Influence of the dissipative contact models on the dynamic response of multibody systems

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Abstract

The constitutive law used to describe the contact-impact events plays a crucial role in engineering applications. This paper studies the influence of different contact force models on the simulation of the dynamic response of multibody systems. The main issues associated with the most common continuous elastic and dissipative contact force models are presented and discussed. The contact force models are used for the simulation of a bouncing ball problem. The dynamic analysis of different multibody systems shows that the prediction of the dynamic behavior of hard and soft contacting surfaces depends on the selection of the contact force model.

Keywords: Contact Forces, continuous models, coefficient of restitution

1. Introduction

The prediction of the dynamic behavior of multibody systems (MBS) involves the formulation of the governing equations of motion and the evaluation of their kinematic and dynamic characteristics of the system. This desideratum is reached when all the necessary ingredients which influence the response of the multibody systems are adequately taken into account [1]. The contact-impact phenomenon is among the most important and complex to model because they are dependent on many factors, such as the geometry of the contacting bodies, the material properties and the formulation law used to represent the interaction among the different bodies that comprise the MBS [2].

In a simple manner, the contact mechanics problem must be studied when two bodies that are initially separated come into contact. For this, either the finite element method or the multibody system approaches can be utilized. On one hand, there is no doubt that the finite element analysis is the most powerful and accurate method to solve contact problems [3]. On the other hand, the method of multibody systems is the most efficient method for the dynamic analysis of the gross motion of mechanical systems [4].

Regardless of the method considered to describe the contact problems of the colliding bodies, it is necessary to model and analyze the contact process. This issue involves two main steps, namely: (*i*) the contact detection and (*ii*) the evaluation of the contact forces, which are the result of collisions between bodies [5, 6].The contact detection is an important issue in contact modeling of colliding bodies, which deals with the determination of when, where and which points are in contact. A significant task in contact detection is to check whether the candidate contact points or surfaces are in contact or not, using the evaluated relative penetration of the bodies. The efficiency and accuracy of this step depends on the level of complexity of the contacting surfaces [7], the number of potential colliding bodies [8], and the kinematics of the bodies [9]. In turn, the evaluation of the contact forces can be performed by using different approaches introduced over the last few decades [10, 11]. In the present study, several continuous contact force models are tested, for which the local deformations and normal contact forces are treated as continuous events and introduced into the equations of motion of the multibody systems as external generalized forces.

The contact-impact phenomena are characterized by abrupt changes in the values of system variables, most commonly discontinuities in the system velocities [12]. Other effects directly related to the impact phenomenon are those of vibrations propagation through the system [13], local elastic and plastic deformations at the contact zone [14], frictional energy dissipation [15] and wear [16, 17]. Impact is a prominent phenomenon in many mechanical systems, such as mechanisms with intermittent motion [18] and mechanisms with clearance joints [19]. Furthermore, during an impact event, the multibody systems can exhibit discontinuities in geometry and some material properties can be modified or influenced by the impact itself [20]. Therefore, in order to correctly model and analyze multibody systems in general, appropriate contact force models must be adopted. The suitable representation of the contact mechanics for multibody systems is a big challenge, namely for cases related to contact-impact of soft materials [21]. It is eminently difficult to find methods and algorithms which can model the highly complex phenomenon of contacting bodies realistically and efficiently enough for multibody systems simulations [22, 23]. From the modeling methodology point of view, several different methods have been introduced to model the contact response in multibody systems. As a rough classification, they can be divided into contact force based models and methods based on geometrical constraints, each of them showing advantages and disadvantages for each particular application [24, 25].

The main purpose of this work is to present a comparative study on several compliant contact force models in the context of multibody system dynamics. In the sequel of this process, fundamental characteristics of the well known elastic and dissipative contact force models are analyzed. The similarities of and differences between the contact force models are studied for hard and soft contacts by means of the use of high and low values of restitution coefficient for the contacting surfaces. Results for a classical bouncing ball problem are presented and used to discuss the main assumptions and procedures adopted throughout this work.

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2. Elastic contact force models

The simplest elastic contact force model is represented by a linear spring element, in which the spring embodies the elasticity of the contacting surfaces. This linear contact force model, also known as Hooke's law, can be expressed as [26]

$$F_N = K\delta \tag{1}$$

where F_N denotes the normal contact force, K is the spring stiffness and δ represents the relative penetration or deformation of the colliding bodies. The spring stiffness of this model can be evaluated by using a simple analytical formula or obtained by means of the finite element method. In turn, the penetration is determined from the relative position of the contacting bodies.

