Reliability of a silo structure with initial geometric imperfections loaded with pressure below atmospheric and wind

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Abstract

A method of reliability estimation of shell structures with random initial geometric imperfections and random loading is considered. An example of calculations of a silo structure is presented. As the silo cylindrical constructions can be loaded with pressure below atmospheric any geometric discrepancies are very dangerous and can significantly reduce their capacities. The initial discrepancies can be described using deterministic relations and random non-homogeneous correlation functions, the parameters of which are chosen on the basis of the EU codes and according to the engineering knowledge. Wind loading is also described as random. Its magnitude and other characteristic parameters and direction are estimated on the basis of the allowable data. The realizations of the random fields and parameters are generated using a conditional rejection method. To calculate the silo reliability a version of a stratified sampling method is applied.

Keywords: reliability, silo structures, random initial geometric imperfections, wind loading

1. Introduction

Silo structures can be made of very thin metal sheeting. Because of the sheet slenderness they can easily lose both general and local stability. Among various reasons which can lead to the reduction of the silo load capacity, initial geometric imperfections are one of the most important. To describe the fields deterministic and random initial geometric imperfections were generated. The silo structures were loaded with pressure below atmospheric. The negative pressure can be caused by a sudden drop of the arching material or a malfunction of roof filters while emptying the silo [3]. Wind loading was also described using probabilistic functions. Its magnitude and direction are considered as random. On the basis of the Monte Carlo reduction methods the silo's reliability was estimated.

2. Silo structure

A cylindrical aluminium silo of $V = 324 m^3$ capacity was considered. The height of the silo shell is 25m. The shell comprises 10 rings, each 2500mm high. The silo sheets thickness are 10.5mm at the bottom and 4mm at the top (total of 10 different values – see Fig. 1). The internal diameter of the shell is the same everywhere and equals $D_w = 4000 mm$. The roof forms a dome with the maximum height of 536mm and 5.5mm of thickness. The bottom of the silo forms a hopper that is 3340mm high and 10.5mm thick. The silo's shell and the roof are made of AlMg3 aluminium metal sheets with the plasticity limit $R_{e,\min} = 70 MPa$ whereas the silo's hopper is made of AlMgSi1 aluminium, with the plasticity limit $R_{e,\min} = 120 MPa$.

The program MSC Visual Nastran [4] was used to carry out the analysis of the silo. Four-node shell elements of size 250×175 mm were applied where the first value corresponds to the vertical direction.

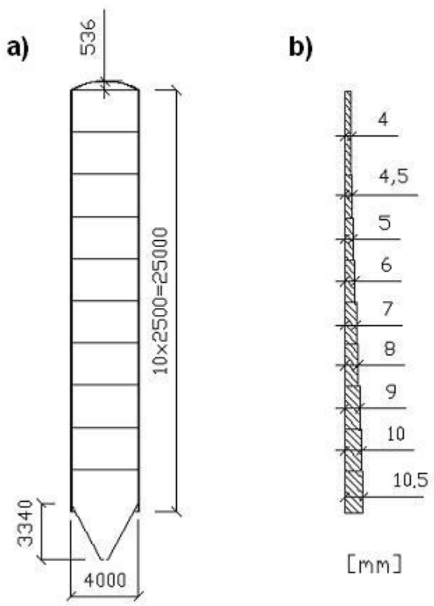

Figure 1: Silo geometry (a) and the sheet thickness (b)

In the areas of the silo's supporting construction, the roof and the hopper, the finite element mesh sizes were appropriately smaller. In total, there were 11232 shell elements and 11304 nodes. The aluminium elasticity modulus $E = 69 GPa$ and Poisson's ratio $\nu = 0.3$ were assumed. A corresponding model was computed in a commercial program SOFiSTiK [5] to assure that the numerical results are trustworthy. The SOFiSTiK model consists of 10432 shell elements and 10494 nodes.

3. Geometric imperfections

3.1. Deterministic description of imperfections in Polish Standards (PN) and in Eurocode 3 (EC)

The first step in the generation of any imperfections in a structural design is to make initial assumptions about their size. The adaptation of imperfections on an idealized lateral surface of the structure needs to rely on a set of normatives, which regulate the probable dimensions of imperfections' that can naturally appear on a calculated silo. Initially, the first assumptions for numerical computations have to be estimated on the basis of formulas given in the design guidelines. Currently, there is a possibility to alterably conduct the dimensioning basing on Polish Standards (PN) or the EC 3.

