

## Moving load passage optimization via semi-active control system

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### Abstract

In this paper the optimal control system for straight passage of moving load is considered. The magnitude of the moving force is assumed to be constant by neglecting inertial forces. The response of the system is solved in modal space. The idea of semi-active control manner for 1D continuum subjected to travelling load was first based on the numerical investigations. Then, the switching control strategy was formulated. The methods of computing the optimal switching times was developed by means of adjoint state. We present the efficient way of calculating such a switching times. The effect of pre-deflected guideway is also considered. Several examples demonstrate the efficiency of the proposed techniques. The controlled system widely outperforms passive solutions.

Keywords: optimization, beams, sandwich structures, vibrations, industrial problems

### 1. Introduction

Technological processes such as cutting (flame, plasma, laser, textile, waterjet, glass cutting) or bonding (glueing, welding, soldering) require precisely straight passage. Active methods of control are, unfortunately, energy-consuming and complicated in practical applications. Moreover, a poor control system can supply energy in the antiphase and in extreme cases can damage the structure. We will focus our research on semi-active systems composed of dampers, which require lower energetic effort.

Most of the active and semi-active developed methods lead to feedback controls determined by state-space measures. In the case of a continuous system, such an approach is typically complex due to observer design. The alternative method is pre-computed open loop control. This is particularly useful in problems with a well defined excitation. In linear mechanical systems semi-active control methods usually result in switching operations, where parameters to be controlled (damping, stiffness) are switched between two or more values. Typically, the optimal switching pattern results in a large number of switchings. If the error occurs and the switching pattern is shifted in time domain, then such a complicated control may immediately drive the system to undesired or even unstable state. The aim of the approach presented in the paper is to design an effective and safety switching method with reduced number of switchings. The paper is an extension of the previous work published in the articles [1], [2], [3].

### 2. Mathematical model

We consider the system composed of simply supported Bernoulli-Euler beam, supported by a set of control dampers (Figure 1). The magnitude of the moving force  $P$  is taken as constant by neglecting the inertial forces. This is under the assumption that the mass accompanying the travelling load is small compared to the mass of the beam. The speed of travelling load

is  $v$ . The action of the massless dampers is proportional to the relative velocities of displacements at given points.

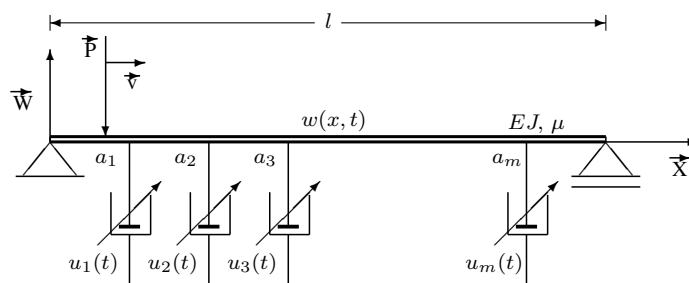


Figure 1: Euler - Bernoulli beam system supported by active viscous dampers.

For such a system, we can write equations of motion as follows:

$$\begin{cases} EI \frac{\partial^4 w(x, t)}{\partial x^4} + \mu \frac{\partial^2 w(x, t)}{\partial t^2} = P \delta(x - vt) + \\ - \sum_{i=1}^m u_i(t) \frac{\partial w(x, t)}{\partial t} \delta(x - a_i), \\ w(x = 0, t) = 0, \quad w(x = l, t) = 0, \\ w(x, t = 0) = 0, \quad \dot{w}(x, t = 0) = 0. \end{cases} \quad (1)$$

Here,  $w(x, t)$  is the transverse deflection of the beam in the point  $(x, t)$ . The parameters of beam are  $EI$ ,  $\mu$  and  $l$  that stand for bending stiffness, constant mass density per unit length and total length of the beam, respectively.

Respecting the boundary conditions we can use a Fourier series method to transform PDE 1 to a system of ODEs.

### 3. Formulation of optimal control problem

For the sake of optimal control problem formulation we consider the system given in the state space form. The straight passage can be achieved by minimizing the relevant norm of beam deflection under a moving load. The problem can be formulated as follows

$$\text{Minimize } J = \int_0^{t_f} [w(vt, t)]^2 = \tag{2}$$

$$\int_0^{t_f} \left[ \sum_{k=1}^{n/4} y_{4k-3}(t) \sin \frac{k\pi vt}{l} \right]^2 dt, \tag{3}$$

$$\text{subject to } \dot{\mathbf{y}}(t) = \mathbf{A} \mathbf{y}(t) + \sum_{i=1}^m \mathbf{B}_i \mathbf{y}(t) u_i(t) + \mathbf{f}(t),$$

$$y_{4k-3}(0) = V_1(k, 0), y_{4k-2}(0) = \dot{V}_1(k, 0),$$

$$y_{4k-1}(0) = V_2(k, 0), y_{4k}(0) = \dot{V}_2(k, 0), k = 1, 2, \dots, n/4$$

$$u_i(t) \in [0, u_{max}], \forall t \in [0, t_f], i = 1, 2, \dots, m. \tag{4}$$

To solve the problem we introduce the adjoint state and calculate the derivative of cost functional. Then, the numerical procedures based on gradient values can be evaluated.

