Identification of the parameters of the fractional rheological models of viscoelastic dampers using particle swarm optimization

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Abstract

An identification method for determination of the parameters of the rheological models of dampers made of viscoelastic material is presented. The models have three or four parameters and the model equations of motion contain derivatives of the fractional order. The results of experiments are approximated using the trigonometric function in the first part of the procedure while the model parameters are determined as the solution to an optimization problem. The particle swarm optimization method is used to solve the optimization problem. The validity and effectiveness of the suggested identification method have been tested using artificial and real experimental data.

Keywords: vibrations, viscoelasticity, material properties, identification, evolutionary methods

1. Introduction

Viscoelastic (VE) dampers have often been used to reduce excessive oscillations of building structures. A good mathematical model of dampers is required for the dynamic analysis of structures with VE dampers embedded in them. The rheological properties of the viscoelastic material dampers are made are the main factors which determine the dampers dynamic behaviour. The complex modulus of VE dampers depends on temperature and the frequency of vibrations.

Several rheological models were proposed to describe the dynamic behaviour of VE dampers, see [1, 2]. The simple rheological models are usually used to describe the rheological properties of VE dampers (see [3]) mounted on structures. However, these simple models are not able to correctly describe the dynamic behaviour of VE dampers. A correct description of the VE dampers requires generalised rheological models which increases the size of the problem arising in the analysis of structures with dampers.

Rheological models of dampers with fractional derivatives have received considerable attention. The reason is their ability to correctly describe the behaviour of VE dampers using a small number of parameters. These models have been used in [2].

An important problem connected with the fractional models, is the estimation of model parameters. The identification procedures for the fractional models are proposed in [2, 4].

A new method for identification of the parameters of the fractional model of VE dampers with three or four parameters is presented in this paper. It is assumed that the behavior of viscoelastic materials from which dampers is made can be described using the linear theory of viscoelasticity. Small strains of dampers are assumed. The results of dynamic tests are used to identify the parameters of a damper model. The validity, accuracy and effectiveness of the procedures have been tested using both artificial and real experimental data.

2. Description of rheological models

Rheological models with three or four parameters are considered. We introduce a fractional element called the springpot, which satisfies the following constitutive equation:

$$u(t) = c\, D_t^{\alpha} q(t) \ , \tag{1}$$

where c and α, $0 < \alpha \le 1$, are the springpot parameters and $D_t^{\alpha} q(t)$ is the fractional derivative of the Riemann-Liouville type of the order α of $q(t)$ with respect to time t.

The Kelvin type four-parameter model is a combination of the spring element and the fractional Kelvin element (see Fig. 1). The Maxwell model consists of the spring element and the fractional Maxwell element connected in parallel (see Fig. 2). Two fractional rheological models with three parameters, i.e., the fractional Kelvin model and the fractional Maxwell model are also considered. In the Kelvin model the spring and the springpot are connected in parallel while in the fractional Maxwell model the spring and the springpot are in series.

The equation of motion of the fourth parameters models are:

$$u(t) + \tau^{\alpha} D_t^{\alpha} u(t) = k_0 q(t) + k_{\infty} \tau^{\alpha} D_t^{\alpha} q(t) \ , \tag{2}$$

where k_0, k_{∞}, τ^{α} are model's parameters.

The equation of motion of the three-parameter models is:

$$u(t) = kq(t) + k\tau^{\alpha} D_t^{\alpha} q(t) \ , \tag{3}$$

$$u(t) + \tau^{\alpha} D_t^{\alpha} u(t) = k\tau^{\alpha} D_t^{\alpha} q(t) \ , \tag{4}$$

for the Kelvin and Maxwell model, respectively.

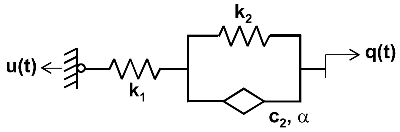

Figure 1: The Kelvin type four-parameter model of VE dampers

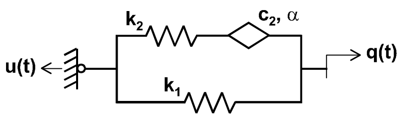

Figure 2: The Maxwell type model of VE dampers

It can be demonstrated that this model fulfils the second law of thermodynamics if $0 \le \alpha \le 1$, $\tau > 0$ and $k_{\infty} > k_0 > 0$.

If the damper's steady state vibration is analysed then

$$u(t) = u_c \cos \lambda t + u_s \sin \lambda t \ , \ q(t) = q_c \cos \lambda t + q_s \sin \lambda t \ , \tag{5}$$

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and the parameters u_c, u_s, q_c, q_s fulfil the relations:

$$u_c = z_1(\lambda)q_c + z_2(\lambda)q_s \ , \quad u_s = -z_2(\lambda)q_c + z_1(\lambda)q_s \ . \qquad (6)$$

3. Description of identification method

We assume that, during the experiments, two functions $u_e(t)$ (function of force in a time domain) and $q_e(t)$ (function of displacement in a time domain) were obtained. The damper is several times harmonically excited and in each case the excitation frequency λ_i, ($i = 1,2,..,n$), is different. The steady state response of the damper is measured, which means that the experimental damper displacements $q_{ei}(t)$ and the experimental damper forces $u_{ei}(t)$ are known for each excitation frequency. All experiments are performed at a constant ambient temperature and changes of damper's temperature during the tests are negligible.

