Dynamic analysis of the electro-mechanical interaction between the drive systems and the electric motors

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Abstract

In the paper dynamic electro-mechanical interaction between the working machine drive system and the electric driving motor is considered. The investigations are performed by means of the circuit model of the asynchronous motor as well as using simple spring-mass models and a precise structural hybrid model of the drive system. In the computational examples there are analyzed and mutually compared dynamic electro-mechanical responses obtained for the drive system start-ups and steady-state operations by the use of the abovementioned models. The results determined here for various excitations generated by the driven machine working tool are qualitatively analysed from the viewpoint of response frequency components induced during system transient and steady-state operation. There are also studied sensitivity to excitations by the motor using frequency response functions and the stability aspect indicating a possibility of negative damping caused by the electro-magnetic flux between the stator and the rotor.

Keywords: coupled fields, vibrations, dynamics, sensitivity, stability

1. Introduction

The drive systems of machines and vehicles are commonly driven by electric motors of various types, e.g. asynchronous motors, synchronous motors, direct-current motors or stepping motors. Torsional vibrations of the drive system usually result in significant fluctuation of rotational speed of the rotor of the driving electric motor. Such oscillation of the angular velocity superimposed on the average rotor rotational speed cause more or less severe perturbation of the magnetic flux and thus additional oscillation of the electric currents in the motor windings. Then, the generated electromagnetic torque is also characterized by additional variable in time components which induce torsional vibrations of the drive system. According to the above, the mechanical vibrations of the drive system become coupled with the electrical vibrations of the currents in the motor windings. An importance of the electromechanical coupling effects taken into consideration is particularly significant when possibly very exact results are required for investigation of extremely responsible drive systems or for analyses of their sufficiently precise motions as well as in order to elaborate for them proper active vibration control algorithms. Nevertheless, because of relatively high computational efforts this problem has been usually studied in a simplified form using separate electrical and mechanical models, as e.g. in [2], or by means of more or less advanced electrical models coupled with relatively rough spring-mass drive-system models, as e.g. in [1].

In the presented paper there is considered an influence of electro-mechanical coupling effects on dynamic responses of the typical working machine geared drive system driven by the asynchronous motor during its start-ups and steady-state operation. Similarly as in [4], the purpose of this study is realized by the use of circuit model of the electric motor and the structural precise model of the working machine drive system.

2. Main assumptions and formulation of the problem

In this paper dynamic investigations of the entire drive system are performed by means of the one-dimensional hybrid structural model consisting of continuous visco-elastic macro-elements and rigid bodies. In this model by the torsionally deformable cylindrical macro-elements of continuously distributed inertial-visco-elastic properties there are substituted successive cylindrical segments of the stepped shafts and coupling disks, as presented in Figure 1. The rigid bodies represent inertias of the gear wheels and of the driven machine working tool. Apart of numerical simulations of coupled electro-mechanical vibrations, this model is employed here also for torsional eigenvalue and stability analyses of the drive train.

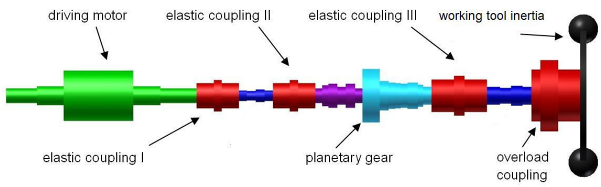

Figure 1: Hybrid mechanical model of the drive system

Torsional motion of cross-sections of each visco-elastic macro-element is governed by the hyperbolic partial differential equations of the wave type. Mutual connections of the successive macro-elements creating the stepped shaft as well as their interactions with the rigid bodies are described by equations of boundary conditions formulated for the macro-element extreme cross-sections. These equations enclose geometrical conditions of conformity for rotational displacements as well as equilibrium conditions for external and internal visco-elastic torques. The solution for forced vibrations has been obtained using the

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analytical approach demonstrated in details e.g. in [3]. Solving the differential eigenvalue problem for the orthogonal system and an application of the Fourier solutions in the form of series lead to the set of modal ordinary differential equations:

$$\ddot{\xi}_m(t) + (\beta + \tau\omega_m^2)\,\dot{\xi}_m(t) + \omega_m^2 \xi_m(t) = \frac{1}{\gamma_m^2} Q_m(t), \quad m = 0,1,2,... \tag{1}$$

where ω_m are the successive natural frequencies of the drive system, β denotes the coefficient of external damping assumed here as proportional one to the modal masses γ_m^2, τ is the shaft material retardation time and $Q_m(t)$ are the modal external excitations. A fast convergence of the applied Fourier solution enables us to reduce the number of the modal equations to solve in order to obtain a sufficient accuracy of results in the given range of frequency. Here, it is necessary to solve only 10÷20 modal equations (1), even in cases of very complex mechanical systems, contrary to the classical one-dimensional beam finite element formulation leading usually to large numbers of motion equations corresponding each to more than one hundred or many hundreds degrees of freedom (if the artificial and often error-prone model reduction algorithms are not applied).