### 4. Numerical results

By applying the Maximum Pontryagin Principle [5] we can easily check that the optimal solution of 4 is driven by so called bang-bang controls. Below we present the numerical examples in case of two (Figure 3), three (Figure 4) and four (Figure 2) active dampers. By the passive case we mean the constant damping set on maximum value.

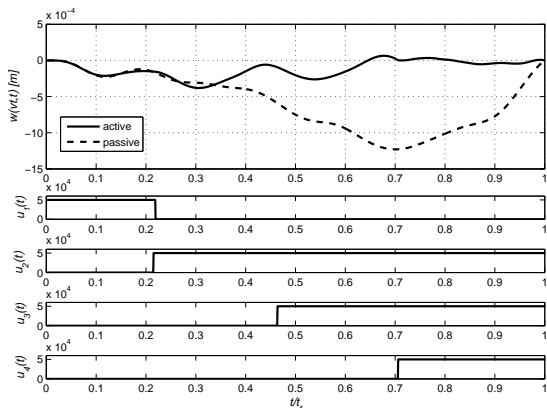


Figure 2: Extremal deflection trajectory and controls. Four active dampers.

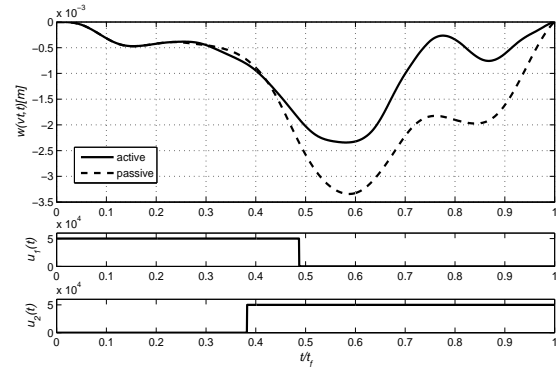


Figure 3: Extremal deflection trajectory and controls. Three active dampers.

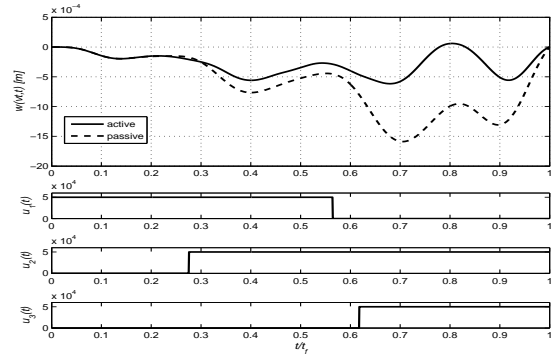


Figure 4: Extremal deflection trajectory and controls. Three active dampers.

A performance of bang-bang control method has been verified for different system parameters and different number of active dampers. The dampers placements significantly affect the shape of the carriage trajectory. The best efficiency of proposed control method is obtained at high travel speeds. The controlled system can efficiently decrease the mass of the guideway. The advantage of control strategy is its simplicity for practical design.

### References

- [1] Pisarski, D. and Bajer, I. Cz., Semi-active Control of 1D Continuum Vibrations Under a Travelling Load, *Journal of Sound and Vibration*, 329(2), pp. 140-149, 2010.
- [2] Pisarski, D. and Bajer, I. Cz., Smart Suspension System for Linear Guideways, *Journal of Intelligent and Robotic Systems*, DOI: 10.1007/s10846-010-9450-7
- [3] Pisarski, D. and Bajer, I. Cz., On the Semi-Active Control of Carrying Structures Under a Travelling Load, *Vibrations in Physical Systems*, 24, pp. 325-330, 2010.
- [4] Mohler, R. R., Bilinear control processes, *Academic Press*, New York, 1973.
- [5] Pontryagin, L. S., Boltyanskii, V. G., Gamkrelidze, R. V., Mishchenko, E. F., The mathematical theory of optimal processes, *Fizmatgiz*, Moscow, 1961.