Shell-beam model of thin-walled space structures for geometrically nonlinear analysis

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Abstract

In the paper, the finite shell-beam model for geometrically nonlinear analysis of thin-walled space structures with open cross-section has been proposed. The standard discretization using beam thin-walled elements is connected with the space discretization of the frame joints. The case of special construction of joints, where complete warping transmission is ensured, has also been considered. Examples confirmed the effectiveness of the proposed FEM analysis.

Keywords: steel structures, structural mechanics, large deformations, numerical analysis

1. Introduction

In the paper, it is assumed that the thin-walled structure can be divided into parts which can be treated as the 3D geometric objects (nodes of the frame, sites of applied loads and constraints applied at some points of the cross section), and which remain 1D geometric objects. With such an approach to the finite element method (FEM) discretization, a problem arises of how to couple the different parts of the structure.

It seems natural is to employ the condition of continuity of the translational displacements on faces, which are shared by both the shell and the thin-walled beams

$$\boldsymbol{u}^{(\mathrm{s})}(\boldsymbol{x}) = \boldsymbol{u}^{(\mathrm{b})}(\boldsymbol{x}), \tag{1}$$

where $\boldsymbol{u}^{(\mathrm{s})}(\boldsymbol{x})$ and $\boldsymbol{u}^{(\mathrm{b})}(\boldsymbol{x})$ are vectors of the node displacements of the shell and the beam, respectively.

The equation (1) can be applicable in the two different ways. In the first method, the so-called *transition elements* are defined between the beam and and the shell node [2].

In the second method, presented in the paper, the *space joint element* is formulated using only translational degrees of freedom on faces connecting the node with the thin-walled beam. It is an extention of the method presented previously in [3], for geometrical nonlinear analysis. It is assumed that the material is linearly elastic. The Total Lagrange Approach and Newton-Raphson method are used.

2. Thin-walled finite element

The thin-walled element has been developed introducing Vlasov's assumption and taking into account second order terms of the finite rotation [4]. The vector of displacements $\boldsymbol{u}^{(\mathrm{b})}$ has the following form

$$\boldsymbol{u}^{(\mathrm{b})} = \left\{ u_x^{(\mathrm{b})} \;\; u_y^{(\mathrm{b})} \;\; u_z^{(\mathrm{b})} \right\}. \tag{2}$$

where

$$u_x^{(\mathrm{b})} = u_{x_0}^{(\mathrm{b})} + z \cdot \varphi_y^{(\mathrm{b})} - y \cdot \varphi_z^{(\mathrm{b})} + \omega \cdot \theta^{(\mathrm{b})} + \\ + \left(\frac{y}{2} - \frac{y_S}{2} \right) \cdot \varphi_x^{(\mathrm{b})} \varphi_y^{(\mathrm{b})} + \left(\frac{z}{2} - \frac{z_S}{2} \right) \cdot \varphi_x^{(\mathrm{b})} \varphi_z^{(\mathrm{b})}, \tag{3}$$

$$u_y^{(\mathrm{b})} = u_{y0}^{(\mathrm{b})} - (z - z_S) \cdot \varphi_x^{(\mathrm{b})} + \\ - \frac{y}{2} \cdot \left(\varphi_x^{(\mathrm{b})2} + \varphi_z^{(\mathrm{b})2} \right) - \frac{z}{2} \cdot \varphi_y^{(\mathrm{b})} \varphi_z^{(\mathrm{b})} + \frac{y}{2} \cdot \varphi_x^{(\mathrm{b})2}, \tag{4}$$

$$u_z^{(\mathrm{b})} = u_{z0}^{(\mathrm{b})} + (y - y_S) \cdot \varphi_x^{(\mathrm{b})} + \\ - \frac{y}{2} \cdot \varphi_y^{(\mathrm{b})} \varphi_z^{(\mathrm{b})} - \frac{z}{2} \cdot \left(\varphi_x^{(\mathrm{b})2} + \varphi_y^{(\mathrm{b})2} \right) + \frac{z}{2} \cdot \varphi_x^{(\mathrm{b})2}. \tag{5}$$

The tangent matrix of the element and the vector of internal forces have been computed using the symbolic operations of MATLAB and the regural expressions of PERL language.