To measure the indentations one can use simple surveys in certain positions of both main directions of silo's lateral surface – either meridional or circumferential direction.

In calculations done on the basis of design codes it is initially assumed, that the limit size of a dent t_{v0} equals 2cm, whereas the maximum length of coverage of this indentation l_m equals 2m, as suggested by the formula: $t_{v0} = 0.01 \cdot l_m$.

In case of areas, where the meridian compressive stresses appear, indentation measurements along the cylinder's generatrix (in meridian direction) have to be conducted in both of the main directions, using surveys of a length ℓ_{gX} determined by the formula (1):

$$l_{gx} = 4\sqrt{rt} \tag{1}$$

The depth of the initial dent is assessed as dependant on the relative indentation parameter U_{0X} , defined by the formula (2):

$$U_{0x} = \frac{\Delta w_{0x}}{\ell_{gx}} \le U_{0,\max} \tag{2}$$

Therefore: $\Delta\omega_{0X} = 2cm$ (as assumed in the first estimation).

So, the length of an indentation along the cylinder's generatrix equals, for example:

$\ell_{gX} = 4\sqrt{Rt} = 4 \cdot \sqrt{2 \cdot 0{,}004} = 0{,}357m$ – dla t = 0,004m

After calculating the revised depth of the initial indentation $\Delta\omega_{0X}$, one can obtain: $\Delta w_{0X} \le U_{0,\max} \cdot \ell_{gX}$, resulting in the data for a given section of the silo, as presented in Table 1.

Thus, according to Eurocode 3 – based calculations, one obtains a maximum length of a meridian indentation equal to: $\Delta\omega_{0X,C-class} = 0{,}57cm \div 0{,}93cm$ for a silo of a C – class erection standard, which is a lesser value, than the initially assessed one.

In case of areas, where the circumferential compressive stresses or shear stresses appear, indentation measurements along the cylinder's perimeter (in circumferential direction) have to be carried out using surveys of a length $\ell_{g\theta}$ determined by the formula (3):

$$\ell_{g\theta} = 2{,}3 \cdot (\ell^2 Rt)^{0{,}25} \text{, but with respect to: } \ell_{g\theta} \le R \tag{3}$$

where: ℓ is the meridian length of shell's segment, $\ell = 2\pi R = 2\pi \cdot 2m = 12{,}566m$.

According to PN-B-03202:1996 the calculations mentioned above are done using a different formula (4):

$$l_m = \frac{2{,}3R}{\sqrt{\frac{R}{l}} \cdot \sqrt{\frac{R}{t}}} \le R \tag{4}$$

Yet, this is just a recast version of the Eurocode formula, and it's giving identical results.

The depth of the initial dent is in this case assessed as dependant on the relative indentation parameter $U_{0\theta}$, defined by the formula (5):

$$U_{0\theta} = \frac{\Delta w_{0\theta}}{\ell_{g\theta}} \le U_{0,\max} \tag{5}$$

After calculating the revised depth of the initial indentation $\Delta\omega_{0\theta}$, one may obtain: $\Delta w_{0\theta} \le U_{0,\max} \cdot \ell_{g\theta}$, resulting in the data for a given section of the silo, as presented in Table 2 for the first section of construction in question.

Thus, according to Eurocode 3 – based calculations, one obtains a maximum length of a circumferential indentation equal to: $\Delta\omega_{0\theta,C-class} = 3{,}90cm \div 4{,}97cm$ for a silo of a C – class erection standard, which is a greater value, than the initially assessed one.

Summarizing, the methods of calculation of the indentations' dimensions are identical in Polish and European design codes and are giving the same results. However, in the Eurocode 3 additional conditions that may increase the depth of the indentation are stated (showing more consistency with the realities of engineering).

Therefore it is shown, that the first estimation – the depth of the dent t_{v0} equal to 2cm and the indentation's scope length in both main cylinder directions respectively equal to 0.6m and 2m – may be implemented in the process of computational models' generation in numerical programs, as a fine starting point for further, more accurate deliberations.

3.2. Random description of imperfections

Geometric initial imperfections of any shell or plate can be described by means of two-dimensional random fields. Here, three random fields of initial imperfections were analyzed: non-correlated field (the white noise field), homogenous correlated field, as well as non-homogenous correlated field. Due to the lack of data describing fields of real imperfections, the parameters of the correlate functions were assumed a priori; however, these reflect the standard values.

