Parallel cracks propagation in orthotropic polymer matrix composite subjected to tension

Tomasz Sadowski [1,*], Livu Marsavina[2] and Eduard Craciun[3]

[1]*Department of of Solid Mechanics, Lublin University of Technology*
ul. Nadbystrzycka 40, 20-618 Lublin, Poland
e-mail: t.sadowski@pollub.pl

[2]*Department of Strength of Materials, Politehnica University of Timisoara*
Blvd. M. Viteazu, Nr.1, Timisoara 300222, Romania
e-mail: msvina@mec.upt.ro

[3]*Department of Mathematics, "Oviduis" University of Constanta*
Mamaia 124, 900540 Constanta, Romania
e-mail: mcraciun@univ-ovidius.ro

Abstract

Parallel cracks in an orthotropic polymer matrix composite is an important case in engineering practise. Two parallel cracks of different length where studied under uniaxial tension state in order to estimate: stress, strains and displacement distributions. In the first approach - a mathematical model of cracks in the orthotropic composite was proposed. Starting from the boundary conditions, constitutive equations and far field loading one can obtain the representation of the incremental displacement, stresses and strains fields using two complex potentials (Guz method). Using the Cauchy's integral theory the considered fields in the vicinity of the single crack and co-linear cracks tips where determined. The second approach – numerical by application of Finite Element Method - allows for estimation: the fields in the vicinity of the crack tips, critical values which cause cracks propagation and cracks interaction in the considered polymer matrix composite. The presented results were verified experimentally.

Keywords: Polymer matrix composite; parallel cracks; cracks interaction, uniaxial tension

1. Introduction

In the classical approach the problem of parallel cracks propagation is reduced to a set of integral equations, which are solved numerically, but the solution is complex and has slow convergence. Kachanov [1, 2], for example, assumed that traction in each crack can be represented as a sum of an uniform component and a non-uniform component. The interaction among the cracks is only due to the uniform components. These assumptions simplify considerably the mathematical approach and allow for 'closed-form' solutions to be obtained for some cases. The improvement of this method was proposed by Li et. al. [2] for closely spaced multiple cracks. Gorbatikh et al. [4] developed a method that predicts Stress Intensity Factors (SIF) at tips of two-dimensional cracks at distances that may be extremely small, up to 1% of the crack lengths. Other approximate method approach was proposed by Petrova et all. [6] for modelling of interaction of different sets of cracks.

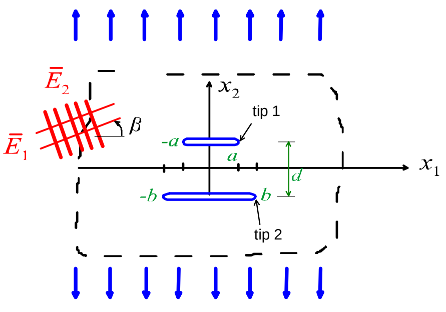

Figure 1: Two parallel unequal cracks

In the present paper we consider two unequal cracks in orthotopic an elastic composite Fig.1. We suppose that the material is unbounded and is subjected to normal uniform loading *p*. The analytical model (Guz's representation theorem [7]) and numerical one where formulated. Additional experimental tests confirm obtained theoretical results.

1. Mathematical modelling

As it was shown by Guz [7] the elastic state of the body can be expressed by two analytical complex potentials $\Psi_j(z_j)$ defined in two complex planes z_j, $j=1,2$. We denote by u_1, u_2 the involved components of displacements and by θ_{11}, θ_{12}, θ_{21} and θ_{22} the stresses.

According to Guz's representation formulae, we have:

$$u_1 = 2\,\mathrm{Re}\{b_1\Phi_1(z_1) + b_2\Phi_2(z_2)\} \tag{1}$$

$$u_2 = 2\,\mathrm{Re}\{c_1\Phi_1(z_1) + c_2\Phi_2(z_2)\} \tag{2}$$

$$\sigma_{11} = 2\,\mathrm{Re}\{a_1\mu_1^2\Psi_1(z_1) + a_2\mu_2^2\Psi_2(z_2)\} \tag{3}$$

$$\sigma_{12} = -2\,\mathrm{Re}\{\mu_1\Psi_1(z_1) + \mu_2\Psi_2(z_2)\} \tag{4}$$

$$\sigma_{21} = -2\,\mathrm{Re}\{a_1\mu_1\Psi_1(z_1) + a_2\mu_2\Psi_2(z_2)\} \tag{5}$$

$$\sigma_{22} = 2\,\mathrm{Re}\{\Psi_1(z_1) + \Psi_2(z_2)\} \tag{6}$$

In the above relations $z_j = x_1 + \mu_j x_2$, for $j = 1,2$ and :

$$\Psi_j(z_j) = \Phi'_j(z_j) \tag{7}$$

a_j, b_j, c_j, $j=1,2$ are parameters.

