# Metamodel-based approaches for solving robust design optimization problems – Software development perspective*

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Abstract

The main idea behind the robust design optimization (RDO) is that the merit or quality of a design is represented not only by the mean value but also by the variability of its performance. Therefore, in the framework of RDO, improvement of the mean level of the design performance should simultaneously lead to a reduction of its variance. Since RDO of realistic engineering problems is a computationally expensive task, in most of the cases the design objective as well as the constraint functions are substituted by analytical metamodels. Apart from the employed scatter analysis technique, the strategy of creating metamodels is a crucial component of an efficient RDO solution. The paper presents three such strategies pointing out their advantages and weaknesses. The emphasis is put on computer implementation of various metamodel-based approaches showing that the choice of object oriented programming paradigm greatly facilitates developing a software for RDO. The presentation is concluded with a numerical example of the robust design optimization of a thinwalled beam subjected to impact loading.

*Keywords: robustness, optimization, software, stochastic phenomena*

## 1. Introduction

The optimal designs resulting from "classical" deterministic structural design optimization can exhibit high sensitivity to unavoidable parameter variations. Therefore, an optimization approach developed with a purpose to provide solutions of any practical importance should incorporate in its formulation conditions, which account for potentially dangerous stochastic scatter of structural responses. Such a formulation is offered by robust design optimization (RDO), see. e.g. [1]. By its definition the optimal design obtained using the RDO approach do not only minimizes or maximizes selected nominal characteristics of the structure but also renders the structure not sensitive (robust) with respect to material parameter imperfections and with respect to a scatter of external loading. RDO leads to designs that maintain their quality and assumed functionality in a wide range of operating conditions.

The above mentioned reasons are not the only motivation for developing RDO techniques. In the case of optimizing a part of a complex system, the part is often treated as a stand alone structure. However, in order to perform structural analysis of this part it is necessary to account for the behavior of the remaining parts. This requires either simulating the entire integrated system (which can be not cost efficient) or making an assumption concerning the ranges of possible interactions between the part under study and the rest of the system. The latter directly leads to the RDO formulation. The similar situation occurs when several groups of designers simultaneously create a complex system, independently optimizing its elements/subsystems. Quite often, due to the complexity of the design and the time constraints imposed on the design process, the designer teams have to conduct their optimization tasks without the complete knowledge of the other elements, which responses constitute inputs to the respective subsystems under optimization.

As it is shown below, the formulation of RDO problem is much more complex when compared to deterministic formulation. Usually, mainly due to computationally intensive scatter analysis embedded inside the optimization loop, the overall cost of RDO computations is very high. It is therefore a common practice in optimization of realistic engineering structures represented by complicated finite element models to substitute these models by the so-called metamodels – explicit and computationally inexpensive approximating functions defined using results of planned numerical experiments.

A non-standard formulation and a wide choice of possible metamodel-based solution strategies may complicate computer implementation of RDO. Hence, the main goal of the current study is a development of efficient object-oriented programming techniques that facilitate this implementation.

## 2. Formulation of the RDO problem

Mathematically the RDO problem can be formulated as the following multi-criteria optimization problem (see [2]):

$$\text{find: } \mathbf{d}, \boldsymbol{\mu}_{\mathbf{X}}, \tag{1}$$

$$\text{minimizing: } \big\{\mathbb{E}[f(\mathbf{d},\mathbf{X},\mathbf{P})], \sigma[f(\mathbf{d},\mathbf{X},\mathbf{P})]\big\}, \tag{2}$$

subject to:

$$\mathbb{E}[g_i(\mathbf{d},\mathbf{X},\mathbf{P})] - \beta_i\,\sigma[g_i(\mathbf{d},\mathbf{X},\mathbf{P})] \geq 0, \quad i = 1,\dots,k_g, \tag{3}$$

$$\sigma[c_l(\mathbf{d},\mathbf{X},\mathbf{P})] \leq {}^u\sigma_l, \quad l = 1,\dots,k_c, \tag{4}$$

$${}^l d_j \leq d_j \leq {}^u d_j, \quad j = 1,\dots,n_d, \tag{5}$$

$${}^l\mu_{X_r} \leq \mu_{X_r} \leq {}^u\mu_{X_r}, \quad r = 1,\dots,n_X, \tag{6}$$

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where $\mathbf{d} = \{d_1, \ldots, d_{n_d}\}$ are deterministic design variables, $\mathbf{X} = \{X_1, \ldots, X_{n_X}\}$ are random design variables, $\mathbf{P} = \{P_1, \ldots, P_{n_P}\}$ are random parameters, $\boldsymbol{\mu}_{\mathbf{X}}$ is the vector of mean values of the variables $\mathbf{X}$, $\mathbb{E}[\cdot]$ and $\sigma[\cdot]$ stand for expectation and standard deviation, respectively, functions $f$ and $g_i$, $i = 1, \ldots, k_g$, are the objective and constraint functions, as in the deterministic optimization problem, respectively, $c_l$, $l = 1, \ldots, k_c$, represent constraints on standard deviations of selected responses and inequalities (5) and (6) are side constraints for design variables $\mathbf{d}$ and $\boldsymbol{\mu}_{\mathbf{X}}$, respectively. The factors $\beta_i > 0$, $i = 1, \ldots, k_g$, are the prescribed feasibility indices and ${}^u\sigma_l$, $l = 1, \ldots, k_c$, denote the upper limits for the standard deviations in (4). Separation of the random variables into $\mathbf{X}$ and $\mathbf{P}$ variables corresponds to their different "nature" in the optimization task and the possibility to control their probability distribution parameters during the RDO process. Random parameters $\mathbf{P}$ are in fact uncontrollable noise variables i.e. they are only a source of the observed scatter of structural responses. On the other hand, mean values $\boldsymbol{\mu}_{\mathbf{X}}$ of variables $\mathbf{X}$ are treated as design variables and they can change in the optimization process.