In the first case the initial imperfections were described by means of non-correlated random field (i.e. the white noise). Geometric deviations are defined in each point irrespectively of the remaining ones by means of uniform distribution. For the purpose of the calculations, three values of the deviations t_{v0} in relation to the cylindrical surface of the shell were assumed: 50, 25, and 20mm. The roof's and hopper's geometry, as well as the lower edge of the silo below the level of the support area were assumed as ideal. Standard deviation of the field described by the uniform distribution can be calculated according to the formula (6), (for $t_{v0} = 25$mm):

$$\sigma = \frac{2t_{v0}}{\sqrt{12}} = \frac{2 \cdot 0.025}{\sqrt{12}} = 0.0144\text{m} \tag{6}$$

The model including such initial imperfections is purely theoretical as, in reality, a model containing deformations of such scale and intensity would never be approved for exploitation. However, an analysis of such a construction enables us to define differences in limit load, which appear with different models of imperfections.

Table 1: Calculated depth of the initial dent $\Delta\omega_{0X}$, presented for the first section of the silo (from 0m to 2.5m below the top edge)

				A – class silo:		B – class silo:		C – class silo:	
sect.	t [cm]	R [m]	ℓ_{gX} [cm]	$U_{0,\max}$	$\Delta\omega_{0X}$	$U_{0,\max}$	$\Delta\omega_{0X}$	$U_{0,\max}$	$\Delta\omega_{0X}$
1	0.4	2.00	35.78	0.006	0.21 cm	0.010	0.36 cm	0.016	0.57 cm

Table 2: Calculated depth of the initial dent $\Delta\omega_{0X}$, presented for the first section of the silo (from 0m to 2.5m below the top edge)

					A – class silo:		B – class silo:		C – class silo:	
sect.	t [mm]	R [m]	$\ell_{g\theta}$ [cm]	$\ell_{g\theta p}$ [cm]	$U_{0,\max}$	$\Delta\omega_{0\theta}$	$U_{0,\max}$	$\Delta\omega_{0\theta}$	$U_{0,\max}$	$\Delta\omega_{0\theta}$
1	4	2	243.84	200.00	0.006	1.46 cm	0.010	2.44 cm	0.016	3.90 cm

Generating correlated random fields requires an advanced software. A method proposed in [6] that enables generating two-dimensional and three-dimensional truncated Gaussians random fields was used. The fields can be described by means of homogenous and non-homogenous correlated functions. In the algorithm the sequence method that uses an expanding area of generated points – random variables – was applied. In this way, discrete fields of large area (thousands of points) can be generated. Conditional probability distributions as well as the rejection method were employed. The input data of the algorithm are: a covariance matrix, expected values of random variables as well as the envelope of the random values determined in every point of the field. The envelope of the random field used in this algorithm enables generating sets of random variables with boundary conditions corresponding precisely to the analyzed engineering object also on the basis of experimental data or measurements of actual constructions. The algorithm generates circular data, which is of particular importance when analyzing a silo.

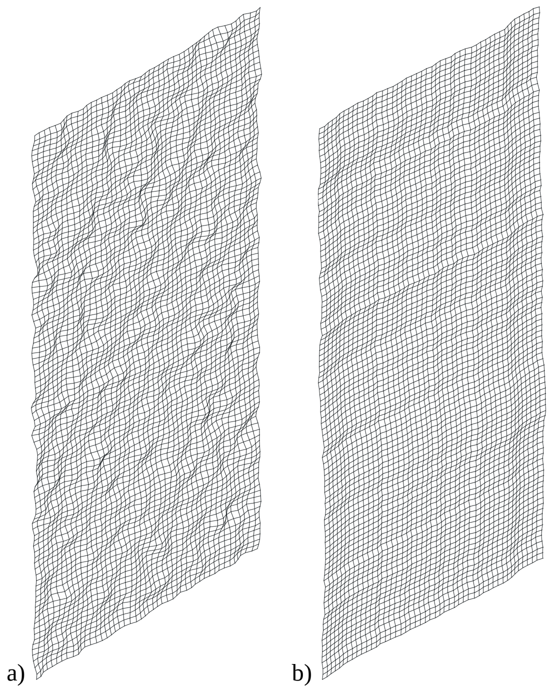

Figure 2: Realizations of the silo initial of the initial geometric imperfection: a) homogeneous field, b) non-homogeneous field

In this paper two random fields were generated. For lack of the appropriate data the correlation function is usually chosen arbitrarily. Apparently the function should confirm that the correlation between random variables vanishes when the random point distance increases.

