the *Grains Neighbourhood Areas* algorithm (GNA) which creates models of the material in the way similar to the algorithm "larger first", proposed by Van Mier and Van Vliet[10], however GNA works much more quickly. In the proposed method three random numbers generators based on probability distribution function are used: uniform, normal (Gauss) and Fuller. The generator of the Fuller distribution was obtained from the cumulative function for Fuller sieve curve. Diameters of grains which are located in the space of the model are calculated by the Fuller generator. The generator of the uniform distribution is used for receiving the angle in the polar coordinate system which describes direction of grain location. The generator of the uniform distribution is used also for determining the distance of next grains in the case of A-type samples and Gauss generator in case of B-type samples.

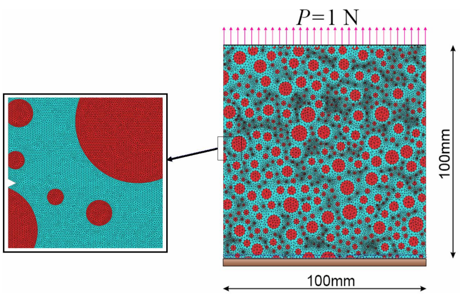

Figure 1: Boundary conditions and the random distribution of grains in the model sample

Every new grain is located in the neighbourhood of the previous grain. The area of the neighbourhood is defined as the circle with the set radius, divided in 6 sectors. In every vacant sector location of next random grains are tried. The process of positioning the grain assumes that polar coordinates in every sector are changing in the interval (α, r): $0° < \alpha \leq 60°$, $R_{min} \leq r \leq R_{max}$. If the generated grain location, are not colliding with the grain already existing in the model, the attempt is recognized as successful otherwise a next attempt is taken. The number of attempts N is one of parameters of the algorithm and it decides on the degree of packing of material. The structure received in this way is discretized in order to receive FE mesh.

2. Analysis of cracking

Analysis of cracking was performed using the authors' computer program *CrackPath3*, in which the technique of moving windows with the high density of the FE mesh was applied. This technique assumes the high density of the FE mesh in surroundings of the crack tip and the rare mesh in area away from the crack.

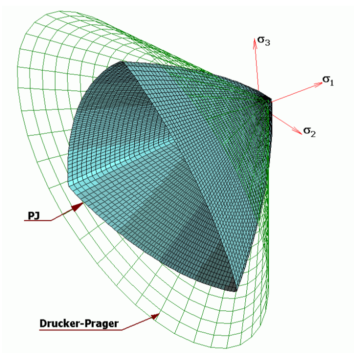

Figure 2: The limit surface associated with PJ criterion

Inside the window with fine mesh, material of composite is modeled as precisely as it is possible, while outside this window the composite is modeled as the homogeneous material with elastic characteristics determined in homogenizations procedures. The window with the fine FE mesh is moved with the top of the crack in every computational step or after a few steps (what shortens the computation time), in which position of the crack tip is being estimated (fig. 3). The point in which the crack is initiated is determined at each calculation step using PJ failure criterion described in earlier papers of one of the authors [4,5]. The shape of the limit surface associated with this condition is shown on fig. 2.

The technique of the moving window with fine mesh was presented in previous papers of the authors [7,8]. This simple re-meshing procedure considerably reduces (3 ÷ 4 of times) the

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of the numerical problem to solve what is related to reduction of the number of nodes in FE model.

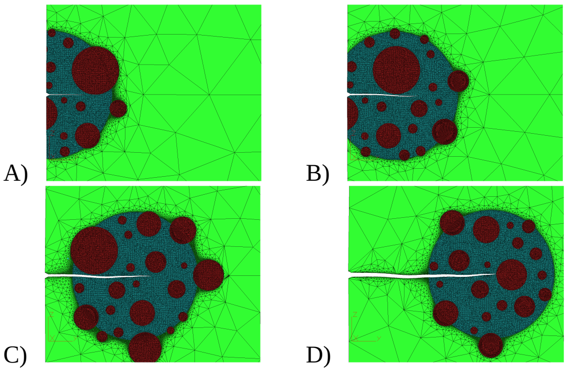

Figure 3: The view of crack propagation in the case of 4 windows with fine FE mesh

Analysis of the crack propagation in described "numerical sample" was made using the author's *CrackPath3* computer program and the technique of the fine mesh moving window. The program calculates the stress field using finite elements methods and then it seeks the point of the crack initiation on the basis of the *JP* criterion [4,5].This is the point of the highest value of the material effort (μ). The the crack is assumed to continue in direction of highest value of the μ ratio. The value of the material effort ratio μ is calculated based on the formula containing stress tensor components and material constants according to the *PJ* failure criterion.

$$\mu = \rho(\sigma) / \rho_o(\sigma) \qquad (1)$$

where ρ and ρ_o are radii in the stress space (see fig. 4)

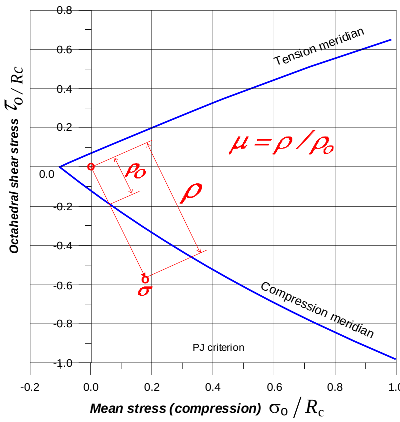

Figure 4: The Definition of the material effort ratio μ

After finding the direction of the crack propagation, a FE mesh is modified in surroundings of the crack tip in order to add the next crack segment with the length equal to the size of the incised element. The procedure is carried on until the demanded number of steps is achieved or the crack stops propagating [7,8].

Other methods of analysis of crack propagation in the heterogeneous materials were described e.g. in papers: Bažant [1], Carpinteri and others. [2], Mishnaevsky [3]. Other method of determining the direction of the crack propagation in polycrystalline material was described in paper of Sukumar and Srolovitz [9].

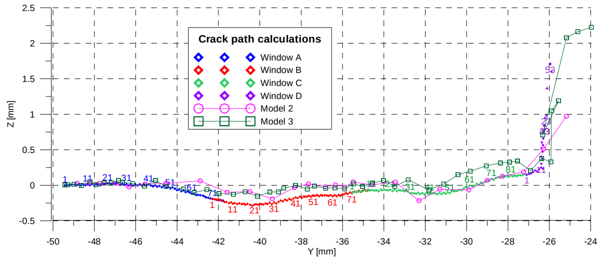

Figure 5: The path of the crack propagation

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