In the formulation (1)–(6), the RDO problem is shown to be a vector optimum problem, in which two criteria, namely the mean value $\mathbb{E}[f(\mathbf{d}, \mathbf{X}, \mathbf{P})]$ and the standard deviation $\sigma[f(\mathbf{d}, \mathbf{X}, \mathbf{P})] = \sqrt{\mathbb{V}\mathrm{ar}[f(\mathbf{d}, \mathbf{X}, \mathbf{P})]}$ of the goal performance are to be minimized. Since the criteria often conflict with each other Pareto optimality is one possible way of defining optimal solutions for such a multicriteria vector optimization problem. Though other methods can be employed for seeking the Pareto optimal solutions, a straightforward scalarization approach is the linear combination method leading to the following weighted objective function:

$$\tilde{f} = (1 - \alpha)\mathbb{E}[f(\mathbf{d}, \mathbf{X}, \mathbf{P})]/\mu^* + \alpha\sigma[f(\mathbf{d}, \mathbf{X}, \mathbf{P})]/\sigma^*. \quad (7)$$

Here $0 < \alpha < 1$ is the weighting factor and $\mu^*$ and $\sigma^*$ are the normalization constants. Very often the robust design objective includes "mean on target" and "minimize variation" robust design goals, formulated as (cf. [3]):

$$\hat{f} = \frac{1 - \alpha}{\mu^*}[\mathbb{E}(f) - M]^2 + \frac{\alpha}{\sigma^*}\sigma(f), \quad (8)$$

where the first component of the objective function corresponds to the "mean on target" ($M$ is the target) and the second to performance scatter minimization.

The key elements of efficient algorithms for solving the RDO problems are: a method for estimating the performance variance and an appropriate metamodeling technique to approximate the implicit objective and constraint functions, see [4]. The possible strategies of realizing this task from the point of view of constructing metamodels are given below.

## 3. Metamodel-based strategies for RDO

A drawback of using sampling techniques for assessing values of statistical moments in (7), (8), (3) and (4) is the inherent numerical noise due to the random estimation error. On the other hand, these techniques are universal and do not exhibit limitations of perturbation approaches. Therefore, for computational efficiency it is crucial to use suitable response surfaces (metamodels) to approximate the implicit functions in the formulation of RDO problem. The following metamodel-based strategies are considered:

**Direct approximation strategy.** This is a standard approach consisting in creating analytical metamodels directly for functions $f$, $g_i$, $i = 1, \ldots, k_g$, and $c_k$, $k = 1, \ldots, k_c$, in the combined space of variables $\mathbf{d}$, $\mathbf{X}$ and $\mathbf{P}$. Hence, the designs of numerical experiments are set up in $n_{dXP} = n_d + n_X + n_P$ dimensional space. Depending on the value of $n_{dXP}$ as well as the nature of the analyzed problem, there is a wide choice of possible metamodel formulations. The metamodels can then be used to estimate moments in (7), (8), (3) and (4) by a selected method.

**Dual response surface strategy (DRS).** Contrary to the previous strategy, here the response surfaces are built not for the functions $f$, $g_i$ and $c_k$ but for their respective statistical moments. Therefore, these are the metamodels approximating mean values and standard deviations of the functions in the RDO problem formulation. The metamodels are defined in $n_{dX} = n_d + n_X$ dimensional spaces. Accounting for the fact that usually $n_{dX}$ is many times smaller than $n_{dXP}$ this strategy allows for better fitting the surfaces to experimental points. Typically in the literature a quadratic polynomial approximation is adopted for this purpose. In the version of the DRS strategy implemented by the author, instead of "global" polynomial response surfaces, there are used the kriging approximation or the moving least squares approach.

**Approximation of weighted objective and robustness constraints.** One way to improve the efficiency of the DRS method is by reducing the number of metamodels, creating them for the weighted objectives (7) or (8) and the constraints (3) and (4). The problem of finding parameters of approximating functions is solved $k_g + k_c + 1$ times, i.e. as many times as for the direct approximation strategy. Of course, an overall time reduction of the RDO process very much depends on computational complexity of the underlying structural analysis. However, in many cases, especially using kriging-based metamodels this strategy can significantly speed up the optimization process.

The advantages as well as weaknesses of the three above strategies will be described in detail in the presentation.

## 4. Object oriented implementation of RDO

The variety of metamodel-based approaches to RDO problem may be seen as an important difficulty in their direct and seamless computer implementation. Different types of random variables and design variables, their mutual dependencies, using the functions alternately in the space of random variables for scatter analysis and in the space of design variables within the optimization problem, all these complicate the programmers task. However, the object oriented programming paradigm provides tools for easy and elegant RDO implementation. In the presentation it will be shown how to design the hierarchy of classes in order to adequately represent all the RDO components and the necessary functionality and how to take advantage of the so-called function polymorphism in developing functions for stochastic analysis as well as for optimization related operations.

## References


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