Analysis of effective properties of piezocomposites by the subregion BEM-Mori-Tanaka approach

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Abstract

Recently, many approaches have been proposed to estimate the effective properties of composites. The most typical are: the self-consistend method and the Mori-Tanaka method. However, they are restricted to simple geometries of phases. Also for complex constitutive laws the analytical results are complicated. On the other hand, the combination of numerical methods and these approaches gives an efficient computational scheme for estimating effective properties of composite materials. In this paper the hybrid subregion boundary element method (BEM) and Mori-Tanaka approach is implemented to solve coupled field equations of linear piezocomposites in the unit cell approach and then to determine the effective properties . To obtain the BEM fundamental solutions, the Stroh formalism is used. The influence of the interface electrical boundary conditions on the effective properties is investigated. The numerical examples will demonstrate an effectiveness of the BEM-Mori-Tanaka approach.

Keywords: boundary element methods, coupled fields, homogenization, material properties

1. Introduction

The overall properties of composite materials are required during the design process. The effective properties can be determined by applying analytical, empirical or numerical methods. Among most popular numerical methods of modelling composites are the finite element method (FEM) and the boundary element method (BEM). The BEM [1] is particularly suitable for modelling composites because of its high accuracy and easy modification of geometry. Effective material properties can be computed numerically by considering a unit cell or a representative volume element of a composite [6].

Several authors performed the static linear elastic analysis of materials with inclusions, which modelled RVEs or unit cells of non-homogeneous materials, by using different formulations of the BEM. In [10] the authors applied the formulation for many identical inclusions to the evaluation of elastic constants of composites with direct bonding between inclusions and matrix, and also with interphases between inclusions and matrix. In [8] the example of a piezoelectric square RVE with a circular rigid insulating fiber was analyzed by the self-consistent – BEM and Mori-Tanaka – BEM approaches to determine the effective properties.

In this work the developed methods are applied to compute effective properties using the unit cell approach for piezoelectric composites. Due to complexity of the piezoelectric constitutive equations, a combination of the Mori-Tanaka method [6] and the subregion BEM formulation was proposed [3]. In piezocomposites piezoelectrics are connected with other materials: conductors, dielectrics and also other piezoelectrics, hence special boundary conditions must be applied on the interfaces, between different materials. Models of nonpiezoelectric materials are obtained by assuming particular material properties.

2. The BEM-Mori-Tanaka method for two-phase piezoelectric composite

The Mori-Tanaka method is one of the micromechanical approaches to estimating the concentration factor [6]. The BEM-Mori-Tanaka method is a numerical procedure used to obtain the effective properties of composites and it can deal with arbitrary shapes of inclusions. In this method, a typical inclusion is embedded in an infinite matrix subjected to a homogenous generalized strain boundary condition. The BEM is applied to calculate the dilute generalized strain concentration factor $\mathbf{A}_{DIL}^{(2)}$ and then the effective properties $\mathbf{C}^*$ of the piezocomposite can be found by using the following equation:

$$\mathbf{C}^* = \mathbf{C}^{(1)} + v_2\left(\mathbf{C}^{(2)} - \mathbf{C}^{(1)}\right)\mathbf{A}_{MT}^{(2)}, \tag{1}$$

where superscripts “(1)” and “(2)” denote the matrix and inclusion phases, v_2 is a volume fraction of the inclusion and $\mathbf{A}_{MT}^{(2)}$ denotes the Mori-Tanaka concentration factor [8]:

$$\mathbf{A}_{MT}^{(2)} = \mathbf{A}_{DIL}^{(2)}\left(\left(1 - v_2\right)\mathbf{I} + v_2\mathbf{A}_{DIL}^{(2)}\right)^{-1}, \tag{2}$$

where $\mathbf{I}$ denotes the identity matrix.

3. Models of materials

In the present formulation, piezoelectric material is modelled as homogeneous, transversal isotropic, linear elastic and linear dielectric. For this material, the piezoelectric effect in the two-dimensional case depends on the nine material constants. These constants are: four elastic constants, three piezoelectric constants and two dielectric constants.

The linear dielectric, transversal isotropic and linear elastic material (for example the composite graphite/epoxy) may be described by the four elastic constants, two dielectric constants. In this case the piezoelectric effect does not occur, then the piezoelectric constants are equal to zero.

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The isotropic, linear elastic and conducted material cannot be analyzed directly using the Stroh formalism [7]. The Stroh formulation requires the distinct eigenvalues, therefore only a quasi-isotropic material can be used. The difference between the solution based on a quasi-isotropic and pure isotropic model is negligible [7]. Another problem is a fact, that dielectric permittivity constants for the conductors are non-measurable. For a practical computations it can be assumed that the dielectric constants for the conducting material are equal to the vacuum dielectric constant.