The initial geometric imperfections were described by means of the following homogenous correlated function (7):

$$K(\Delta x_1, \Delta x_2) = \sigma^2 \exp\left(-(\beta\Delta x_1)^2 - (\gamma\Delta x_2)^2\right) \qquad (7)$$

where: Δx_1 and Δx_2 are distances between the points of the field along the horizontal and vertical axis, σ is a standard deviation describing the variability of the field, and β and γ are the decay coefficients. In the example the following parameters were assumed: $\sigma = 0.01443\text{m}$, $\beta = \gamma = 2,2\text{m}^{-1}$.

For comparison the same standard deviation as in the non-correlated field (6) was taken. The following range of truncation of the Gaussian distribution was applied:

$$\pm 3\sigma = \pm 3 \cdot 0.0144 = 0.0433\text{m} \qquad (8)$$

A set of realizations – initial geometric imperfections of the silo – were generated. The field of imperfections, for which the calculations were made, is illustrated by Figure 2a. The introduced deformations can describe the dented metal sheeting of the construction. It should be pointed out, though, that the imperfections as shown in the illustration are of bigger scale, as, in reality, only in some places, their amplitudes exceed the value of 40mm (8). For such type of imperfections the maximum value of negative pressure $p_{\text{hom}} = 2.282kPa$.

The second analysed field was non-homogenous field described by the following function (9):

$$K(\Delta x_1, \Delta x_2) = \sigma \cos(\alpha\Delta x_1) \exp(-\beta\Delta x_1 - \gamma\Delta x_2) \qquad (9)$$

where the following parameters were assumed: $\sigma = 0.0144\text{m}$, $\alpha = 0.2\text{m}^{-1}$, $\beta = \gamma = 0.005\text{m}^{-1}$. In this case the circular shape of the construction is taken into account by means of the cosine function. A model of a field of imperfections is presented in Figure 2b.

On the basis of generated imperfections one can conclude that this kind of a field enables modelling deformations related to the joints made, fusions in the metal sheeting and damages existing on whole part of the tank.

2000 realizations of initial imperfections were generated. Then, they were classified according to the average amplitude of the displacements. For the purposes of the calculations three fields were chosen: one with the lowest amplitude, one with an average amplitude and one with the highest average amplitude.

4. Negative pressure loading acting on silos with geometrical imperfections

One of the variants of the load which was adopted for analysis is the pressure below atmospheric, an important factor with a strong tendency to weaken the silo's shell structure and is, in some extreme cases, even able to lead to the loss of silo's stability. The negative pressure in the silo chamber may occur due to various different reasons: a sudden subsidence of material residing at the top levels of the chamber, roof filters malfunction during an emptying of a leak – proof silo or an explosion of dust from materials stored inside the chamber. The value of negative pressure created because of the dust explosion is a sum of influences of many various factors, such as the material type, its concentration and the ignition energy of its dust, also the place where the explosion took place, the type and size of the security flaps and the explosion pressure growth factor.

The first stage of the analysis involved calculations made for a silo of a perfect shell geometry. The value of the maximum pressure that was obtained, i.e. $p_{ideal} = 2.717kPa$ constituted a reference point for further calculations, in which different types of geometric imperfections created additional factor reducing the load capacity of the ideal shell.

The next stage was to analyze the influence of deviations in the form of local dents of the shell on its load capacity. It was assumed that the remaining part of the shell was not subjected to any deformations.

The dent was estimated according to the standard design code formula (1) ÷ (4) where $l_m = 2000mm$ and the depth of the dent corresponding to this deformation was $t_{v0} = 20mm$. Calculations were made for ten different height options, on which the deformation was located. The middle points of the dents were placed on the following heights: 2.5, 6.25, 8.75, 11.25, 13.75, 16.25, 17.5, 18.75, 20.0, 21.5, 22.5, and 23.5m. The lowest load value $p_{dent1} = 2.155$ kPa was obtained when the dent is placed on the height of 21.5m. It can be explained by the fact that this area of the shell is built of the thinnest metal sheeting. Besides that, the strengthening influence of the silo's roof is not dominating.

