Influence of mechanical vibrations on acoustic attenuation of ducts

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Abstract

Noise transmission problems where sound propagates in flow ducts or pipes occur in a variety of contexts. Some of the most common applications are ventilation and air-conditioning systems, exhaust systems, and aero-engine turbofan inlet and bypass ducts. External boundaries can channel sound propagation, and in some cases can create build-up or attenuation of acoustic energy within a confined space. Pipes or ducts acts as guides of acoustic waves and the net flow of energy. In this article the prediction of sound transmission loss in duct with hard boundary walls is examined. Mechanical model of acoustic duct is proposed as well. The difference between the sound energy on inlet side of the duct and that radiated from the outlet side is called the transmission loss. The larger the transmission loss, the smaller the amount of energy passing through and consequently, less noise heard. Transmission loss depends on frequency. Finite element analysis (FEA) is the computational technique used for the calculations. FEA may be used to develop model for frequency dependent transmission loss, to calculate transmission loss in specific critical applications, and to provide a research tool to better understand transmission loss.

Keywords: computational acoustic, mechanical system, transmission loss

1. Introduction

A lot of devices with duct systems like air-condition, heating equipment, frig and so on are being used in people's life. External boundaries can channel sound propagation, and in some cases can create buildup or attenuation of acoustic energy within a confined space. Pipes or ducts acts as guides of acoustic waves, and the net flow of energy, other than that associated with wall dissipation, is along the direction of the duct [2].

There are lot of papers presenting experimental and numerical methods to predict sound transmission loss in silencers and acoustic ducts (e.g. [4]).

On this consideration, in this paper it is proposed an efficient practical numerical method for calculation of attenuation of sound of ducts/pipes based on FEM. The transmission loss estimation by the proposed numerical method was tested by comparison with the experimental one on an sound attenuation in pipe [3]. The method shows its viability by presenting the reasonably consistent anticipation of the experimental result. One can observe mechanical behaviour of pipe's medium for lower frequencies (high transmission loss) and wave behaviour for higher frequencies.

2. Acoustic governing equations

Theory in this section is based on handbook of acoustic [2]. A single equation for the acoustic part of the pressure results in formula

$$\frac{\partial^2 p}{\partial t^2} = c^2 \nabla^2 p \tag{1}$$

where ρ is the mass density, c is speed of sound in considered medium and p is the acoustic pressure.

An import and special case is a time-harmonic wave

$$p(\mathbf{x},t) = p(\mathbf{x}) e^{i\omega t} \tag{2}$$

where $\omega = 2\pi f$ is the angular frequency, with f denoting the frequency. Assuming the same harmonic time dependence for the source terms, the wave equation for acoustic waves reduces to an inhomogeneous Helmholtz equation

$$-\frac{\omega^2 p}{\rho\, c^2} = \nabla \cdot \left(\frac{1}{\rho} \nabla p \right). \tag{3}$$

For the acoustic pressure in 2D axisymmetric geometries the wave equation becomes

$$\frac{\partial}{\partial r}\left(-\frac{r}{\rho}\frac{\partial p}{\partial r} \right) + r\frac{\partial}{\partial r}\left(-\frac{1}{\rho}\frac{\partial p}{\partial z} \right) - \left[\left(\frac{\omega}{c} \right)^2 - \left(\frac{m}{r} \right)^2 \right] \frac{rp}{\rho} = 0 \tag{4}$$

where m denotes the circumferential wave number and k_z is the out-of-plane wave number

For simplicity the preferred work choice is to work in non-dimensional frame of reference [3]. Now some dimensionless variables will be introduced in order to make the system much easier to study. This procedure is very important so that one can see which combination of parameters is more important that the others. Putting dimensionless variables into the time-harmonic wave equation we get

$$-\omega'^2 p' - \nabla'^2 p' = 0\,, \tag{5}$$

and into the axisymmetric time-harmonic wave equation we get

$$\frac{\partial'}{\partial' r'}\left(-r'\frac{\partial' p'}{\partial' r'} \right) + r'\frac{\partial'}{\partial' r'}\left(-\frac{\partial' p'}{\partial' z'} \right) - \left[\omega'^2 - \left(\frac{m'}{r'} \right)^2 \right] \frac{r' p'}{\rho} = 0\,. \tag{6}$$

Since now primes will not be written (old variables symbols will be used) but it's important to remember that they are still there.

3. Acoustic model of pipe

In this paper the pipe of length L and radius r with rigid walls is analysed. Figure 1 shows sound power transmission [3], where J_i - sound power from source, J_r - sound power reflected from pipe inlet, J_{r^*} - sound power reflected from pipe outlet (if $L \to \infty$ then $J_{r^*} \to 0$) and J_t - sound power transmitted by outlet outside pipe.

Figure 1: A description of sound power transmission in the pipe with outlet to free space (no coming back wave from outside)

The difference between the sound energy on one side of the pipe and that radiated from the second side (both expressed in decibels) is called the sound transmission loss. Transmission loss (TL) is given by $TL = 10\log\frac{W_i}{W_t}$ where W_i denotes the incoming power at the inlet, W_t denotes the transmitted (outgoing) power at the outlet. The incoming and transmitted sound powers are given by equations

$$W_i = \int_A \frac{p_0^2}{2\rho c} dA\,, \quad W_t = \int_A \frac{|p|^2}{2\rho c} dA\,, \tag{7}$$

where p_0 represents the applied pressure source amplitude and A - the area of boundary where waves are incoming to the pipe and outgoing from the pipe.