One primary weakness associated with this contact force model is the quantification of the spring constant, which depends on the geometric and material characteristics of the contacting bodies. Furthermore, the assumption of a linear relation between the penetration and the contact force is at best a rough approximation, because the contact force is affected by the shape, surface conditions and mechanical properties of the contacting bodies. In addition, the contact force model given by Eq. (1) does not account for the energy loss during an impact event.

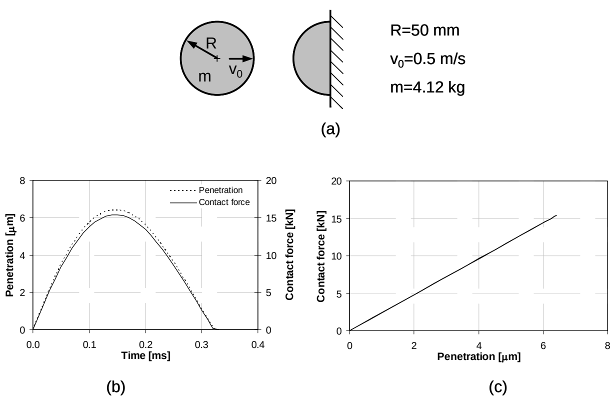

Figure 1: Externally colliding spheres modeled by Hooke contact law (a) Impact scenario of two spheres; (b) Penetration and contact force versus time; (c) Force-penetration relation

For this linear contact force model, Fig. 1 shows the penetration δ, the normal contact force F_N and the force-penetration relation of two externally colliding spheres. The spheres are identical and have the same radius of 50 mm. The left sphere has an approaching initial velocity of 0.5 m/s, while the right sphere is stationary. A relative spring stiffness of 2.4×10^9 N/m is utilized for the results in Fig. 1. The spheres are considered to be made of steel with the Young's modulus and the Poisson's ratio of 207 GPa and 0.3, respectively.

The well known contact force model for representing the collision between two spheres of isotropic materials is based on the work by Hertz, utilizing the theory of elasticity [27]. The Hertz contact theory is restricted to frictionless surfaces and perfectly elastic solids. The Hertz law relates the contact force with a nonlinear power function of penetration and is expressed as [28]

$$F_N = K\delta^n \tag{2}$$

where K is a generalized stiffness parameter and δ has the same meaning as defined above. The exponent n is equal to 3/2 for the case where there is a parabolic distribution of contact stresses, as in the original work by Hertz [29]. For materials such as glass or polymer, the value of the exponent n can be either higher or lower, leading to a convenient contact force expression which is based on experimental work, but that should not be confused with the Hertz theory [30].

The generalized stiffness parameter K is dependent on the material properties and shape of the contact surfaces. For two spheres in contact, the generalized stiffness parameter is a function of radii of the spheres i and j and the material properties as follows [31].

$$K = \frac{4}{3(\sigma_i + \sigma_j)}\sqrt{\frac{R_i R_j}{R_i + R_j}} \tag{3}$$

in which the material parameters σ_i and σ_j are given by

$$\sigma_l = \frac{1-\upsilon_l^2}{E_l}, \quad (l=i, j) \tag{4}$$

and the quantities υ_l and E_l are the Poisson's ratio and Young's modulus associated with each sphere, respectively. For contact between a sphere i and a plane surface body j, the generalized stiffness parameter depends on the radius of the sphere and the material properties of the contacting surfaces, and can be expressed as

$$K = \frac{4}{3(\sigma_i + \sigma_j)}\sqrt{R_i} \tag{5}$$

It is important to note that, by definition, the radius is negative for concave surfaces (such as mechanical joint clearances) and positive for convex surfaces (such as external impacts) [29]. Figure 2 depicts the penetration δ, the normal contact force F_N and the force-penetration relation for two externally colliding spheres modeled by the Hertz contact force law. The generalized stiffness is equal to 2.4×10^{10} N/m$^{3/2}$ for the calculations used to generate the plots. The impact scenario is the same as described for the Hooke law example presented in Fig.1.

