Synthesis of Line and Torsional Stiffness Parameters for Legs of 6DOF Parallel Mechanism

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Abstract
A novel algorithm for synthesis of the stiffness parameters of 6DOF mechanism legs with U-P-R-U joints is presented. A substitute compliance of any leg is modelled by a linear spring in P-joint and torsional spring in R-joint. The formulated algorithm was exemplified on the multilink suspension mechanism with given structure and dimensioning. Substitute compliances are defined for the suspension rods with joints made as cylindrical elastomeric bushings. Using the described approach it is possible to determine the bushings rates for different positions of the mechanism and load conditions. Numerical example is given.

Keywords: multibody dynamics, inverse problems, optimization

1. Introduction

In the paper a novel algorithm for synthesis of some stiffness parameters of a 6DOF parallel mechanism with known kinematical model is presented. The considered mechanism, shown in Fig. 1, belongs to the group of Stewart/Gough platforms Ref. [2], which have found a broad application in robotics, motion simulators, machining tools, measuring machines, and car suspensions among the others. Depending on the mechanism application, its effective compliance (reduced to end-effector) is influenced by various design parameters. The substitute compliance of the legs usually play the most significant role in this regard due to compliance of drive systems and joints (e.g. with elastomeric inserts).

Using the algorithm described herein, a designer can faster predict values of stiffness parameters of the mechanism legs (links), having defined some required characteristics of the mechanism spatial stiffness. Core of the procedure for synthesis of longitudinal compliance of the mechanism legs was described in Ref. [1]. In this paper the stated problem is further extended and generalized by taking into account also a torsional stiffness of the legs.

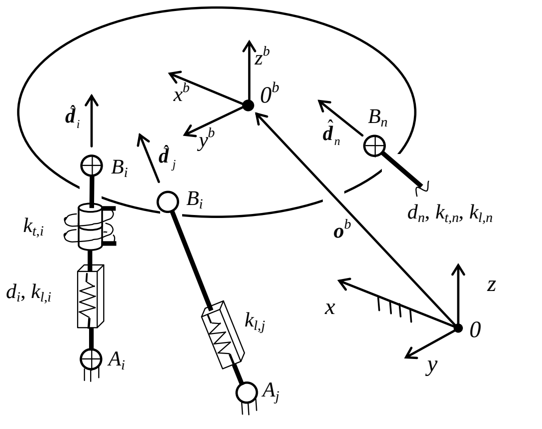

Figure 1: Rigid platform constrained by n-legs

2. Statement of the stiffness synthesis problem

Some legs (with i-index on Fig. 1) can be considered as a serial kinematic chain with U-R-P-U joints, where longitudinal and torsional compliance of the whole leg are modelled by a line (translational) spring in the P-joint and torsional spring in the R-joint, respectively. The U-joints are free to move without any resistance. Other legs (with j-index) can be treated as a degenerated case, e.g. with line-spring in P-joint only.

It is assumed that all the joints, links and platforms are ideal and fully dimensioned, the line and torsional springs are decoupled, non-preloaded, and described by a linear load-deflection characteristics. The given pose of moving platform is initially in static equilibrium. When the load state (wrench) exerted on the platform is changed with a selected magnitude and in quasi-static fashion, the moving platform undergoes an elastic displacement (twist) dependent on the structure compliance. Relation between twist ($\hat{\boldsymbol{t}}$) and wrench ($\hat{\boldsymbol{w}}$) is described by the mechanism stiffness matrix ($\boldsymbol{K}$), Ref. [2]:

$$\hat{\boldsymbol{w}} = \boldsymbol{K}\hat{\boldsymbol{t}} \tag{1}$$

The stiffness matrix (six-by-six) is defined as follows:

$$\boldsymbol{K} = \boldsymbol{J}^T \mathrm{diag}(\boldsymbol{k})\,\boldsymbol{J} \tag{2}$$

where: $\boldsymbol{J}$ denotes the mechanism Jacobian matrix, dependent on geometry only; $\boldsymbol{k}$ – vector of the legs' stiffness coefficients, composed of two parts, i.e. ($\boldsymbol{k}_\alpha$) coefficients that are to be determined and ($\boldsymbol{k}_\beta$) that are known.