4. The subregion BEM

Composites can be modelled using different approaches of the boundary element method. The BEM is a well-known procedure [1, 3]. This approach is based on the division of the primary region into several homogeneous subregions. For each subregion one can write equations of the BEM.

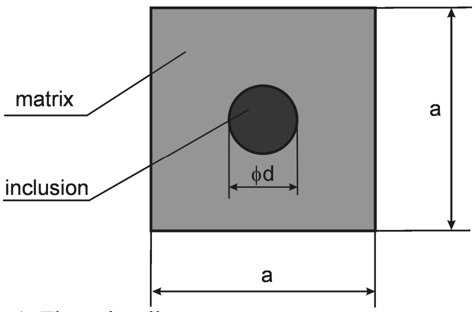

Figure 1: The unit cell

Let the unit cell shown in Figure 1 be composed of two parts, which occupy the subdomains Ω_1 (the matrix, subscripts "1" and "3") and Ω_2 (the inclusion, the subscript "4") with boundaries Γ_1, Γ_2. The boundary Γ_{12} denotes the common boundary (subscripts "3" and "4"). To determine overall properties of the piezoelectric composite the uniform strain and the uniform electric field boundary conditions are applied on the boundary of the unit cell [8]. Two load cases are considered: 1) the uniform strain boundary conditions with zero electric field, and 2) the uniform electric field with no strains, are applied along the boundary.

The BEM equations for each subregion have a form:

$$\begin{aligned}&\Omega_1 : \begin{bmatrix}\mathbf{H}_{11} & \mathbf{H}_{13} \\ \mathbf{H}_{31} & \mathbf{H}_{33}\end{bmatrix}\begin{bmatrix}\mathbf{U}_1 \\ \mathbf{U}_3\end{bmatrix} = \begin{bmatrix}\mathbf{G}_{11} & \mathbf{G}_{13} \\ \mathbf{G}_{31} & \mathbf{G}_{33}\end{bmatrix}\begin{bmatrix}\mathbf{T}_1 \\ \mathbf{T}_3\end{bmatrix} \\ &\Omega_2 : \mathbf{H}_{44}\mathbf{U}_4 = \mathbf{G}_{44}\mathbf{T}_4\end{aligned} \tag{3}$$

In equation (3) $\mathbf{H}_{ii}$, $\mathbf{G}_{ii}$, $\mathbf{U}_i$ and $\mathbf{T}_i$ denote the parts of the $\mathbf{H}$ and $\mathbf{G}$ BEM matrices and parts of the vectors $\mathbf{U}$ and $\mathbf{T}$, which contain the generalized displacements and tractions, respectively. Generalized quantities denote mechanical and electrical quantities, which are coupled due to the piezoelectric effect.

To obtain the final set of equations, the interface compatibility and equilibrium equations, must be implemented:

$$\begin{aligned}&\mathbf{U}_3 = \mathbf{U}_4 \\ &\mathbf{T}_3 = -\mathbf{T}_4\end{aligned} \tag{4}$$

Multiplying both sides of (3) by the inverse of G and implementing (4) one can obtain the final system of equations for the generalized displacements:

$$\begin{bmatrix}\mathbf{A}_{11} & \mathbf{A}_{13} \\ \mathbf{A}_{31} & \mathbf{A}_{33} + \mathbf{A}_{44}\end{bmatrix}\begin{bmatrix}\mathbf{U}_1 \\ \mathbf{U}_3\end{bmatrix} = \begin{bmatrix}\mathbf{T}_1 \\ \mathbf{O}_3\end{bmatrix}, \tag{5}$$

where $\mathbf{A}_{ij} = \left(\mathbf{G}^{-1}\mathbf{H}\right)_{ij}$; $\mathbf{O}_3$ is a null matrix of the appropriate dimensions. The system of equations (5) can be rearranged using the prescribed boundary conditions.

When the connection between two piezoelectric or dielectric material is analyzed, the conditions (4) are still valid, but when the piezoelectric material is bonded with the other nonpiezoelectric material, the electric boundary conditions on the interface may be discontinuous [9]. For a piezoelectric – conductor interface we have the equilibrium of the tractions and the continuity of the displacements. The additional condition is the ideal electric conductor condition. The ideal electric conductor condition is applied to the whole boundary Γ_2 – the constant electric potential solution follows from the Maxwell equations for the ideal conductor.

5. Summary

The BEM was used to solve an electroelastic boundary value problem within a unit cell of a piezocomposite and to determine the effective properties using the Mori-Tanaka approach. The subregion boundary element formulation will be used to model different piezocomposites. The following deformable and quasi-rigid inclusions will be considered: piezoelectric, dielectric and conducting. The numerical examples will demonstrate an efficiency of the BEM solutions.

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