Computational FE^2 scheme for heterogeneous shell structures

Thierry J. Massart[1], **Benoit C.N. Mercatoris**[2,*], **Peter Berke**[3,†], **Benoit Piezel**[4,‡]
Building, Architecture and Town Planning, CP 194/2, Université Libre de Bruxelles
Avenue F.D. Roosevelt 50, 1050 Brussels, Belgium
[1]*e-mail: thmassar@batir.ulb.ac.be,* [2]*e-mail: bmercato@ulb.ac.be,*
[3]*e-mail: pberke@batir.ulb.ac.be,* [4]*e-mail: bpiezel@batir.ulb.ac.be*

Abstract

A two-scale framework is presented for the failure of heterogenous shell structures, in which membrane-flexural couplings may appear. The texture and the behaviour of such materials and their constituents result in complex behaviours. This is the case for damage-induced anisotropy properties with localisation of damage in quasi-brittle materials, which are difficult to model by means of macroscopic closed-form constitutive laws; or for bending-driven deformation mechanisms in fiber reinforced composites such as woven composites. A periodic computational homogenisation procedure is presented for the in-plane and the out-of-plane behaviour of planar shells; and its incorporation in a FE^2-type computational scheme will be discussed by means of two physical applications. First, the deformation mechanisms of a woven-type fiber-reinforced composite classically used in conveyor belts is analysed. Secondly, the flexural cracking of periodic masonry plates is scrutinised using a localisation-enhanced framework, based on microstructure-informed embedded strong discontinuities incorporated in the shell description.

Keywords: multi-scale modelling, shell behaviour, flexural failure, damage, localisation

1. Introduction

The formulation of macroscopic constitutive relations for the behaviour of heterogeneous shells is complex, due to their textured microstructure which may considerably influence their overall mechanical behaviour. The number of material parameters needed to identify membrane and flexural effects as well as their coupling may quickly make closed-form approaches too complex and costly, especially when non-linear effects need to be incorporated. Non-linear descriptions are however crucial in a broad range of situations, such as for instance the behaviour of woven composites where flexural failure accompanied with damage may play an important role. When the constituents exhibit a quasi-brittle behaviour, the description may become even more complex as this may result in initial and damage-induced anisotropic flexural properties, accompanied with localisation of damage. This is the case in structures made of cracking-sensitive materials such as masonry. The multi-scale computational strategies aim at solving this issue by deducing a homogenised response at the structural scale from a representative volume element (RVE), based on constituents properties and averaging theorems.

2. Multi-scale modelling of shells structures

Computational homogenisation approaches allow identifying homogenised continuum properties from the constituents constitutive behaviour of a heterogeneous mesostructure. This allows the set-up of nested computational procedures in which a sample of the mesostructure is used to determine numerically the local macroscopic material response. The definition of such a nested scheme essentially requires the definition of: (i) a fine scale constitutive description for the constituents, (ii) the definition of a representative mesostructural sample, (iii) the choice of a macroscopic representation, and (iv) the set-up of scale transitions linking macroscopic and fine scale quantities. Scale transitions for homogenisation towards a Kirchhoff-Love shell behaviour were recently proposed in [1, 2]. In [1], the microstructure is represented by a unit cell on which a strain-periodic displacement field is imposed. The constituents inside the unit cell may be modelled using any closed-form formulation, depending on the physics to represent. As a result, the response of a coarse scale point under any loading program may be computed. Such a unit cell is sketched in Figure 1 for the case of a woven composite.

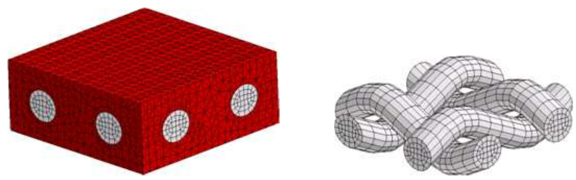

Figure 1: Periodic RVE for a woven composite (left) full RVE mesh with (right) fiber reinforcement discretisation. The unit cell consists of a single period RVE

Based on such a periodic unit cell with periodic boundary conditions and averaging theorems, displacements at controlling points as well as the corresponding reaction forces allow controlling the shell generalised strains (average deformations and curvatures tensors) and stresses (resultant forces and moment tensors) as represented in Figure 2 [3].

