First results on shear band analysis using a finite element method with r-adaptivity

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Abstract

A finite element method with mesh adaptivity is developed and applied to problems of material instability. Basis of the computational method for large elastic-plastic deformations is the energy criterion of path stability [5, 6]. At a bifurcation point, the stable deformation path is automatically selected if a time-independent, incrementally non-linear material model is used. The J_2 corner theory [1] is applied as a material model. To achieve a good approximation of the first bifurcation instant higher degree polynomials are used as shape functions. Moreover, in order to reproduce analytically predicted discontinuities of the velocity gradient along the boundaries of localization bands the mesh is allowed to move relative to the material when the incremental energy is minimized.

Keywords: shear band analysis, material instability, strain localization, finite element method, r-adaptivity

1. Introduction

One way to numerically analyse localization phenomena in elastic-plastic solids is to use an incrementally non-linear material model combined with the energy criterion of plastic path instability [5, 6]. On this basis, in recent years the formation and post-critical evolution of shear bands have been successfully studied by using a finite element discretization with linear elements [3, 4]. While satisfactory results have been obtained with triangulations adapted to analytical results, simulation results for the first bifurcation instant and corresponding fields become more or less useless on arbitrary meshes constructed without apriori knowledge of the expected deformation pattern. The approximation of the first bifurcation instant could be considerably improved with a meshfree discretization method [2]. Apart from its computational cost it is, however, a drawback of this technique that discontinuities of the velocity gradient can not be reproduced using standard moving least squares shape functions.

In the present work, the finite element discretization used in [3, 4] is developed further by

- applying higher order elements and
- a realization of r-adaptivity.

It will be shown that even on initially arbitrary meshes the accuracy of the numerically determined bifurcation instant and the corresponding velocity fields are considerably improved.

2. Theoretical framework

Constitutive rate equations of a simple time-independent material undergoing isothermal deformations are laid down in the form

$$\dot{\mathbf{S}} = \frac{\partial U}{\partial \dot{\mathbf{F}}} = \mathsf{C}\dot{\mathbf{F}}, \qquad \mathsf{C} = \frac{\partial^2 U}{\partial \dot{\mathbf{F}} \partial \dot{\mathbf{F}}}, \qquad U = \frac{1}{2}\dot{\mathbf{S}} \cdot \dot{\mathbf{F}}, \tag{1}$$

where $\mathbf{S}$ is the first Piola-Kirchhoff stress, $\mathbf{F}$ is the deformation gradient, and a superimposed dot denotes the (forward) material time derivative with respect to a time-like parameter t. A central dot indicates an inner product. The velocity-gradient potential U is assumed to be continuously differentiable and positively homogeneous of degree two in $\dot{\mathbf{F}}$ so that the instantaneous moduli $\mathsf{C} = \mathsf{C}^T$ depend on the *direction* of $\dot{\mathbf{F}}$ in a non-linear and piecewise continuous manner.

We consider quasi-static deformations of a material body with kinematically constrained boundary, either totally by a given boundary motion or partially by periodicity conditions. Due to the potential form of the constitutive law (1), the rate boundary value problem in velocities is governed by Hill's variational principle which requires stationarity of the functional

$$J(\mathbf{v}) = \int_V U(\nabla \mathbf{v})\, dV \tag{2}$$

when prescribed nominal body forces and nominal surface tractions are absent. V denotes the body volume and ∇ the gradient in a given reference configuration.

Moreover, according to Petryk's work on path stability (e.g. [5, 6]), the velocity field of a stable deformation process is obtained by minimizing the incremental energy on the set of kinematically admissible fields, which, under the present conditions, amounts to minimizing the velocity functional (2).

3. Numerical approach

The computational domain is covered with a triangular mesh. The coordinates of the mesh vertices relative to a fixed global coordinate system are $\mathbf{X}_i$, $i = 1, \ldots, NV$. Index i indicates the global vertex number and NV is the total number of vertices. Within each triangle, the velocity field is interpolated by

$$\mathbf{v}(\mathbf{X}) = \sum_{j=1}^{NS} \mathbf{v}_j \varphi_j(\mathbf{X}), \tag{3}$$

where $\mathbf{v}_j$ denote the nodal values, j is the local index of nodes introduced to construct (higher degree) polynomials being shape functions φ_j and NS is the number of shape functions within each triangle depending on the polynomial degree.

In order to evaluate Petryk's stability criterion numerically, the functional (2) is minimized with respect to nodal values $\mathbf{v}_j$ *and* vertex positions $\mathbf{X}_i$ subjected to inequality constraints serving as bounds for the quality of each triangle. The resulting discretization procedure can be understood as a subparametric finite element method with simultaneous r-adaptivity.