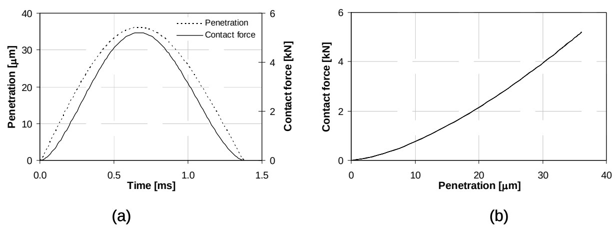

Figure 2: Externally colliding spheres modeled by Hertz contact force law: (a) Penetration and normal contact force *versus* time; (b) Relation between normal contact force and relative penetration depth

It is apparent that the Hertz contact law given by Eq. (2) is limited to contacts with elastic deformations and does not include energy dissipation. This contact force model represents the contact process as a non-linear spring along the direction of collision. The advantages of the Hertz model relative to Hooke law reside on its physical meaning represented by both its nonlinearity and by its relation between the generalized stiffness and geometry and material properties of the contacting surfaces. Although the Hertz law is based on the elasticity theory and some studies have been performed to extend its application to include energy dissipation.

In fact, the process of energy transfer is an extremely complex task of modeling contact-impact events. When an elastic body is subjected to cyclic loads, the energy loss due to internal damping causes a hysteresis loop in the force-penetration diagram, which corresponds to energy dissipation, during the contact-impact phenomena.

3. Dissipative contact force models

As mentioned above, the pure elastic contact force models suffer from the limitation that they cannot represent the energy loss during the contact process. In order to overcome this drawback, more advanced contact force models must be taken into account. One of the first dissipative contact force models was proposed by Kelvin and Voigt [31]. This model considers a linear spring in conjunction with a linear damper element. These two elements are associated in parallel, and the contact force model expressed as

$$F_N = K\delta + D\dot{\delta} \tag{6}$$

where the first term of the right-hand side is referred to as the elastic force component and the second term accounts for the energy dissipation during the contact event. In Eq. (6), D is the damping coefficient of the damper and $\dot{\delta}$ represents the relative normal contact velocity, and the remaining terms having the same meaning as it was described in the previous section.

The use of the contact law given by Eq. (6) to the impact of two externally spheres implies the outcome illustrated in Fig. 3, in which the penetration, the contact force history and the force-penetration relation are presented. A stiffness value of 2.4×10^9 N/m and a damping coefficient of 3000 Ns/m have been utilized for the calculations. As it can be observed in Fig. 3a, this model has some weaknesses, namely the fact that the contact force at the beginning of the contact is not continuous due to the existence of the damping component. This particular issue is not realistic because when the contact begins, both elastic and damping force components must be null. Moreover, at the end of the restitution phase, the penetration is null, the relative contact velocity is negative and, consequently, the resulting contact force is also negative. This situation does not make sense from the physical point of view, in the measure that the bodies cannot attract each other [2].

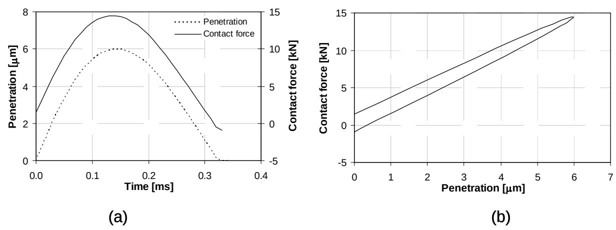

Figure 3: Externally colliding spheres modeled by Kelvin and Voigt contact law: (a) Penetration and contact force versus time; (b) Force-penetration relation

Hunt and Crossley [32] demonstrated that the linear spring-damper contact force model does not represent the physical nature of the energy transferred during the contact process. Instead, they represented the contact force by the Hertz law with a nonlinear viscous-elastic element which can be written as

$$F_N = K\delta^n + D\dot{\delta} \tag{7}$$

In order to guarantee that the damping force component satisfies both boundary conditions at the time of initial contact and at the time of separation, the damping coefficient D is chosen such that the damping force is in phase with the penetration velocity, but proportional to the penetration [33]. Hunt and Crossley proposed to evaluated the damping coefficient as

$$D = \chi\delta^n \tag{8}$$

where χ is called the hysteresis damping factor given by

$$\chi = \frac{3K(1-c_r)}{2\dot{\delta}^{(-)}} \tag{9}$$

in which K is the generalized stiffness parameter, c_r denotes the coefficient of restitution and $\dot{\delta}^{(-)}$ represents the initial contact velocity. Thus, after some mathematical manipulation, the expression for the Hunt and Crossley contact force model has the following form

$$F_N = K\delta^n\left[1+\frac{3(1-c_r)}{2}\frac{\dot{\delta}}{\dot{\delta}^{(-)}}\right] \tag{10}$$

With this model, the energy loss during the contact is assumed to be associated with the material damping of the contacting bodies, which would dissipate energy in the form of heat. The Hunt and Crossley force model expresses the damping as a function of penetration, which sounds physically reasonable. Furthermore, this model does not present discontinuities at the initial instant of contact and at the end of contact, i.e., it starts and ends with the zero value.