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[*] F.R.S.-FNRS post-doctoral fellow
[†] F.R.S.-FNRS post-doctoral fellow
[‡] F.R.S.-FNRS post-doctoral collaborator

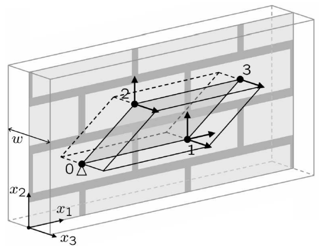

Figure 2: Periodic unit cell for masonry with controlling points

3. Upscaling for localising behaviour

In order to account for damage localisation effects at both scales, the scale transition procedures have to be adapted accordingly. When both fine and coarse scale descriptions follow similar kinematical assumptions, these adaptations have been proposed recently [4]. This approach can be extended to shell formulations, where higher order kinematical quantities such as curvatures appear at the structural scale. This requires adaptations within the structural scale description as well as in the scale transitions. Any localisation enhancement at the coarse scale to represent failure requires a criterion to detect localisation and to determine its orientation. This criterion should be based on computationally homogenised results. For thin shell structures, the detection of the structural scale localisation can be based an extended acoustic tensor concept [5]. The use of this tensor can be shown to extract mesostructurally motivated average localisation orientations for various coupled flexural-membrane loading paths [3].

At the macroscopic scale, an embedded strong discontinuity model finite element formulation is used [6]. Once structural localisation is detected, the shell macroscopic scale displacement field is enriched by jumps along a discontinuity line, the orientation of which is deduced from the acoustic tensor-based criterion. The weak form of equilibrium is then solved together with a weak continuity condition on the stress along the discontinuity line. The material response between the generalised stresses across the discontinuity to the jumps is required. Further to the recently proposed discontinuity approach where the material behaviour is defined by closed-form laws [6], the bulk and discontinuity material behaviours are deduced here from fine scale unit cell computations. The material behaviour of the coarse scale discontinuity is extracted from a further damaging unit cell, denoted as Localising Volume Element (LVE), by means of an enhanced upscaling procedure. This extraction requires the definition of average generalised strains to be applied on the LVE from the coarse scale displacement and rotation jumps. An enhanced upscaling procedure based on an approximate energy consistency has been proposed recently for the in-plane case [4] and is extended to the out-of-plane case [7].

Such a multi-scale scheme can be implemented using parallel computation tools. Structural multi-scale computations using these tools will be illustrated for the woven composite and the structural masonry applications. The methodology will be assessed by comparing multi-scale simulation results to fine scale computations used as a reference. Such an illustration is given in Figure 3 for the case of stair-case out-of-plane failure mode propagation on a square masonry shell subjected to out-of-plane forces. The overall response of the wall of the full fine scale and multi-scale computations are in good agreement.

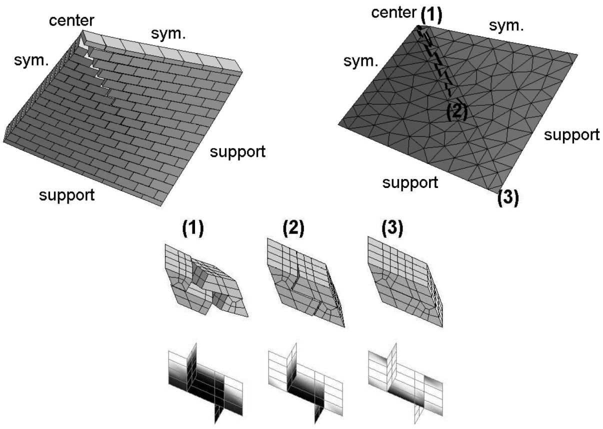

Figure 3: Configuration at peak load of a wall subjected to a central load (displacements magnified by a factor of 500): (top-left) deformed configuration of the complete 3D fine scale computation, (top-right) deformed configuration of the multi-scale shell computation and (bottom) related deformed unit cells.

4. Conclusions

The multi-scale methodology proves to be a valuable tool for the investigation of heterogeneous shell structures. In particular, it allows to account for the strong coupling between the structural response and the underlying mesostructural features of the material. Specific enhancements are however needed in order to account properly for the consequences of the failure characteristics of the constituents.

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