Analysis and design of thermo-mechanical interfaces

Krzysztof Dems[1] **and Zenon Mróz**[2]
[1] *Technical University of Łódź K-411, ul. Żeromskiego 116, 90-924 Łódź, Poland.*
[2] *Institute of Fundamental Technological Research, ul. Pawińskiego 5B, 02-106 Warsaw, Poland.*

Abstract

The thermo-mechanical structure with internal interface of varying shape and material properties is considered. The different thermal and mechanical nature of this interface is discussed. Next the relevant optimality conditions for arbitrary thermal and mechanical behavioral functional with respect to shape and material properties are formulated and expressed using the computer oriented approach based on finite element method. Several examples of interface design illustrate the applicability of proposed approach to control the structural response under the applied thermal and mechanical loads.

Key Words: Sensitivity analysis, optimization, heat transfer, thermo-elasticity

1. Introduction

The present paper constitutes an extension of previous author's works in the area of sensitivity analysis and optimal design of material interfaces for a wide class of boundary-value problems (cf. [1] - [5]). The multi-phase structure is subjected to the thermal or thermo-mechanical boundary conditions and its response is specified in terms of the thermal and mechanical functionals. Their variation is considered with respect to shape of interface separating different phases of structural material. The optimality conditions are derived for the assumed cost function and behaviour constraints. The present analysis will be confined to geometrically linear theory, assuming thermally anisotropic material and nonlinear stress-strain relations.

2. Formulation of problem

In this paper we shall discuss the optimal design of a composite multi-phase structure, for which the internal interface separating these phases can undergo shape modification, as shown in Fig. 1. The material properties of this interface can also be modified during optimization process. The shape variation of structural domain Ω associated with internal boundary modification can be defined as an infinitesimal transformation process

$$\Omega \Rightarrow \Omega^t : \mathbf{x}^t = \mathbf{x} + \mathbf{v}^p(\mathbf{x},\mathbf{b})\delta b_p \qquad (1)$$

where $\mathbf{v}_p(\mathbf{x},\mathbf{b})$ denotes a transformation velocity field associated with shape parameter b_p treated as time-like parameter. Such description of domain modification allows us to use the material derivative (or rate) concept in deriving the desired sensitivities of considered functional.

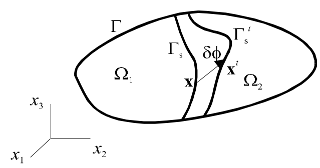

Figure 1: A structure occupying domain Ω with varying interface Γ_s (b)

An interface shown in Fig. 1 can represent either the internal surface separating two materials with different thermal and mechanical properties or can be treated as a discrete material interface inclusion of vanishing thickness with structure and properties differing from the base material structure. The latter interface can be regarded as surface on which some thermal and mechanical state fields exhibit discontinuities.

The behaviour of structure subjected to thermal and mechanical loads can be described by the sets of equations describing heat transfer within structure domain and mechanical response due to applied load, supplemented with proper sets of boundary conditions. Some additional thermal and mechanical conditions must be also specified along the interface separating the phases of considered structure.

3. Classification of interfaces

Now we classify the types of thermal and mechanical interfaces within the structure.

Consider first the interface separating materials with different thermal or/and mechanical properties. For thermal response of structure the continuity of temperature and normal heat flux across interface is assured, while the mechanical response of structure is characterized in this case by continuity of displacements and internal tractions along interface.

More difficult conditions appear when the interface constitutes a thin inclusion surface with different thermal and/or mechanical properties than the material of structure. When the conductivity coefficient of interface is considerably smaller than the coefficients of structural material, then the interface can be considered as isolating surface on which the discontinuity of temperature fields occurs and continuity of normal heat flux is preserved. On the other hand, when the conductivity of interface is considerably greater than the conductivity of structural material, then the interface plays the role of conductor transferring heat along its surface. In this case, we can assume the continuity of temperature field across the interface whereas the normal heat flux is allowed to suffer discontinuity.

Similar behavior of interface inclusion can be observed for the mechanical system. Depending on the ratio of material stiffness properties of interface and base material, the interface within elastic structure can be considered as reinforcing or softening surface. In the former case, when the interface is stiffer than the base material and plays a role of discrete reinforcing surface, the discontinuity of internal tractions is observed, whereas the displacement field retains continuity. On the other hand, for a weak interface (in mechanical sense) it can be considered as softening surface preserving the continuity of internal tractions which are proportional to the jump in displacements across interface.