Lankarani and Nikravesh [13] used the general trend of the Hertz contact law incorporated with a hysteresis damping factor to propose a continuous contact force model for the contact analysis of multibody systems. The hysteresis damping factor, which accommodates the energy dissipation during the contact, is a function of the impact velocity, material properties and the coefficient of restitution, and is written as

$$\chi = \frac{3K(1-c_r^2)}{4\dot{\delta}^{(-)}} \tag{11}$$

Based on the analysis of Hunt and Crossley [32] that separates the contact force into elastic and dissipative components, the model proposed by Lankarani and Nikravesh is

$$F_N = K\delta^n\left[1+\frac{3(1-c_r^2)}{4}\frac{\dot{\delta}}{\dot{\delta}^{(-)}}\right] \tag{12}$$

which is quite similar to the model proposed by Hunt and Crossley.

The contact force model given by Eq. (12) is valid for the cases in which the dissipated energy during the contact is relatively small when compared to the maximum absorbed elastic energy. That is, the relation is valid for the values of the coefficient of restitution close to unity. The consequences of this simplifying premise are discussed in detail by Lankarani and Nikravesh [10]. In later papers, Lankarani and Nikravesh [33] and Shivaswamy and Lankarani [34] proposed a new approach for contact force model, in which the local plastic deformations were also incorporated.

Analyzing Eqs. (9) and (11), it can be observed that for a perfectly elastic contact; i.e. c_r=1, the hysteresis damping factor assumes a zero value, while for a perfectly plastic contact, i.e. c_r=0, the hysteresis damping factor does not assume an infinite value as it would be expected. This fact is not surprising because, for instance, Lankarani and Nikravesh [13] derived their model for high values of restitution coefficient, and therefore, this model would be only valid for hard contacts, such as those between metals. For lower values of the coefficient of restitution, they utilized local plasticity of the contact surfaces as the dominant mode of energy dissipation. The values of the coefficient of restitution for soft materials are low or medium, which clearly indicates that the hysteresis damping factor given by Eqs. (9) and (11) cannot be used. Therefore, a more accurate expression for this factor is required.

More recently, Flores et al. [35] described a contact force model for hard and soft materials. This contact force model was developed with the foundation of the Hertz law together with a hysteresis damping parameter that accounts for the energy dissipation during the contact process. An expression for χ was obtained by relating the kinetic energy loss by the impacting bodies to the energy dissipated in the system due to internal damping. Considering the kinetic energy before and after impact, the energy loss ΔE can be expressed as a function of the restitution coefficient c_r and initial impact velocity $\dot{\delta}^{(-)}$ as

$$\Delta E = \frac{1}{2} m \left(\dot{\delta}^{(-)}\right)^2 \left(1 - c_r^2\right) \tag{13}$$

where m is the equivalent mass of the two spheres given by

$$m = \frac{m_i m_j}{m_i + m_j} \tag{14}$$

On the other hand, the energy loss can also be evaluated by the integration of the contact force around the hysteresis loop. Flores et al. [35] considered that the dissipated energy is due to internal damping, which was evaluated by modeling the contact process as a single degree of freedom system, yielding

$$\Delta E \simeq \frac{1}{4} \chi \left(1 - c_r\right) \dot{\delta}^{(-)} \delta_{max}^{\frac{5}{2}} \tag{15}$$

Substituting Eq. (15) in Eq. (13) and after some mathematical manipulation, an expression for the hysteretic damping factor χ can be expressed as [35]

$$\chi = \frac{8K(1 - c_r)}{5 c_r \dot{\delta}^{(-)}} \tag{16}$$

Consequently, the contact force model is given by

$$F_N = K \delta^n \left[1 + \frac{8(1 - c_r)}{5 c_r} \frac{\dot{\delta}}{\dot{\delta}^{(-)}}\right] \tag{17}$$

