

## Estimation of the finite element solution accuracy to the strand temperature field in the continuous casting process of steel.

Zbigniew Malinowski\* and Beata Hadala

Department of Heat Engineering and Environment Protection, AGH University of Science and Technology

al. Mickiewicza 30, 30-059 Kraków., Poland

e-mail: malinows@agh.edu.pl

e-mail: Beata.Hadala@agh.edu.pl

### Abstract

Three dimensional model of heat transfer in the continuous casting process has been developed and tested. Accuracy of the temperature field computation has been estimated based on the heat balance computed for strand cooling. Nonlinear, Hermitian shape function and weighting functions have been employed in the model. Effect of the solidification heat on the strand temperature has been analyzed. The heat balance in the control volume has been used in order to improve the solutions accuracy.

Keywords: heat transfer, finite element methods, error estimation

### 1. Introduction

Heat transfer in the continuous steel casting process is difficult to model. It is due to interaction of heat conduction and heat convection caused by a liquid steel flow. The problem can be treated as steady-state and can be well described by the convection-diffusion heat transfer equation. The finite element method is most widely used to solve the problem. However, in the case of convection dominated processes convergence is not obvious. The latent heat in the convection-diffusion heat transport equation makes the solution much more complicated [2]. Several methods have been proposed to overcome this difficulties. One of the most popular formulation uses the non symmetric weighting functions [3]. The method is efficient but in some cases solutions to the temperature field are not satisfactory. Stable and good looking results may not be accurate. It makes the interpretation of the computation results more complex. One of the possibilities to validate the solution accuracy to the temperature field gives the heat balance in the control volume.

### 2. Heat transfer model

The heat transfer while continuous casting of steel is a very rapid process, that involves cast metal, casting equipment and environment. The strand temperature field while cooling in casting mould, in secondary cooling zones and in air has been computed using steady solution to the convection-diffusion heat transport equation. To model such a process it is necessary to formulate suitable boundary conditions for each type of cooling which take place in the continuous casting line. Numerical model has been based on the weighted residual Galerkin method. This method yields the set of linear equations in the form

$$(K_{nn} + W_{nn})P_n = G_n \quad (1)$$

where  $n$  is number of unknowns and  $P_n$  are unknown parameters. Several formulations depending on the choice of weighting and shape functions can be developed to define the heat conductivity matrix  $K_{nn}$ , the heat convection matrix  $W_{nn}$

and the heat load vector  $G_n$ . In the developed three dimensional finite element model nonlinear Hermitian shape function and weighting functions have been employed [3]. In computational tests it has been noted that significant influence on solution stability has the solidification heat. Internal heat source  $q_v$  at element nodes can be calculated from

$$q_v = Q_s \frac{dV_s}{d\tau} \quad (2)$$

where  $Q_s$  is heat of solidification,  $V_s$  represents volume of solid phase and  $\tau$  is time. The solid phase volume has been computed from

$$V_s = 1 - \exp^{-K \frac{T_{li} - T}{T_{li} - T_{so}}} \quad (3)$$

where  $K$  is solidification kinetics constant,  $T$  is temperature,  $T_{li}$ ,  $T_{so}$  are liquidus and solidus temperature, respectively. In Eqn. (2) derivative of  $V_s$  with respect to time can be estimated from the finite difference approximation

$$q_v = Q_s \frac{\Delta V_s}{\Delta \tau} \quad (4)$$

With this approximation realistic results can be obtained in many technical problems in which internal heat source  $q_v$  is low if compared to the boundary heat flux. It is possible to use more complex method and calculate derivative of  $V_s$  taking into consideration solidified particle movement

$$\frac{dV_s}{d\tau} = \frac{\partial T}{\partial z} \frac{\partial V_s}{\partial T} \frac{\partial z}{\partial \tau} \quad (5)$$

Performing partial differentiation with respect to particle path, temperature and time we have

$$q_v = -Q_s \frac{\partial T}{\partial z} v_z \frac{K}{T_{li} - T_{so}} \exp^{\frac{T_{li} - T}{T_{li} - T_{so}}} \quad (6)$$

In Eqn. (6)  $z$  represents distance measured along the particle path and  $v_z$  is particle velocity. It should be noted that Eqn. (4) adds additional term to the load vector  $G$ . In the case of Eqn. (6) additional term has been added to the heat convection matrix  $W$ .

### 3. Results of computations

The computation tests have been performed to model heat transfer in the continuous casting line. The strand cross section of 100 mm × 100 mm and the mould length of 800 mm have been assumed for computations. The chemical composition of steel has been assumed as: 0.20% C, 0.50% Mn, 0.15% Si, 0.25% Cr and 0.35% Ni. Strand moves with the constant casting speed of 0.53 m/s. The inlet temperature of liquid steel is 1550°C. In Fig. 1 temperature variation in the strand axis has been presented. If heat of solidification is approximated by Eqn. (4) it destabilize the solution. Rapid solidification of steel in the mould results in unexpected temperature drop and temperature rise in the mould. Below the mould temperature oscillations have been obtained in the strand axis. The temperature variation resulting from heat source approximation by Eqn. (4) are not acceptable. If heat of solidification is ignored in the finite element model the temperature distribution in the strand axis is stable but significant portion of energy in the system is neglected. The developed new procedure defined by Eqn. (6) for the solidification heat handling has given stable solution to the temperature field. However, it is not obvious that the results are correct in full.