For cases where the dent is of elliptic shape additional calculations were made. The same length of the dent along the perimeter $l_m = 2000$ mm as well as a much shorter generating line of the shell close to the standard value of 600 mm were assumed. The calculations were made for three localisations of the dents: 20.0, 21.5, and 22.5 m. The maximum values of negative pressure were: $p_{wg2a} = 2.375$, $p_{wg2b} = 2.355$ and $p_{wg2c} = 2.388kPa$, so they were higher than in the case of a symmetric deformation but lower than in the case of the ideal shell.

The next option introduced an initial deformation in the form of two dents situated on the height of h = 21.5 m, where the placed deformation was least favourable. The dents were located on two opposite points of the cross section. The maximum value of negative pressure reached $p_{wg3} = 2.173kPa$, so it appeared higher than in the case of a single dent.

A detailed analysis of imperfections including a single dent or a group of a few deformations require making an inventory of real cases of such deformations.

The maximum negative pressure that was obtained for non-correlated field of imperfections, with the maximum value of imperfections ±50 mm, reached $p_{wn1} = 7.390kPa$, and for ±25 mm $p_{wn2} = 4.704kPa$. Additionally, calculations for three silo models with maximum imperfections equalling 20 mm, being close to the ones described by norms were made: $p_{wn3a} = 4.1709$, $p_{wn3a} = 4.2038$, $p_{wn3c} = 4.203545kPa$.

In all the cases mentioned above, the value of the negative pressure was much higher than in the case of the ideal surface of the shell. This being so, one can conclude that that chaotically distributed dents strengthen the shell.

For the case when the imperfections field is described by a homogeneous correlation function the calculations are performed 3 times. The negative pressure, which the silo is able to withstand, with such indentations initially input, was equal to $p = 2.282kPa$. Slightly greater values are obtained for the imperfections field described by a heterogeneous correlation function. 2000 generations of imperfections values are generated. These results are then classified in terms of the average amplitude of displacements. Three fields of the lowest, average and maximum mean amplitude are chosen for the calculations. The limit negative pressure values are as follows: $p_1 = 2,55kPa$, $p_2 = 2,544kPa$, $p_3 = 2,4704kPa$.

5. Wind loading acting on silos with geometrical imperfections

Second type of loads, analyzed in the present work is the pressure/suction of the wind. Wind load may be, according to EC3 normatives, adopted for the calculations in three forms – as an accurate loading, expressed with a trigonometric functions series formulas (a); as a segment loading, where the previous variant is simplified to three zones of different values of wind pressure p_i (b); or expressed as a loading of only one simplified value p_0, independent from the angle of wind pressure direction (c). These approaches are illustrated in Fig. 3.

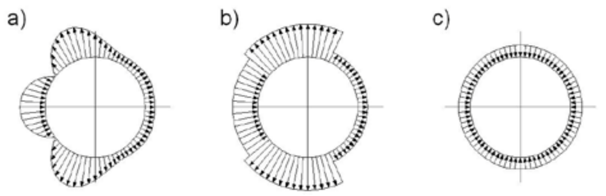

Figure 3: Types of wind loading [1]: a) exact description – trigonometric functions, b) segments loading, c) simplified homogeneous loading

The first type of loading which is being considered is the wind loading on the ideal surface of the silo, with no dents. In this example, on a model with an idealized geometry, three forms of wind loading were imposed: an exact trigonometric description, segment and simplified. A set of equations needed for the accurate model of the wind, which was used in the most exact numerical calculations are as follows:

Defining the wind load characteristic value as:

$$p_k = q_k \cdot \beta \cdot C_e \cdot C \quad (10)$$

a) if $z \in \langle 0m;10m \rangle$, then the equation states as follows:

$$C_e C(z,\alpha) = 1,0 \cdot (-0,356 + 0,322\cos\alpha + 0,636\cos 2\alpha + +0,501\cos 3\alpha + 0,058\cos 4\alpha - 0,128\cos 5\alpha - 0,034\cos 6\alpha) \quad (11)$$