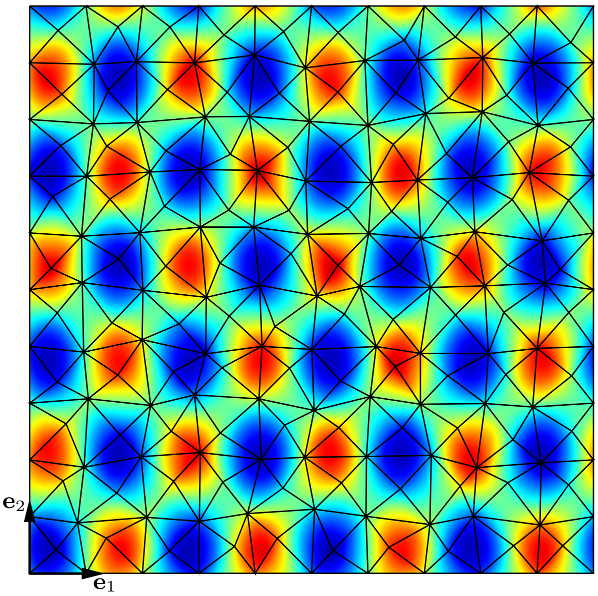

Figure 1: Distribution of the $w_{,22}$ component of the velocity-gradient of the bifurcation mode when (2) is minimized with respect to nodal velocities $\mathbf{v}_j$ only.

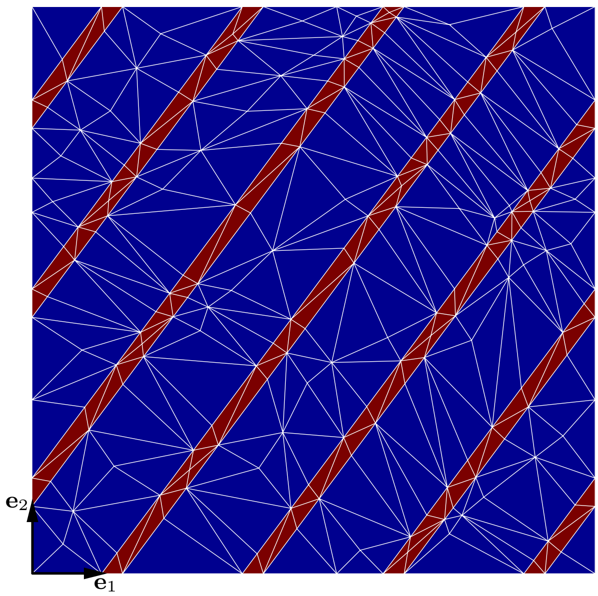

Figure 2: Distribution of the same field as in Fig. 1 and the rearranged mesh when (2) is minimized with respect to both nodal velocities $\mathbf{v}_j$ and vertex coordinates $\mathbf{X}_i$.

4. Computational examples

The constitutive rate equations (1) are specified by the J_2 corner theory [1]. An initially homogeneous material body is subjected to overall plane strain isochoric compression, such that the non-zero components of the deformation gradient $\bar{\mathbf{F}}$ are

$$\bar{F}_{11} = 1 - t, \quad \bar{F}_{22} = (1-t)^{-1}, \quad \bar{F}_{33} = 1, \tag{4}$$

on an orthonormal basis. Computational examples will be presented for domains with kinematically constrained boundaries and for rectangular domains with periodic boundary conditions. Periodic boundary conditions are of interest in order to simulate the behaviour of an infinite continuum subject to a given mean deformation with gradient $\bar{\mathbf{F}}$. For this case numerical results can be compared with theoretical findings from Petryk's local stability analysis. Numerically determined quantities are

- the critical instant of primary bifurcation $t^\star$ when the velocity field $\bar{\mathbf{v}}$ corresponding to uniform deformation ceases to minimize the discretized functional (2) and
- the bifurcation mode $\mathbf{w}$ so that $\mathbf{v} = \bar{\mathbf{v}} + \mathbf{w}$ is the actual minimiser of (2) at the critical instant $t^\star$.

5. Sample simulation results

Figures 1 and 2 represent results from a sample computation with periodic boundary conditions. For the underlying material data the critical instant of ellipticity loss is $t^\star_{th} = 0.2744$. The considered domain is a square at $t^\star_{th}$. With cubic polynomials as shape functions φ the critical instant is numerically detected at $t^\star = 0.2769$ on the non-regular mesh depicted in Fig. 1. The theoretical value $t^\star_{th}$ is thus approximated with an error of only 0.9% whereas using linear shape functions on the same mesh would increase this error to 42.6%. Minimizing the discretized velocity functional at $t^\star$ with respect to nodal velocities $\mathbf{v}_j$ only (based on a fixed mesh) yields a velocity gradient distribution as depicted in Fig. 1 for the $w_{,22}$ component.

This distribution as well as the mesh geometry are drastically changed (Fig. 2), when the nodal coordinates are no longer fixed, i.e. when the minimization is performed with respect to both nodal velocities $\mathbf{v}_j$ *and* vertex coordinates $\mathbf{X}_i$.

Fig. 2 shows the distribution of the $w_{,22}$-component of the velocity-gradient field and the resulting mesh. Obviously, the presented discretization method provides the theoretically predicted rank-1 solution with two different piecewise constant regions of the velocity-gradient.

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