The design requirements include a set of demanded twists ($\hat{\boldsymbol{t}}_d$) for defined wrenches ($\hat{\boldsymbol{w}}_d$), suitably selected for the mechanism application. The goal of synthesis is to find stiffness parameters ($\boldsymbol{k}_\alpha$) of the legs, for which the mechanism will realize the demanded twist with the smallest error. Eqn. (1) can be rearranged to the following form:

$$\hat{\boldsymbol{w}} - \boldsymbol{H}_\beta\,\boldsymbol{k}_\beta = \boldsymbol{H}_\alpha\,\boldsymbol{k}_\alpha \tag{3}$$

Eqn. (3) describes an analytical relation between unknown ($\boldsymbol{k}_\alpha$) and given parameters ($\boldsymbol{k}_\beta$, $\boldsymbol{J}$, $\hat{\boldsymbol{w}}_d$, $\hat{\boldsymbol{t}}_d$), therefore it can be used to solve the formulated problem.

*Footnotes may appear on the first page only to indicate research grant, sponsoring agency, etc. These should not be numbered but referred to by symbols, e.g. *,+. The footnote text may be produced in a small font.

In order to determine α-spring rates ($\boldsymbol{k}_\alpha$), suitable number ($s \geq \alpha$) of independent components of the measured platform twist under specified load must be given. Solution of Eqn. (3) can be directly determined by a linear solver, if only the design matrix ($\boldsymbol{H}_\alpha$) is full rank and well conditioned. However, in order to avoid impractical solutions the problem is solved by using the constrained linear least-squares algorithm, in the form of [3]:

$$\min_{\boldsymbol{k}_\alpha} \frac{1}{2}\left\| \hat{\boldsymbol{w}} - \boldsymbol{H}_\beta \boldsymbol{k}_\beta - \boldsymbol{H}_\alpha \boldsymbol{k}_\alpha \right\|_2^2 \tag{4}$$

Additionally, constraints of min-max type on the design variables are included here.

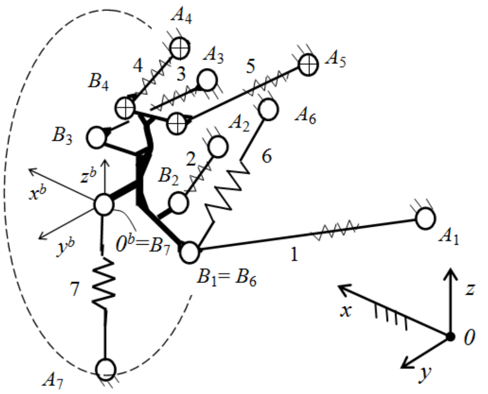

Figure 2: Elastokinematic model of the wheel guiding mechanism with compliant links (no1-5), main spring (no6) and tire (no7)

3. Numerical example

A numerical example is solved for a multi-link mechanism used for guiding a car wheels, Ref.1. Its elastokinematical model is shown in Fig. 2 for a rear (unsteerable) left wheel. The mechanism is composed of wheel carrier, wheel with a pneumatic tire, five rods made of steel sheet and joined by cylindrical elastomeric bushings, and the suspension coil spring. The wheel carrier is constrained by seven elastic legs.

The suspension spring is considered as the compliant leg no6 with the spring rate $k_{l,6}$. The road wheel is instantaneously modelled as the leg no7 (Fig. 2). Radial elasticity of the tire is modelled by a line spring element with the static rate $k_{l,7}$ (wheel ride rate). The spring rates of the tire and the suspension spring are assumed to be known.