Considering now simultaneously the thermo-mechanical system with the interfaces discussed above, we obtain the wide class of different thermal and mechanical structural responses due to the properties of interface. The possible combinations of thermal and mechanical properties of an interface are shown in Table 1.

Table 1: Possible classes of interface

Thermal conductivity of interface	Mechanical properties of interface		
	weak	strong	separating interface
small	isolating softening surface	isolating reinforcing surface	isolating & mechanically separating surface
great	conducting softening surface	conducting reinforcing surface	conducting & mechanically separating surface
separating interface	thermally separating softening surface	thermally separating reinforcing surface	thermally & mechanically separating surface

4. Response behavioral functionals

Let us now define arbitrary thermal and mechanical behavioral functionals which can play the role of objective functional or global constraint in optimization procedure. The thermal functional can be assumed in the general form depending on the temperature and heat flux fields T and $\mathbf{q}$, namely :

$$G_t = \int \Psi_t(T, \nabla T, \mathbf{q}) d\Omega + \int \Phi_t(T, q_n) d\Gamma + \int \hat{\Phi}_t d\Gamma_s \tag{2}$$

where $\hat{\Phi}_t = \hat{\Phi}_t(\bar{T}, <T>, q_n)$ for the case of structure with isolating interface and $\hat{\Phi}_t = \hat{\Phi}_t(T, <\bar{q}_n, <q_n>)$ when the conducting interface appears within structure domain. $\bar{T}$ and $\bar{q}_n$ denote here the average temperature and normal heat flux, respectively, while $<T>$ and $<q_n>$ are the jumps of proper quantities along interface. On the other hand, the mechanical behavioral functional can be considered as depending on displacement, strain and stress fields $\mathbf{u}$, $\boldsymbol{\varepsilon}$ and $\boldsymbol{\sigma}$, and it is given in the form:

$$G_m = \int \Psi_m(\sigma, \varepsilon, \mathbf{u}, \sigma^i) d\Omega + \int \Phi_m(\mathbf{T}, \mathbf{u}) d\Gamma + \int \hat{\Phi}_m d\Gamma_s \tag{3}$$

where now $\hat{\Phi}_m = \hat{\Phi}_m(<\mathbf{T}>, \mathbf{u}, \boldsymbol{\sigma}^i)$ for structure with reinforcing interface and $\hat{\Phi}_m = \hat{\Phi}_m(\mathbf{T}, <\mathbf{u}>, <\mathbf{u}^i>)$ in the case of structure with softening interface.

5. Optimality conditions

The typical optimization problem can be stated, for instance, as follows:

$$\text{Min. } G_t \quad \text{subject to} \quad G_m - G_m^0 \le 0 \ , \ C - C_0 \le 0 \tag{4}$$

where G_m^0 denotes the upper bound on global mechanical behavioral constraint, $C = \int c\, d\Omega$ is a structural cost and C_0 denotes the upper bound on the cost of structure.

Let us assume, for instance, that the problem (4) is concerned with the structure with thermally isolating and mechanically reinforcing interface. In this case, the jump in temperature along interface can induce significant initial thermal stresses and then the global mechanical constraint should be mainly imposed on effective stresses in order to assure the safe response of optimal structure under service load. When the isolating interface behaves as mechanically softening inclusion, then it can cause the desired thermal response, preserving at the same time the proper redistribution of induced thermal stresses, according to the form of imposed mechanical constraint.

On the other hand, assuming the structure with thermally conducting interface, the induction of initial thermal stresses in interface can be avoided, due to continuity of temperature field across this interface, and the mechanical constraint in (4) can not affect essentially the optimal structure response.

The optimality conditions for the problem (4) follow from the stationarity of Lagrange functional and they take the form:

$$\begin{aligned} &\frac{DG_m}{Db} + \xi_1 \frac{DC}{Db} + \xi_2 \frac{DG_t}{Db} = 0 \ , \ C - C_0 + \eta_1^2 = 0 \\ &2\xi_1\eta_1 = 0 \ , \ G_t - G_t^0 + \eta_2^2 = 0 \ , \ 2\xi_2\eta_2 = 0 \end{aligned} \tag{5}$$

where ξ_1 and ξ_2 are the Lagrange multipliers and η_1 and η_2 denote the slack variables. The sensitivities DG_t/Db and DG_m/Db appearing in optimality conditions are derived following the sensitivity analysis already discussed in previous papers ([1] – [5]).

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