In the paper the complex potentials $\Psi_j(z_j)$ were formulated for two parallel cracks, Fig. 1. The cracks interaction problem was analysed estimating the energy release rate criterion *G*

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corresponding to the crack propagation at fourth crack tips: - *a*, -*a*, *b*, -*b*. The cracks start to propagate when:

$$G(-a) = 2\gamma \ , \ G(-a) = 2\gamma \ , \ G(b) = 2\gamma \ , \ G(-b) = 2\gamma \quad (8)$$

where γ is the fracture surface energy of the composite along the cracks propagation line.

2. Numerical modelling

Finite Element Method (FEM) was used to estimate the fracture parameters at the tip of the cracks. The Modified Crack Closure Integral (MCCI) was applied to extract the SIFs and energy release rate (ERR) from the numerical results.

The concept of a closure integral was first used by Irwin to relate the global ERRs to the crack tip SIFs. Rybicki and Kanninen [8] modified the procedure so that only one analysis was necessary. They observed that the stress field in front of the crack-tip is similar to the stress field that would exist over a closed portion of the crack. They proposed determining ERRs from the integrals:

$$\begin{aligned} G_I &= \lim_{\Delta c \to 0} \frac{1}{\Delta c} \int_0^{\Delta c} \sigma_{22}(x_1,0) \cdot u_2(\Delta c - x_1, 0) dx_1 \\ G_{II} &= \lim_{\Delta c \to 0} \frac{1}{\Delta c} \int_0^{\Delta c} \sigma_{12}(x_1,0) \cdot u_1(\Delta c - x_1, 0) dx_1 \end{aligned} \quad (9)$$

where G_I and G_{II} are the interface ERRs, u_1 and u_2 are the opening, respectively sliding displacements of two points from the opposite crack flanks situated at a small distance behind the crack tip Δc. With the quarter-point elements having symmetric nodal positions the crack closure integral can be performed independently for the opening (Mode I) and sliding (Mode II) displacements. This yields decoupled values for G_I and G_{II}, which are used to compute K_I and K_{II}. The determination of the SIF based on MCCI method is implemented in FRANC2D/L [9].

The MCCI method was used successfully for determination of SIF's and ERR in bi-material models with cracks approaching the interface [10], and for kinked cracks from interface [11].

3. Numerical results

In order to validate the analytical predictions two cases: equal and unequal parallel cracks were considered numerically. Fig. 2 presents σ_{22} stress distribution along the cracks line for the case of strong cracks interaction.

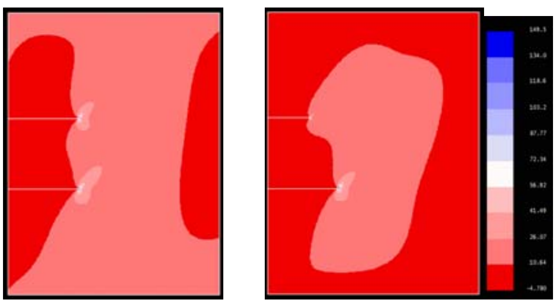

Figure 2: σ_{22} stress distribution in equal and unequal cracks

4. Conclusions

Two methods were applied to the analysis of the two parallel cracks: analytical and numerical.

The mathematical approach bases on two analytical complex potentials $\Psi_j(z_j)$ defined in two complex planes z_j, $j=1,2$. Both potentials were derived for considered cracks configurations. They allow estimating the states of: displacements, strains and stresses. It leads to calculation of the SIFs and cracks interaction.

The numerical solution for calculation of SIFs and ERRs was done with application of the MCCI method. Obtained displacements, strains, stresses distributions and SIFs are similar to the analytical one qualitatively and quantitatively.

The interaction of two parallel cracks depends on considered cracks configuration. If the distance between cracks is much smaller than their lengths, the interaction between the cracks is strong.

The experimental results confirm the theoretical and numerical calculations.

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