4. Mechanical model of pipe

Due to mechanical behaviour of medium in duct/pipe in this subsection mechanical model is proposed. Pipe and domain outside the inlet of pipe is modelled by mechanical system of two: masses, springs and dampers (see Fig. 2).

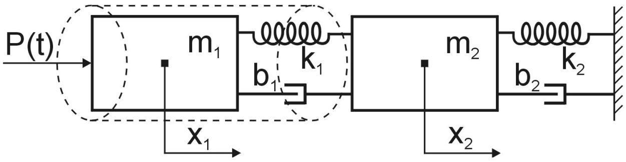

Figure 2: Mechanical system modelling duct with transmission loss frequency-dependent characteristic. Pipe's domain is denoted by dash line

Equations of motion of masses

$$m_1 \frac{\partial^2 x_1}{\partial t^2} + k_1(x_1 - x_2) + b_1 \frac{\partial x_1}{\partial t} = P(t) \tag{8}$$

$$m_2 \frac{\partial^2 x_2}{\partial t^2} + k_1(x_2 - x_1) + k_2 x_2 + b_2 \frac{\partial x_2}{\partial t} = 0 \tag{9}$$

where $P(t) = P_0 e^{i\omega t}$ is extortion force.

Let us assume a time-harmonic motion of masses

$$x_1(t) = x_{10} e^{i\omega t} \quad x_2(t) = x_{20} e^{i\omega t}\,. \tag{10}$$

Using above equations in system of equations of motion we get

$$-m_1\omega^2 x_{10} + k_1(x_{10} - x_{20}) + b_1 i\omega x_{10} = P_0 \tag{11}$$

$$-m_2\omega^2 x_{20} + k_1(x_{20} - x_{10}) + k_2 x_{20} + b_2 i\omega x_{20} = 0\,. \tag{12}$$

Solving above equations for x_{10} and x_{20} we get formulas for amplitudes of harmonic motion of masses.

Transmission loss of mass-spring-damper system we can define as

$$TL_{mech} = 20\log\frac{P_0}{\left|-m_2\omega^2 x_{20}\right|}\,, \tag{13}$$

where argument of logarithm function describes ratio of amplitudes of extortion force and force transmitted from second mass.

5. Numerical results

All acoustic problems considered in this work are governed by dimensionless equations with appropriate boundary conditions [3]. Typical boundary conditions for acoustic model of pipe are: sound-hard boundaries (walls), impedance boundary conditions and radiation boundary conditions (eg. plane wave). Numerical results are obtained using standard computational code COMSOL Multiphysics [1] for air (*ρ=1.25 kg/m³*, *c=343 m/s*). Results for acoustic model shows that sound transmission loss depends on frequency (horizontal axis) and ratio radius to length of pipe.

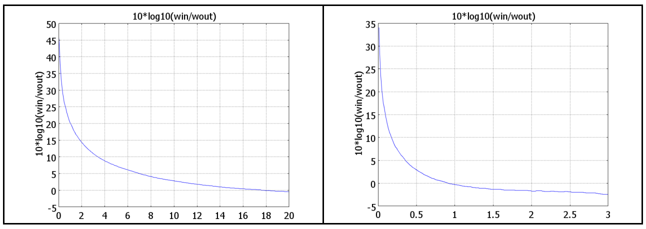

Figure 3: Plot of a transmission loss of pipe with ratio r/L equals: 0.01 (left) and 0.2 (right)

Numerical results for mechanical model

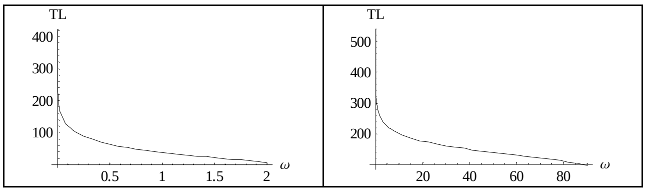

Figure 4: Plot of a transmission loss of mechanical system (left) $m_1 = 10^{-5}, m_2 = 10^{10}, k_1 = 10^{-1}, k_2 = 10^{11}, b_1 = 0, b_2 = 10$, (right) $m_1 = 10^{-5}, m_2 = 10^{3}, k_1 = 10^{-1}, k_2 = 10^{10}, b_1 = 0, b_2 = 10$

6. Conclusions

Numerical results shows that acoustic properties of pipe can be modelled by mechanical system of double mass, spring and damper. This is big challenge to get parameters of mechanical system to receive adequate properties of pipe with outlet to free space.

References

[1] Comsol Multphisycs User's Guide, *Modeling Guide and Model Library*; *Acoustic Module Model*, Documentation Set, Comsol AB, 2007.

[2] Rossing Thomas D. (Ed.), *Handbook of Acoustic*, Springer, LLC New York, 2007.

[3] Strek T., Finite Element Modelling of Sound Transmission Loss in Reflective Pipe, Finite Element Analysis, David Moratal (Ed.), Sciyo, pp. 663-684.

[4] Wilson G.P., Soroka W.W., Approximation to the Diffraction of Sound by a Circular Aperture in a Rigid Wall of Finite Thickness, *The Journal of The Acoustical Society of America*, Volume 37, Number 2, pp.286-297, 1965.