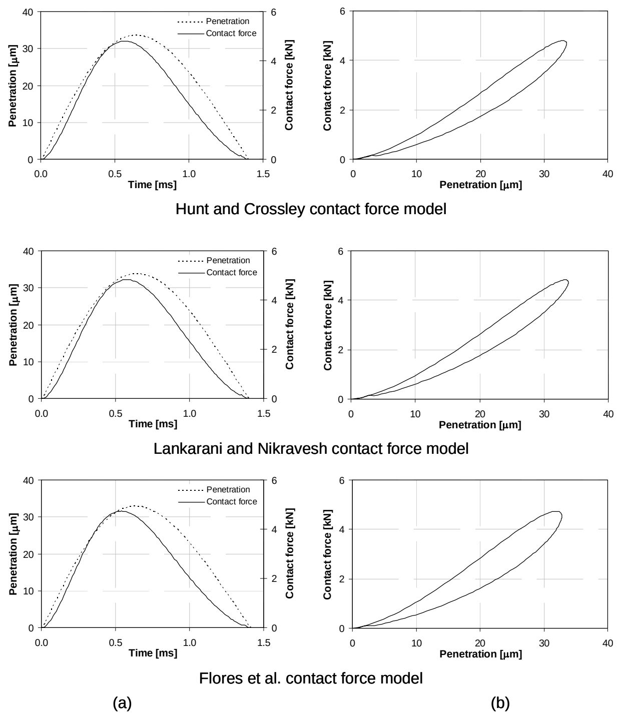

Figure 4: Externally colliding spheres modeled by Hunt and Crossley, Lankarani and Nikravesh, and Flores et al. dissipative contact force models: (a) Penetration and contact force versus time; (b) Force-penetration relation

From the analysis of Eq. (16), it can be concluded that for a perfectly elastic contact, i.e. c_r=1, the hysteresis damping factor assumes a zero value, and when the contact is purely plastic, i.e. c_r=0, the hysteresis damping factor is infinite, which is reasonable from a physical point of view [36]. The penetration and contact force histories and the force-penetration relation for the Hunt and Crossley, Lankarani and Nikravesh, and Flores et al. contact force models are shown in the plots of Fig. 4, which correspond to the externally collision between two spheres presented before. The value of the coefficient of restitution is equal to 0.8. From the analysis of Fig. 4 it can be observed that the compression and restitution phases of the contact process are not equal due to the differences in the energy dissipation between these two phases. This fact is clear and visible by observing the non symmetrical nature of the contact force plots.

4. Results and Discussion

In order to better understand the consequences of the use of different contact force models, the classical bouncing ball problem was considered [36]. Figure 5 shows the bouncing ball system, which consists of a ball falling down due to gravitational action and that impacts with a table moving harmonically in the direction of impact. The geometrical and inertial properties of the system are included in Fig. 5. The generalized stiffness parameter used to model the contact between the ball and table is equal to 140×10^{10} N/m$^{3/2}$. Furthermore, the table is given a harmonic excitation of

$$y_t(t) = 0.5 + 0.1 \sin 7.5t \tag{18}$$

The occurrence of contact between that ball and table is determined by evaluating the relative penetration at any time during the numerical solution of the dynamic equation of motion as

$$\delta = y_b - y_t - R \tag{19}$$

where y_b and y_t represent the y coordinate of the ball and the table centers of mass, respectively, and R is the radius of the ball. Positive values of δ denote that there is no contact between the ball and table. Therefore, the detection of the instant of contact occurs when the sign of deformation changes between two discrete instants in time [24]. This problem was analyzed by using a FORTRAN code named MUBODYNA [37].

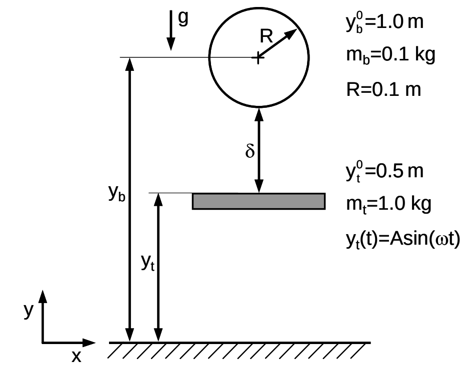

Figure 5: Bouncing ball system

The ball and table approach each other with a nonzero relative velocity in the normal direction until the distance between them has been vanished. Then, two different scenarios can occur, namely: (*i*) the ball and table separate immediately after the collision with a finite positive normal velocity, (*ii*) or the ball and table may remain in permanent contact. The system response scenario depends on the constitutive contact law used to model the contact event, on the value of the coefficient the restitution and on the overall system dynamics.