The identification procedure consists of two main steps. In the first step the experimental results are approximated by a simple harmonic function of time while the model parameters are determined in the second stage of the identification procedure.

In the first step, experimentally measured displacements $q_e(t)$ of the damper and the dampers force $u_e(t)$ are approximated using the functions:

$$\tilde{q}(t) = \tilde{q}_c \cos\lambda t + \tilde{q}_s \sin\lambda t \ , \qquad (7)$$

$$\tilde{u}(t) = \tilde{u}_c \cos\lambda t + \tilde{u}_s \sin\lambda t \ . \qquad (8)$$

The least-square method is used to determine parameters $\tilde{q}_c$, $\tilde{q}_s$, $\tilde{u}_c$ and $\tilde{u}_s$. Part of the measuring results relating to steady state vibrations is used as data in this step.

In the second step, it is assumed that a set of results of the above-described first step of the procedure given by $\tilde{u}_{ci}$, $\tilde{u}_{si}$, $\tilde{q}_{ci}$ and $\tilde{q}_{si}$ and relating to the different excitation frequencies λ_i is known. If the considered rheological model is able to correctly simulate the VE damper behaviour then Eqns (6) must approximately be fulfilled by results of the first step identification procedure, i.e. (for $i = 1,2,...,n$):

$$\tilde{z}_{1i} = \frac{\tilde{u}_{ci}\tilde{q}_{ci} + \tilde{u}_{si}\tilde{q}_{si}}{\tilde{q}_{ci}^2 + \tilde{q}_{si}^2} \ , \qquad \tilde{z}_{2i} = \frac{\tilde{u}_{ci}\tilde{q}_{si} - \tilde{u}_{si}\tilde{q}_{ci}}{\tilde{q}_{ci}^2 + \tilde{q}_{si}^2} \ . \qquad (9)$$

If the rheological model perfectly fits the experimental data then $z_{1i} - \tilde{z}_{1i} = 0$ and $z_{2i} - \tilde{z}_{2i} = 0$ for $i = 1,2,...,n$, where $z_{1i} = z_1(\lambda_i)$, $z_{2i} = z_2(\lambda_i)$. In practice some differences exist and parameters of the model are determined as the solution to an appropriately defined optimization problem.

The mentioned optimization problem is formulated as:

Find the values of parameters of the considered model which minimize the functional

$$J = \sum_{i=1}^{n} \left[(z_{1i} - \tilde{z}_{1i})^2 + (z_{2i} - \tilde{z}_{2i})^2 \right] \ , \qquad (10)$$

and fulfil the constraints: $0 < \alpha \le 1$, $\tau > 0$, $k_\infty > k_0 > 0$ (in the case of the four-parameter models).

The above optimization problem is solved with the help of the particle swarm optimization method (PSO).

A calculation is performed using both artificially generated and real experimental data. For the artificial data the PSO algorithm was applied 5 times and almost exact values of models parameters are obtained for all the considered models of dampers. The PSO procedure was also applied to identify parameters of damper investigated by Makris [2]. The results obtained are presented in Figures 3 and 4 for the considered Maxwell models.

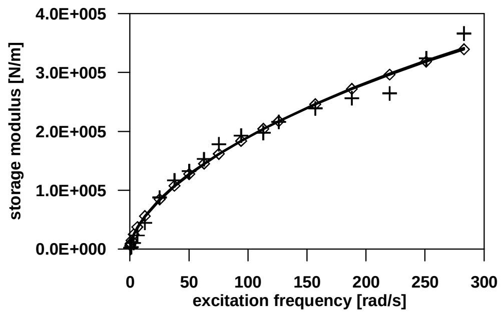

Figure 3 Comparison of the experimentally obtained storage modulus (small crosses) with ones obtained using the three-parameter Maxwell model (solid line with rhombuses) and using the four-parameter Maxwell model (solid line)

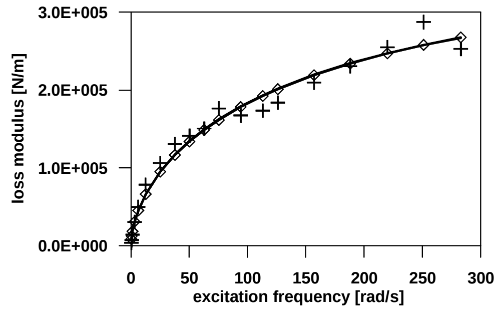

Figure 4 Comparison of the experimentally obtained loss modulus (small crosses) with ones obtained using the three-parameter Maxwell model (solid line with rhombus) and using the four-parameter Maxwell model (solid line)

4. Concluding remarks

The proposed identification method can be effectively used to determine the parameters of a group of rheological models with the fractional derivatives which can be used to modelling of VE dampers. The procedure seems to be quite general. The identification problem is reduced to the nonlinear optimization problem which is solved by means of PSO. Based on the calculation made for the artificial data, it was found that the method is not sensitive to measurements' noises.

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