On Wheel - Rail Elastic Contact Problems for Multi-Layer Structure

Andrzej Chudzikiewicz[1] **and Andrzej Myśliński**[2]

[1] *Institute of Transport, Warsaw University of Technology*
ul. Koszykowa 75, 00 - 662 Warsaw, Poland
e-mail: ach@it.pw.edu.pl

[2] *Systems Research Institute*
ul. Newelska 6, 01 - 447 Warsaw, Poland
e-mail: myslinsk@ibspan.waw.pl

Abstract

This paper deals with the numerical solution of elastic wheel-rail rolling contact problems including friction and wear. Multi-material rail model where Young modulus of rail material near the rail surface is dependent on the depth of the rail according to the exponential law is assumed. The equilibrium state of this contact problem is described by the hyperbolic variational inequality of the second order. This problem is solved numerically using a quasistatic approach. Numerical examples are provided and discussed.

Keywords: dynamics, contact mechanics, multiscale problem, numerical methods

1. Introduction

This paper deals with the numerical solution of the wheel-rail elastic contact problems including friction and wear. The contact of a rigid wheel with an elastic rail lying on a rigid foundation is considered. The friction between the bodies is governed by the Coulomb law [5, 12]. We employ Archard's law of wear [11]. In the model the wear is identified as an increase in the gap between bodies. The elastic rolling contact problems were considered by many authors. For details see the references in monographs [5, 9, 12].

Numerous laboratory experiments or numerical experiments in [1, 2, 6, 10] indicate that the use of a coating material attached to the conventional steel body reduce the magnitude of residual or thermal stresses. However in a conventional coating structure homogeneous materials are used. The abrupt change in the mechanical properties of the materials at the surface coating-substrate interface results in stress concentration or degraded bonding strength [10, 13].

Therefore in this paper we solve numerically the wheel - rail contact problem with friction and wear assuming more complicated model of a coating layer than in [1, 2]. We assume that between the homogeneous coating layer and the homogeneous substrate there exists the graded interlayer which properties depend on its depth according to the exponential law.

In the paper the time-dependent model of this rolling contact problem is introduced. Following [3, 5] we take special features of this rolling contact problem and we use so-called quasistatic approach to transform it into equivalent stationary system and to solve it numerically. Finite element method is used as a discretization method. The numerical results are provided and discussed.

2. Rolling Contact Problem

Consider deformations of an elastic strip lying on a rigid foundation (see Fig. 1). The strip has constant height h and occupies domain $\Omega \in R^2$ with the boundary Γ. A wheel rolls along the upper surface Γ_C of the strip. The wheel has radius r_0, rotating speed ω and linear velocity V. The axis of the wheel is moving along a straight line at a constant altitude h_0 where $h_0 < h + r_0$, i.e., the wheel is pressed in the elastic strip. It is assumed, that the head and tail ends of the strip are clamped, i.e., we assume that the length of the strip is much bigger than the radius of the wheel. Moreover it is assumed, that there is no mass forces in the strip. The body is clamped along a portion Γ_0 of the boundary Γ of the domain Ω. The contact conditions are prescribed on a portion Γ_C of the boundary Γ. Moreover, $\Gamma_0 \cap \Gamma_C = \emptyset$, $\Gamma = \bar{\Gamma}_0 \cup \bar{\Gamma}_C$. We denote by $u = (u_1, u_2)$, $u = u(x, t)$, $x \in \Omega$, $t \in [0, T]$, $T > 0$ a displacement of the strip. Assume $\Omega = \Omega_1 \cup \Omega_2 \cup \Omega_3$ where Ω_1 denotes upper part of the strip. In Ω_2 material parameters depend on the exponential law.

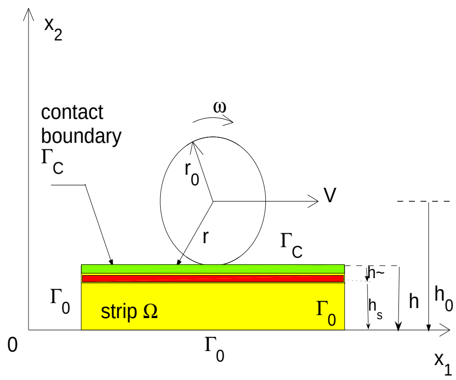

Figure 1: The wheel rolling over the strip.