3. Space joint element

The space joint element with two faces ($k = 1, 2$) is shown in Fig. 1. The element is discretized using shell elements with 6 d.o.f. in the node.

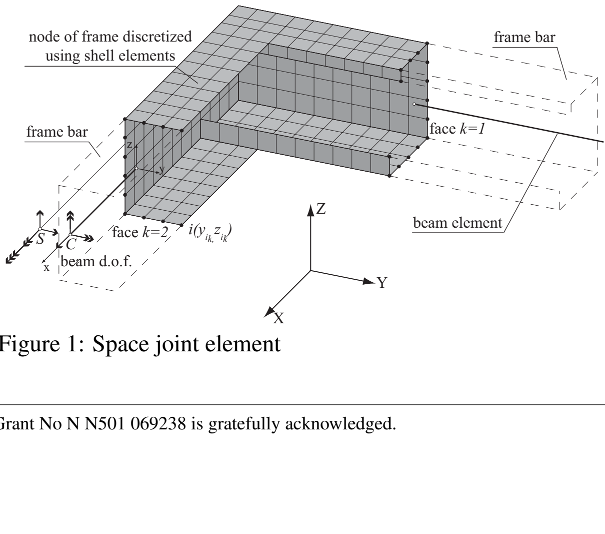

Figure 1: Space joint element

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$$\boldsymbol{B}_{L_{i_k}} = \begin{bmatrix} 1 & 0 & 0 & \left(\frac{y}{2}-\frac{y_S}{2}\right)\cdot\varphi_y^{(b)} + \left(\frac{z}{2}-\frac{z_S}{2}\right)\cdot\varphi_z^{(b)} & z+\left(\frac{y}{2}-\frac{y_S}{2}\right)\cdot\varphi_x^{(b)} & -y+\left(\frac{z}{2}-\frac{z_S}{2}\right)\cdot\varphi_x^{(b)} & \omega \\ 0 & 1 & 0 & -z+z_S+(-y+y_S)\cdot\varphi_x^{(b)} & -\frac{z}{2}\cdot\varphi_z^{(b)} & -y\cdot\varphi_z^{(b)}+\frac{z}{2}\cdot\varphi_y^{(b)} & 0 \\ 0 & 0 & 1 & y-y_S+(-z+z_S)\cdot\varphi_x^{(b)} & \frac{y}{2}\cdot\varphi_z^{(b)}-z\cdot\varphi_y^{(b)} & -\frac{y}{2}\cdot\varphi_y^{(b)} & 0 \end{bmatrix} \tag{6}$$

The dependence between the k node deegres of freedom of the thin-walled element and the translational degrees of freedom of the shell i node on the k face has a general form

$$\boldsymbol{q}_{i_k}^{(s)} = \boldsymbol{B}_{i_k}\left(\boldsymbol{q}_k^{(b)}\right), \tag{7}$$

where $\boldsymbol{q}_k^{(b)} = \{\boldsymbol{u}^{(b)}\ \ \boldsymbol{\varphi}^{(b)}\ \ \theta^{(b)}\}$ is the vector of local d.o.f. of the thin-walled element and $\boldsymbol{q}_{i_k}^{(s)} = \{\boldsymbol{u}^{(s)}\}$ is the vector of local translational d.o.f. of the node shell. The transformation matrix $\boldsymbol{B}_{i_k}$ can be written using Eqs. (3–5). After linearization, the following equation for the increment is obtained

$$\Delta\boldsymbol{q}_{i_k}^{(s)} = \boldsymbol{B}_{L_{i_k}}(\boldsymbol{\varphi}^{(b)})\Delta\boldsymbol{q}_k^{(b)}. \tag{8}$$

In the computation procedure, static condensation has to be employed. Afterwards, it is necessary to carry out transformation from the local to global coordinate system

$$\Delta\boldsymbol{Q}_{i_k}^{(s)} = \boldsymbol{A}_{i_k}\Delta\boldsymbol{Q}_k^{(b)}, \tag{9}$$

where the transformation matrix $\boldsymbol{A}_{i_k}$ is equal to

$$\boldsymbol{A}_{i_k} = \left(\boldsymbol{T}_{i_k}^{(s)}\right)^{\mathrm{T}}\boldsymbol{B}_{L_{i_k}}(\boldsymbol{\varphi}^{(b)})\boldsymbol{T}_k^{(b)}. \tag{10}$$

Using (9), the transformation matrix $\boldsymbol{A}_k$ for the face k, and then for the whole space joint element $\boldsymbol{A}$ with K faces can be formulated.