Table 3: Calculated depth of the initial dent $\Delta\omega_{0X}$, presented for the first section of the silo (from 0m to 2.5m below the top edge)

variant	load form	f_{crit} [-]	p_{crit} [kPa]
A	exact loading (trigonometric function)	0.517	2.59
B	segment loading (3 values of the load)	0.508	2.54
C	simplified loading (1 uniform load value)	0.494	2.47

b) if $z \in \langle 10m; 20m \rangle$, then the equation states as follows:

$$C_e C(z,\alpha) = (0{,}8 + 0{,}02 \cdot z) \cdot (-0{,}356 + 0{,}322\cos\alpha + 0{,}636\cos 2\alpha + +0{,}501\cos 3\alpha + 0{,}058\cos 4\alpha - 0{,}128\cos 5\alpha - 0{,}034\cos 6\alpha) \quad (12)$$

c) if $z \in \langle 20m; 30m \rangle$, then the equation states as follows:

$$C_e C(z,\alpha) = (0{,}9 + 0{,}015 \cdot z) \cdot (-0{,}356 + 0{,}322\cos\alpha + 0{,}636\cos 2\alpha + +0{,}501\cos 3\alpha + 0{,}058\cos 4\alpha - 0{,}128\cos 5\alpha - 0{,}034\cos 6\alpha) \quad (13)$$

The results of calculations of preceding formulas are presented in Table 3.

It is observed, that the change of the form of the load does not significantly affect the results, which partly confirms the design standards procedures, and also partly indicates the rationality of an assumption of the easiest mathematical form of wind pressure in the design process. However, in computations of less complex structures or fragments of larger structures one can use more accurate, detailed wind load forms, as it is shown that they do not change the duration of the model calculations. In further calculations, which are taking into account the possibility of imperfection occurrence, as suggested by the mentioned normatives, a full description of the function of the wind was used.

Another goal of the structural analysis, presented in this article, is to analyze the critical pressure of a silo under the wind load acting directly on the geometric centre of the set point imperfection. An elliptical dent of fixed depth (2cm) and fixed size (60cm high, 200cm wide) will be set in this series of computations on the lateral surface of the silo, on some explicit heights above the ground level, similar to the examples from the above paragraphs, for a better correlation of results.

Critical pressure as dependent on the location of the design code suggested dent on selected heights is shown in Fig. 4.

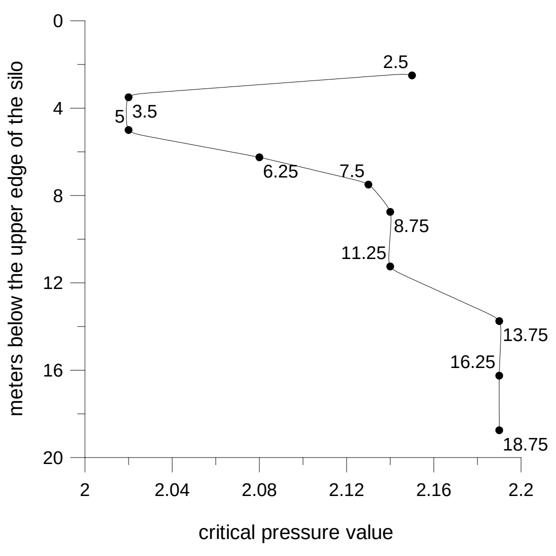

Figure 4: Critical pressure, as dependent on the location of the design code suggested indentation on selected heights

When observing the values of critical pressure for this specific numerical example, obtained from the calculation described above, one can see that a part of the structure, located in the area between 3 and 5 meter below the upper edge of the jacket is the most sensitive area to the initial imperfection occurrence. This is surely because of the smallest thickness of metal sheeting in these sections of the silo, as well as the fact that the wind loading on these heights has the greatest values. Dents below the 13 meter (counting from the upper edge of the jacket) does not affect the carrying capacity of the silo significantly. It behaves under wind loading practically in an identical way, albeit the critical pressure value is significantly lower than for the construction of an ideal geometry. The smallest value of critical pressure is $p = 2{,}02kPa$ in this case.

Another important calculations series, which are presented in this article is the analysis of the critical pressure change in a silo under the wind loading of a fixed value, which is acting on the lateral surface of the silo with a single imperfection on it. The imperfection is set on one level (at 350cm below the top of the silo jacket), but it will change its angular position in relation to the wind pressure direction. The indentation's location will be measured as an angle between the direction of the geometric centre of the elliptical indentation and the direction of the wind. The dent is characterized by the same dimensions as in the previous examples. The level of imperfection location has been chosen deliberately – the impact of the wind at this geodetical level gives the slightest critical pressure ratio.

The relationship between the location of a design code corresponding dent and the silo's critical pressure is presented in Fig.5.