The displacement u of the strip satisfies the evolution equation in $\Omega \times (0, T)$ [1]:

$$\rho \frac{\partial^2 u}{\partial t^2} = A^\star DAu, \tag{1}$$

with boundary and initial conditions:

$$u = 0 \quad \text{on } \Gamma_0 \times (0, T), \tag{2}$$

$$B^\star DAu = F \quad \text{on } \Gamma_C \times (0, T), \tag{3}$$

$$u(0) = \bar{u}_0 \quad u'(0) = \bar{u}_1 \quad \text{in } \Omega, \tag{4}$$

where $u(0) = u(x, 0)$, $u' = du/dt$, $\bar{u}_0$, $\bar{u}_1$ are given functions, ρ is a mass density of the strip material, $\Gamma_0 = \Gamma \setminus \Gamma_C$, the operators A, B and D are defined as follows [3, 5]

$$A = \begin{bmatrix} \frac{\partial}{\partial x_1} & 0 \\ 0 & \frac{\partial}{\partial x_2} \\ \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_1} \end{bmatrix}, \quad B = \begin{bmatrix} n_1 & 0 \\ 0 & n_2 \\ n_2 & n_1 \end{bmatrix}, \tag{5}$$

$$D = \begin{bmatrix} \lambda + 2\gamma & \lambda & 0 \\ \lambda & \lambda + 2\gamma & 0 \\ 0 & 0 & \gamma \end{bmatrix} \tag{6}$$

where $n = (n_1, n_2)$ is outward normal versor to the boundary Γ of the domain Ω, λ and γ are Lame coefficients [3], $A^\star$ denotes a transpose of A. By $\sigma = (\sigma_{11}, \sigma_{22}, \sigma_{12})$ and F we denote the stress tensor in domain Ω and surface traction vector on the boundary Γ, respectively. The surface traction vector $F = (F_1, F_2)$ on the boundary Γ_C is a priori unknown and is given by conditions of contact and friction. Under the assumptions that the strip displacement is small the contact and friction conditions on $\Gamma_C \times (0, T)$ take a form [1]:

$$u_2 + g_r + w \leq 0, \quad F_2 \leq 0, \quad (u_2 + g_r + w)F_2 = 0, \tag{7}$$

$$| F_1 | \leq \mu \, | F_2 |, \quad F_1 \frac{du_1}{dt} \leq 0, \tag{8}$$

$$(| F_1 | - \mu \, | F_2 |) \frac{du_1}{dt} = 0, \tag{9}$$

where $\mu \geq 0$ is a friction coefficient. Under suitable assumptions a gap $g_r = h - h_0 + \sqrt{r_0^2 - (u_1 + x_1)^2}$. Assuming that wear constant $k > 0$ is given the wear w is governed by the equation [12]

$$\frac{dw}{dt} = kVF_2. \tag{10}$$

2.1. *Material properties of graded layer*

In the subdomain Ω_2 the operator D is assumed to depend on the depth of the layer [13]. This dependence is governed by the exponential law [13],

$$P(x) = P_{\Omega_1} \cdot e^{\lambda \frac{x_2}{\tilde{h}}}, \tag{11}$$

where P, P_{Ω_1} denote the height dependent material property (material density or Young modulus), and the layer Ω_1 material property, respectively. $\lambda, \tilde{h} > 0$ are given parameters and x_2 denotes spatial variable. In domains Ω_1 and Ω_3 the operator D takes different constant values. The continuity of the displacements and the stresses along the interfaces $\partial\Omega_1 \cap \partial\Omega_2$ and $\partial\Omega_2 \cap \partial\Omega_3$ are assumed.

2.2. *Quasistatic Formulation*

In general, the original dynamic contact problem (1) - (10) has no solutions. Taking into account the special features of this rolling problem one can formulate it in the framework of the quasistatic approach. The quasistatic approach is based on the assumption that for the observer moving with a wheel the displacement of the rail does not depend on time. Under this assumption the system (1) - (10) is transformed into the equivalent elliptic one. In this approach the inertial term is replaced by the stationary term reflecting the dynamics of the body rather than completely neglected it as in classical quasistatic formulation [5]. In literature [13] are reported existence results for the stationary frictionless contact problem between nonhomogeneous surfaces based on the application of singular integral equations. In mathematical literature one can find also existence results for elliptic boundary value problems with rapidly oscillating or discontinuous coefficients (see references in [7]).

3. Numerical Algorithms

It is well known that the application of the classical finite element method, where material properties are constant, to solution of problems with the functionally graded materials may lead to large numerical errors. A proper approach to solve such problems requires application of nonhomogeneous finite element method containing additional approximation functions in order to interpolate material properties at the level of each finite element. This idea is implemented in the framework of the graded [8] or multiscale [4] finite element methods. Among others the convergence of the multiscale finite element method for elliptic systems is shown in [7].

In the paper the quasistaic problem equivalent to (1) - (10) is solved numerically using the finite element and optimization methods. For details of these methods see [3, 5]. The numerical results are provided and discussed. The calculated distribution of the normal stresses in the contact zone are subject to comparison.

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