Finally, the incremental equilibrum equation of the space joint element has the form

$$\widetilde{\boldsymbol{K}}_T^{(b)}\Delta\widetilde{\boldsymbol{Q}}^{(b)} = \widetilde{\boldsymbol{P}}_g^{(b)} + \widetilde{\boldsymbol{P}}_t^{(b)} - \widetilde{\boldsymbol{F}}^{(b)}, \tag{11}$$

where $\boldsymbol{K}_T^{(s)}$ is the global tangent stiffness matrix, $\boldsymbol{P}_g^{(s)}$ and $\boldsymbol{P}_t^{(s)}$ are the load vectors and $\boldsymbol{F}^{(s)}$ is the vector of internal loads

$$\widetilde{\boldsymbol{K}}_T^{(b)} = \boldsymbol{A}^{\mathrm{T}}\boldsymbol{K}^{(s)}\boldsymbol{A},\quad \widetilde{\boldsymbol{P}}_g^{(b)} = \boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}_g^{(s)},\quad \widetilde{\boldsymbol{P}}_t^{(b)} = \boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}_t^{(s)},$$
$$\widetilde{\boldsymbol{F}}^{(b)} = \boldsymbol{A}^{\mathrm{T}}\boldsymbol{F}^{(s)},\quad \Delta\widetilde{\boldsymbol{Q}}^{(b)} = \boldsymbol{A}^{\mathrm{T}}\Delta\boldsymbol{Q}^{(s)}. \tag{12}$$

As a results, the space joint element is expressed by means of the thin-walled deegres of freedom of the K beams, connected with the space joint element.

4. Example. Large displacements of three-bar frame

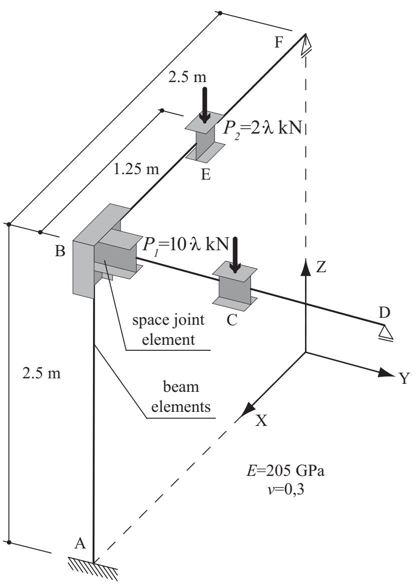

Figure 2: Three-bar frame

The frame is constructed with I100, welded at the node B, and loaded with the concentrated load, as shown in Fig. 2. It is known that the frame with such an orientation of the cross-section cannot be properly modeled using only thin-walled elements [1]. The frame was analyzed adopting two FE shell-beam coupling models: (1) using transients elements and (2) using space joint elements. In order to obtain a proper solution, shown in Fig. 3, both models contained 7446 d.o.f.

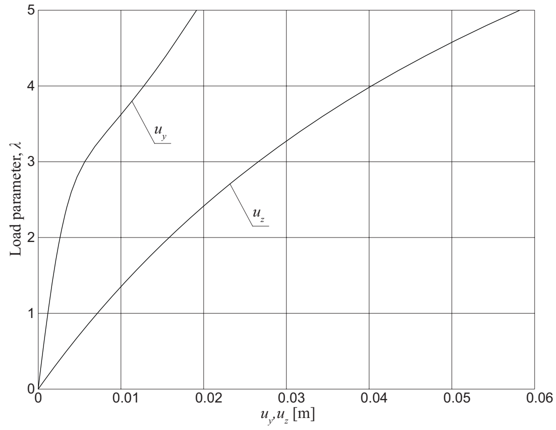

Figure 3: Horizontal and vertical displacements at node C vs load parameter

5. Conclusions

The metod of analysis of thin-walled structures presented in the paper is based on the consistent application of TLA. Problems of defining matrices and vectors for thin-walled element have been overcome using advanced tools of computer science. Results of computations confirmed good effectivenes of the proposed analysis procedure.

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