Figure 6 shows the table and ball motions when the contact is modeled with the pure Hertz law given by Eq. (2). By analyzing the system response, it can be observed that the ball rebounds after each impact with the table. When the impact takes place in the ascendant phase of the table motion, the ball gains energy and reaches a position higher than the one occupied just before the impact.

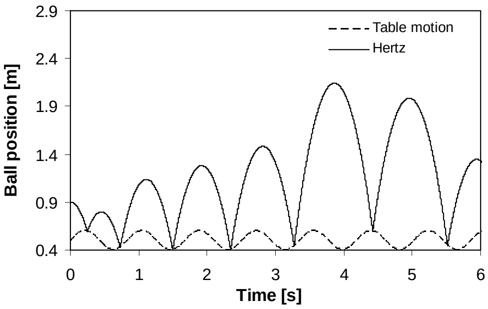

Figure 6: Bouncing ball response for the Hertz contact model

The influence of the use of different dissipative contact force models on the ball motion is illustrated in the plots of Fig. 7a-c. The comparative behavior of the three different contact force models is shown in Fig. 7d.

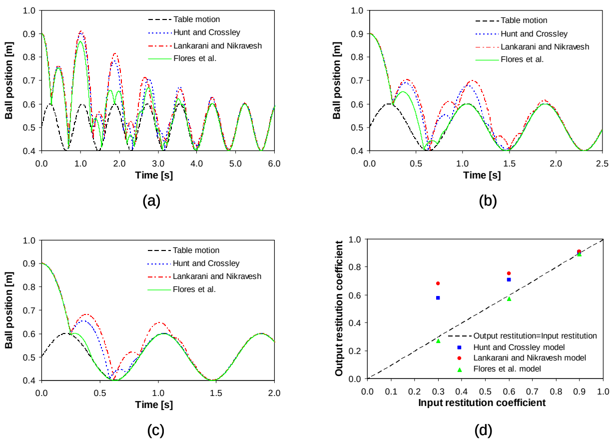

Figure 7: Bouncing ball response for the three dissipative contact force models: (a) c_r=0.9; (b) c_r=0.6; (c) c_r=0.3; (d) Relation between the output and input restitution coefficients

Three difference values for the coefficient of restitution are considered, namely 0.9, 0.6 and 0.3. From the plots of Fig. 7, it can be concluded that the ball losses energy in each impact with the table. Furthermore, after some finite number of impacts the ball remains in contact with the table. This observation is valid for all dissipative contact force models used. For the high values of the restitution coefficient, the Hunt and Crossley model exhibits a response similar to the case simulated with the Lankarani and Nikravesh model. In turn, the system response presents less and lower intensity rebounds when the Flores et al. approach is considered, as the permanent contact between the ball and table is reached fast. This effect is more evident for the lower values of the coefficient of restitution, as it can be observed in Fig. 7c. In general, for lower values of the coefficient of restitution, the Hunt and Crossley and Lankarani and Nikravesh contact force models overestimate the rebounding energy, when compared with the Flores et al. model.

The comparative behavior of the three different contact force models is shown in Fig. 7d in terms of the input restitution coefficient and the resulting output responses as predicted by the system utilizing the different contact force models. The plots are relative to the first impact between the ball and the table. Obviously, the closer the data points to the 45-degrees dashed line, the more closely the contact force model used represent the contact-impact process.

5. Conclusions

The contact-impact phenomena are characterized by large forces that are applied and removed in a short time period. The knowledge of the peak forces developed in the impact process is of paramount importance for the dynamic analysis of multibody systems and has consequences in the design process. The numerical description of the collision phenomenon is strongly dependent on the contact force model used to represent the interaction between the system components. The methodology used to describe multibody systems with impacts is based on the development of different contact force models that include energy dissipation, with which a continuous analysis of the system undergoing external impacts is performed. The basis of the continuous formulation for contact dynamics is to explicitly account for the deformation of the bodies during the impact. A comparative study of different compliant contact force models was presented in this study. Several elastic and dissipative contact force models utilized in the contact of multibody dynamics were revisited. Two simple planar multibody systems that involved contact-impact scenarios were used as examples of application to demonstrate the main consequences of the use of different contact force models. It was shown that the Hunt and Crossley and Lankarani and Nikravesh force models have similar behavior, mainly for high values of restitution coefficient. It was also demonstrated the that these two force models overestimate the penetration and contact forces developed, when compared with the Flores at al. approach. This fact is true in particular for the cases in which the coefficient of restitution present low values.

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