The suspension mechanism can undergo small displacements of the wheel carrier under action of an external load due to elastic deflections of elastomeric bushings in the rods, and the rods itself. Radial stiffnesses of the rod-bushings are reduced to the rod longitudinal axis (Fig. 2) with substitute spring rates $k_{l,i}$ (i = 1, 2…5) [kN/m]. Torsional stiffness of the serial chain bushing-rod-bushing is determined as an equivalent torsional stiffness of the leg with substitute spring rates $k_{t,i}$ (i = 1, 2…5) [Nm/rad]. Considering the rods design, it is assumed that out of five rods only the rod no4 and 5 have a significant torsional stiffness. The remaining legs are without torsional stiffness (i.e. $k_{t,i}$ = 0 for i = 1, 2, 3, 6, 7). The following vector with seven design variables is to be determined:

$$\boldsymbol{k}_\alpha = [k_{l,1}\, k_{l,2}\, k_{l,3}\, k_{l,4}\, k_{l,5}\, k_{t,4}\, k_{t,5}]^T \tag{5}$$

Three road manoeuvres (M1, M2 and M3) are considered where a load changes with respect to design position are described be defined wrenches and elastic displacements of the 5-link mechanism with base line parameters are described by respective twists, Tab.1.

The synthesis goal is to reduce some components of the wheel carrier displacement, which are responsible for proper cooperation of the suspension with ABS system ($\hat{\boldsymbol{t}}_{15}$- carrier pitch angle during braking) and car stability ($\hat{\boldsymbol{t}}_{16}$- wheel toe angle during braking), Ref. [1]. These components have to high magnitude for the base line. Other significant components of the displacement ($\hat{\boldsymbol{t}}_{26}$ and $\hat{\boldsymbol{t}}_{36}$) should be kept without changes.

The values of the legs' stiffness parameters (5) obtained by using the proposed algorithm (4) are given in bottom of Tab.1.

Table. 1. Base line, desired and obtained data for 5-link susp.

Base line $\boldsymbol{k}_\alpha$ = 1e004*[0.7000 0.1849 1.3867 0.2343 0.1647 0.0573 0.0573]^T M1: pure braking $\hat{\boldsymbol{w}}_1$ = [-1000 0 0 0 280 0]^T $\hat{\boldsymbol{t}}_1$ = [0.0004 0.0003 0.0021 -0.0024 0.0125 -0.0009]^T M2: pure cornering $\hat{\boldsymbol{w}}_2$ = [0 -1000 0 -280 0 0]^T $\hat{\boldsymbol{t}}_2$ = [0.0005 0.0001 -0.0009 -0.0041 0.0032 -0.0002]^T M3: pure driving $\hat{\boldsymbol{w}}_3$ = [1000 0 0 0 0 0]^T $\hat{\boldsymbol{t}}_3$ = [0.0011 0.0001 -0.0007 -0.0021 0.0053 0.0002]^T
Desired Reduce: $\hat{\boldsymbol{t}}_{15}$ and $\hat{\boldsymbol{t}}_{16}$; while sustain: $\hat{\boldsymbol{t}}_{26}$ and $\hat{\boldsymbol{t}}_{36}$
Obtained $\boldsymbol{k}_\alpha$ = 1e007*[0.6377 0.2301 1.5746 0.2031 0.1760 0.0515 0.1719]^T $\hat{\boldsymbol{t}}_{15}$ = 0.0119 (9.5% reduce); $\hat{\boldsymbol{t}}_{16}$ = -0.0005 (40% reduce); $\hat{\boldsymbol{t}}_{26}$ = 0.00003; $\hat{\boldsymbol{t}}_{36}$ = 0.0002.

4. Conclusions

In the presented numerical example seven new parameters, describing line and torsional stiffnesses (5) of links of the wheel suspension mechanism, are determined using the proposed synthesis algorithm. Two components of the wheel carrier elastic displacements are reduced by ca. 10% ($\hat{\boldsymbol{t}}_{15}$) and 40% ($\hat{\boldsymbol{t}}_{16}$), what beneficially influences the car active safety. The designer can effectively predict the stiffness parameters for improving the mechanism elastokinematic characteritics.

The described approach can be also useful for various applications of Stewart/Gough platform.

References

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