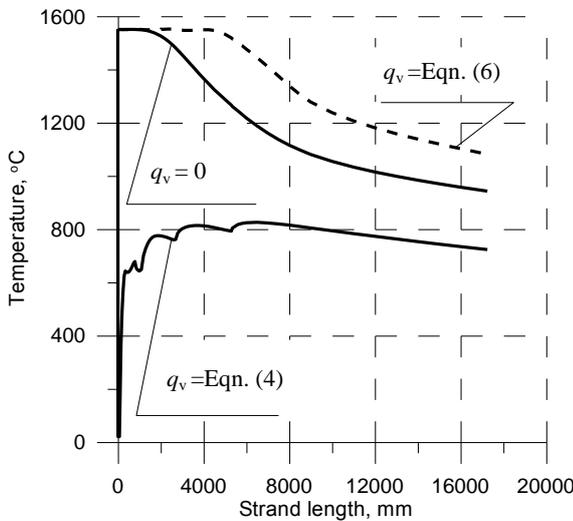


Figure 1: Temperature distribution in the axis of the continuously cast strand for various schemes of solidification heat handling.

In the case of steady - state heat flow it is possible to validate the solution accuracy based on the heat balance in the control volume given by

$$\dot{Q}_i - \dot{Q}_o + \dot{Q}_v - \dot{Q}_b = \Delta\dot{Q} \quad (7)$$

Equation (7) represents energy balance for the continuously cast strand. The difference between heat transfer rate at entry  $\dot{Q}_i$  and exit  $\dot{Q}_e$  to the control volume should be equal to the rate of energy generated inside the strand  $\dot{Q}_v$  minus heat transfer rate to environment  $\dot{Q}_b$ . Thus, the term  $\Delta\dot{Q}$  should be equal zero, otherwise the solution to the convection-diffusion heat transport equation is not accurate and the solution error can be calculated from

$$\Delta T_e = \Delta\dot{Q} / \int_{S_o} \rho c_p v_z dS \quad (8)$$

where  $\rho$  is density and  $c_p$  specific heat.

In Fig. 2 temperature distributions at the side surface of the strand for three tests have been presented. Stable and reasonable results have been obtained in all the tests. Some temperature variations along the strand in the secondary cooling zone are typical for the continuous casting process and result from the change in water cooling flow. All curves are similar and it would be not easy to asses which one is correct. If the heat of solidification is neglected, the lowest curve has been obtained. In this case the solution error calculated from the heat balance indicates that the temperature on the strip surface is too low of about 300°C. The new scheme of the internal heat generation given by Eqn. (6) has resulted in much better temperature approximation presented in Fig. 2 as the middle curve. However, the strand surface temperature is still to low of about 180°C. The error can be readily determined from Eqn. (8) if the heat balance is added to the finite element model. Several methods of weighting functions scaling have been developed [2] but the choice is les or more arbitrary. In the developed model the heat balance discrepancy  $\Delta\dot{Q}$  has been calculated and the weighting functions in the flow direction have been determined from

$$W_i = \left(1 - \beta \frac{\Delta\dot{Q}}{\dot{Q}_i}\right) N_i \quad (9)$$

where  $\beta$  is a correction factor which has to be iteratively adjusted.

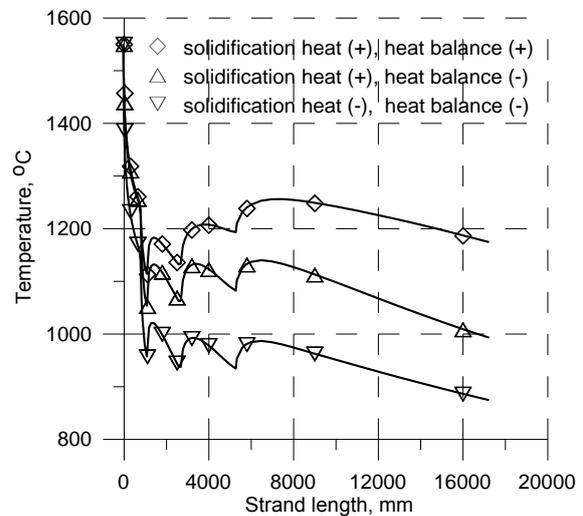


Figure 2: Temperature distributions at the side surface of the continuously cast strand for different finite element models.

### References

- [1] Hadała, B., Malinowski Z., Accuracy of the Finite Element Solution to Steady Convection-Diffusion Heat Transport Equation in Continuous Casting Problem, *Informatyka w Technologii Materiałów*, 9, pp. 302-308, 2009.
- [2] Pao, C.V., Numerical Methods for Time-Periodic Solutions of Nonlinear Parabolic Boundary Value Problems, *Journal of Numerical Analysis*, 2, pp. 647-667, 2001.
- [3] Zienkiewicz, O.C. and Taylor, R.L., *The Finite Element Method*, Vol. 3: *Fluid Dynamics*, fifth ed., Butterworth-Heinemann, Oxford, 2000.