Buckling Analysis of Cellular Beams using the Element-Free Galerkin Method with the Rotational Spring Analogy

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Abstract

The development of local buckling in the web-post and the compressive regions around the openings of cellular beams is often a critical design consideration. The current buckling assessment of such scenario is still based on simplified models, which are calibrated against detailed finite element models only for specific geometries including layout and range of dimensions, Moreover, the direct use of finite element analysis for the design of cellular beams remains computationally expensive. The main objective of this work is to extend the use of the Element-Free Galerkin (EFG) method combined with the Rotational Spring Analogy (RSA) for efficient buckling analysis of cellular beams, with emphasis on local buckling effects. The EFG is used for the planar response and the out-of-plane material stiffness, whereas the RSA is used for the out-of-plane geometric stiffness. A probing approach is then adopted for efficient buckling analysis, which is demonstrated through an application example.

Keywords: meshless methods, Galerkin methods, buckling, steel structures, beams

1. Introduction

Cellular beams, with the ability of withstanding gravity loads over relatively long clear spans while allowing service integration within the beams depth, have become an increasingly common form of construction. The increase in beam depth results in greater flexural rigidity and strength to weight ratio. However, the presence of web holes leads to the development of local buckling, which is typically most critical in the web-post and in the compressive regions around the openings [6,7].

Since the design of cellular beams via detailed numerical models, for example Finite Element Analysis (FEA), continue to be computationally demanding, the current available design approach of such beams is still based on simplified models, which are calibrated against detailed FEA models only for specific geometries including layout and range of dimensions. The main objective of this work is to extend the use of Element-Free Galerkin (EFG) method developed by Belytschko [1] combined with the Rotational Spring Analogy (RSA) proposed by Izzuddin [2,3] for efficient buckling analysis of cellular beams, with emphasis on local buckling effects.

2. Background of the developed approach

This work is concerned with the out-of-plane buckling of cellular beams under planar actions. Consideration is given hear to typical simply supported beams under uniformly distributed loading (UDL) applied to the top flange, though the method is much more general. The critical load factor is evaluated by obtaining the geometric stiffness (K_G) and material stiffness (K_E) of the considered domain, and then solving via an iterative rank 2 reduced eigenvalue problem [3]. While K_E can be determined directly using the meshless method, K_G is simply from the planar stress field according to RSA. Thus, there is a need to establish the planar response so as to obtain K_G. Benefitting from the repetitive cellular beams profile (Fig. 1), the planar response of the whole beam is simplified through the idea of dividing the system into unit cells and considering these under representative actions [5].

2.1. Unit cell analysis

The in-plane analysis of a unit cell is carried out using the EFG method. This approach is chosen for three key reasons:

1. it is a meshless method that can be easily applied to irregular domains,
2. it ensures external equilibrium at sub-domain level between internal loading and boundary actions, and
3. it facilitates the application of the RSA; for example, the same fixed integration points used for determining the material stiffness can be employed for the equivalent rotational springs so as to determine the geometric stiffness.

The last two points present particular benefits of EFG over the meshless local Petrov-Galerkin (MLPG) method [5] in which rigid body testing modes are not exactly represented and where quadrature points are specific to the testing sub-domain.

2.2. System planar response

The planar response of the whole cellular beam system is established by assembling the responses of individual cells expressed in terms of a reduced number of freedoms. Each cell resembles a super-element employing four nodes located at the centroid of the tees, as illustrated in Fig. 1. Upon assembly, the system is solved globally using a standard discrete solution, and a realistic unit-based planar stress distribution is then obtained for each cell.

2.3. Geometric stiffness

In the RSA, higher order kinematic descriptions required in the conventional approach to buckling analysis are avoided by introducing equivalent rotational springs distributed over the area [2]. The rotational stiffness of these springs is equivalent to the value of the normal stress at a specific location. Having obtained the distribution of planar stresses (σ_x, σ_y and τ_{xy}) over the domain, K_G is simply determined from:

$$\mathbf{K}_G = \int_\Omega \mathbf{T_{xy}}^T \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix} \mathbf{T_{xy}} \; t \, d\Omega \quad (1)$$

Where t is the thickness, Ω is the area, and $\mathbf{T_{xy}}$ is a geometric transformation matrix relating spring rotations to freedoms.