In the case of the symmetrical three-phase asynchronous motor electric current oscillations in its windings are described by the six circuit voltage equations, which can be found e.g. in [5]. Next, they are transformed into the system of four Park's equations and then the electromagnetic torque generated by such a motor can be expressed by the following formula

$$T_{el} = \frac{3}{2} pM\left[(i_\beta^s i_\alpha^r - i_\alpha^s i_\beta^r)\cos p\vartheta - (i_\alpha^s i_\alpha^r + i_\beta^s i_\beta^r)\sin p\vartheta\right], \tag{2}$$

where M denotes the relative rotor-to-stator coil inductance, p is the number of pairs of the motor magnetic poles, ϑ denotes the rotation angle between the rotor and the stator and i_γ^q, $\gamma=\alpha,\beta$, are the electric currents in the rotor for $q=r$ and in the stator for $q=s$ reduced to the electric field equivalent axes α and β, [5]. From the abovementioned system of Park's equations as well as from formula (2) it follows that the coupling between the electric and the mechanical system is non-linear in character, which leads to very complicated analytical description resulting in rather harmful computer implementation. Thus, this electromechanical coupling has been realized here by means of the step-by-step numerical extrapolation technique, which for relatively small direct integration steps for equations (1) results in very effective, stable and reliable results of computer simulation.

3. Computational examples and final remarks

In the computational examples there are compared dynamic electro-mechanical responses of the considered drive system during its start-up and steady-state operation under resonant excitation generated by the driven machine working tool, where the mechanical part has been represented by means of the traditional simple one-, two- and four-spring-mass models as well as by the precise hybrid model. Figure 2 illustrates the exemplary electro-mechanical dynamic response in the form of time-histories of the electric currents in the selected phase of the stator (grey line) and rotor (black line) winding (Fig. 2a), asynchronous motor torque (black) and the driven machine retarding torque (grey) (Fig. 2b) as well as of the rotational speeds (Fig. 2c) and elastic torques (Fig. 2d) in the input (black) and output (grey) shaft, respectively. The plots in Fig. 2 demonstrate evident coupling between mechanical and electric response of the considered system. From the analogous results obtained using the abovementioned simple spring-mass models

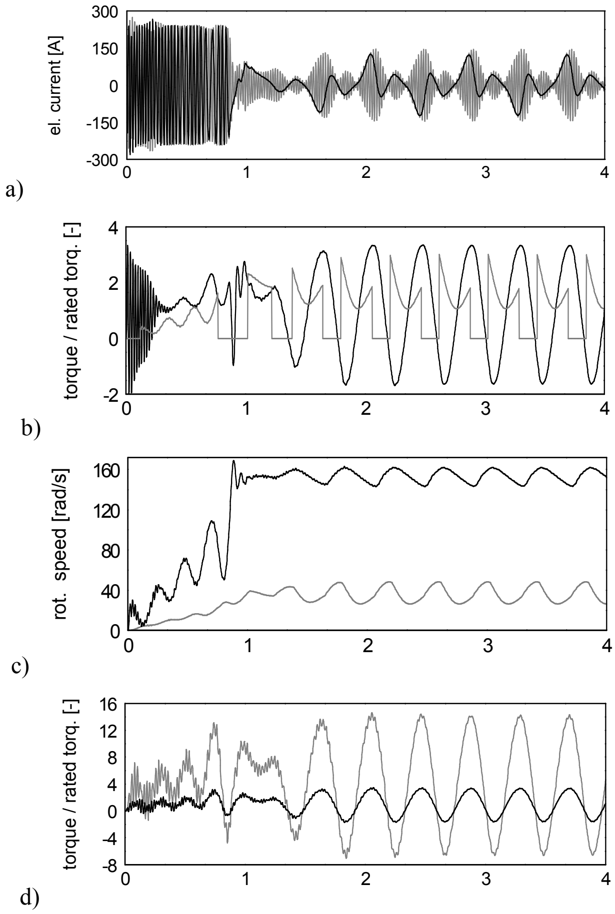

Figure 2: Drive system electro-mechanical dynamic response

of the mechanical part it follows that the reliable electro-mechanical responses can be yielded only by the hybrid model, which sufficiently accurately describes structure and geometry of the real object. Then, such results are qualitatively analysed from the viewpoint of dynamic response frequency components induced during system start-up and steady-state operation. Moreover, there are studied sensitivity to excitations caused by the motor using frequency response functions and the stability aspect indicating a possibility of negative damping generated by the electro-magnetic flux between the stator and the rotor.

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