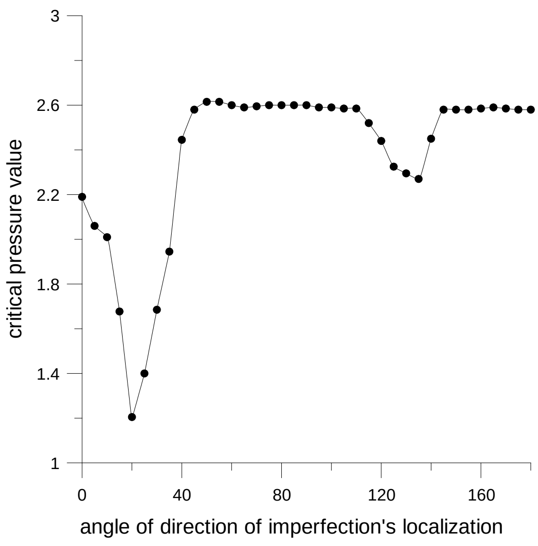

Figure 5: The relationship between the location of a design code corresponding dent and the silo's critical pressure

When observing the values of critical pressure for the example described above one can see that there are two zones on the silo's structure where the wind interacts with the indentation placed on a different angle than the wind load direction's angle clearly to the disadvantage of the construction. These are the ranges where the wind blows directly in the centre of the indentation, on its edge or nearly beyond it (angular difference of the directions equals $0^o - 40^o$) and in the range where the wind suction is drastically reduced (angular difference of the directions equals $110^o - 140^o$). For other ranges of angles between the dent direction and wind direction no clear effect of geometric imperfections on capacity of the silo is visible – the silo behaves the same way as a design with a perfect geometry. The smallest value of pressure which the construction is able to withstand equals to $p = 1,21 kPa$, which is a value significantly lesser than those obtained in the previous series of calculations.

The final stage of the analysis of the silo will consider whether there is a discernible influence the value of wind loading (controlled with its relevant components of characteristic load) and the results of the critical pressure. The following values are determined to be variant: q_k, β and C_e. The loaded silo has a perfect geometry. For a full mapping of the extreme situations the load factor 1.0 is taken as an equity for the surface wind loading producing a critical pressure value equal to $p = 2,59 kPa$.

The critical pressure values, as dependent on the value of the wind load, normalized on the basis of previous assumptions is shown in Fig. 6.

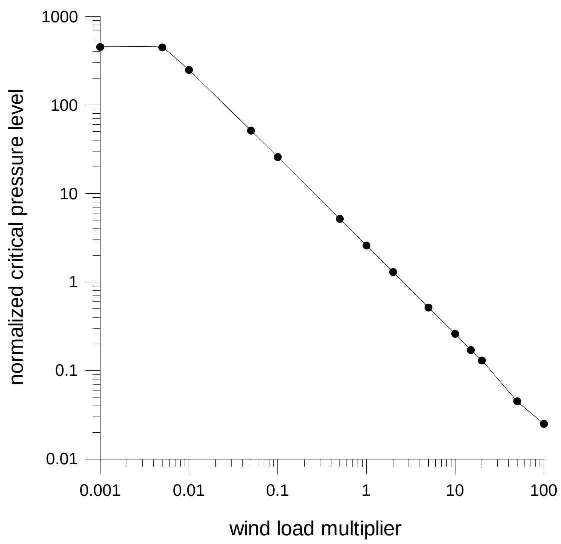

Figure 6: Critical pressure, as dependent on the value of the wind load, normalized on the basis of proper assumptions

When observing the values of critical pressure acquired in the calculations illustrated above, one can see that the in the range of the possible wind loading values, the silo's critical pressure is nearly fully proportional to the level of the normalized load multiplier. When the load multiplier is reduced a hundred times in comparison with the normalized factor 1.0 the failure state – causing pressure is so large that any further reduction in the value of the wind factor has no effect on the stability of the structure. However, at each step of a successive wind value increase the critical pressure, causing the failure of the structure can be expected to lessen.

6. Conclusions

The presented results of the numerical calculations revealed that the initial geometric imperfections significantly influence the mechanical behavior of silo structures. The imperfections can be modeled using data described by the design codes and probabilistic methods. The calculations, aided by the statistical analysis of measured wind parameters have proved the capability of the stochastic methods. The formulation of a methodology for the identification and description of the initial geometric imperfections and random wind loading can ensure a better and much safer design.

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