2.4. Out-of-plane material stiffness

The out-of-plane bending stiffness of the cellular beam is obtained by applying the EFG method with Kirchhoff's theory for thin plates [4]. Since the focus of the present work is to examine local buckling effects, significant computational efficiency can be achieved by limiting the area of to a specific portion of the beams. Assuming that the planar displacements are small, the out-of-plane K_E can be determined with reference to the undeformed geometry.

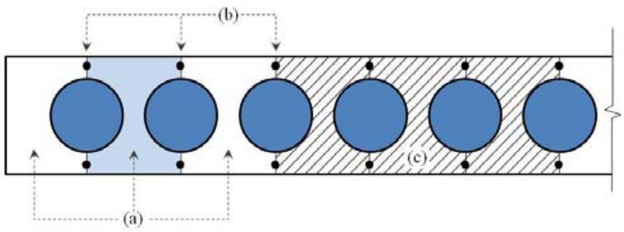

Figure 1: (a) Individual unit cells, (b) 4-noded cell super-elements, (c) Local region for out-of-plane buckling analysis

3. Buckling analysis strategy

This work is aimed at efficient and accurate local buckling analysis of cellular beams. To reduce the size of the problem, discrete buckling assessment is performed within a local region that consists of at most three unit cells, so as to account for local buckling at the cell boundary. The lowest buckling load factor is then determined by shifting the local region along the beam.

3.1. Iterative rank 2 reduced eigenvalue problem

The solution of the full eigenvalue problem for the local buckling can be computationally demanding, even for a model focussed on a local region. This issue is addressed in this work by using two probing modes, an initial assumed mode and its complementary mode, which are then used to formulate a rank 2 eigenvalue problem [3] that accelerates convergence of iterations to the lowest buckling mode of the local region.

3.2. Shifting the local region

Except for the two beam ends, the local regions consisting of three cells (Fig. 1) are identical over the length of a regular cellular beam. This brings significant computational benefits in that the out-of-plane K_E remains the same as the local region is shifted along the beam. Different initial probing modes are used for each assessment to obtain the most likely buckling mode for the particular portion. The critical load factor for the whole beam is then taken as the lowest value considering all regions.

4. Application example

A comparison against detailed FEA performed using ADAPTIC is made for verification purposes, considering elastic behaviour. The employed beam is a symmetric cellular beam fabricated from UB762×267×197 for the upper and lower web-flange sections with 800 mm diameter holes and 920mm spacing. The top and bottom flanges are assumed to be prevented from out-of-plane movement due to external restraints from a floor slab on the top and considerable tensile forces in the bottom flange. The results show that the critical mode is web-post buckling at the left end, where favourable comparison is achieved against the detailed FEA model (Fig. 2).

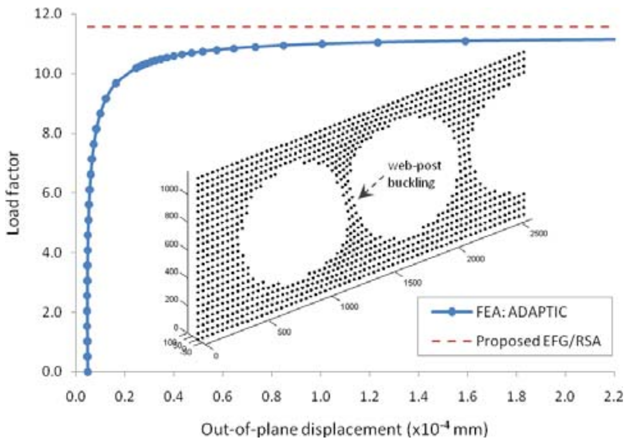

Figure 2: Lowest buckling load factor compared to FEA results

5. Conclusion

This paper presents an efficient method for local buckling analysis of cellular beams, combining the Element-Free Galerkin method with the Rotational Springs Analogy. The new approach brings particular benefits in terms of efficiency and accuracy, and is not only applicable to regular cellular beams with circular holes but also more generally to other forms of cellular beam with different hole shapes and arrangements.

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