

## Development of CL+Computer method conformably to nonlinear dynamic problems

Pavel Mossakovsky<sup>1</sup>, Fedor Antonov<sup>2</sup> and Lilia Kostyreva<sup>2</sup>

<sup>1</sup>Research Institute of Mechanics of Lomonosov Moscow State University  
Michurinsky av. 1, 119192 Moscow, Russia  
email: [ctm@imech.msu.ru](mailto:ctm@imech.msu.ru)

<sup>2</sup>FSUE “MMBPP “Salut”  
Budionny av. 16, 105118 Moscow, Russia  
email: [antonof@gmail.com](mailto:antonof@gmail.com)

<sup>3</sup>Department of mechanics and mathematics, Lomonosov Moscow State University  
Leninskie Gory 1, 119991 Moscow, Russia  
email: [kostyle@inbox.ru](mailto:kostyle@inbox.ru)

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### Abstract

Accurate problem definition and solution of most problems related to the investigation of an impact interaction between deformable bodies involves considerable mathematical difficulties. The most important of them are the necessity to consider dynamic hardening, adiabatic thermal softening, influence of the stress state, possible connectivity loss due to local fracture of the material and complexity of the boundary conditions formulations on the contact surfaces.

This work is devoted to the experimentally-computational method for solving essentially nonlinear dynamic strength problems, which is implemented as an iteration procedure ideologically close to the A.A. Ilyushin Complex Loading plus Computer method. Proposed method does not impose any significant restrictions on the classes of thermomechanical processes which arise in the problem and fairly general boundary conditions, including contact, are allowed, as well as local and macro-fracture.

*Keywords: contact mechanics, dynamics, failure, impact, numerical analysis, plasticity*

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### 1. Introduction

Iteration method for Complex Loading (CL+Computer) was originally proposed by A.A. Ilyushin in the mid sixties of the last century as an experimental and computational procedure for solving plasticity boundary problems under complex loading conditions[1]. Formulated in very general terms, the method seemed too abstract and difficult to implement in practice at that time, so that it was not widely used. The essence of the method consisted in the classification of plastic deformation trajectories obtained by the numerical solution of boundary value problem and further reproduction of the paths that were not a priori included in the class of trajectories admissible for the model used in calculations, on an experimental CL machine. For the material response calculations within subsequent iterations it was supposed to use approximation relations, built on the results of the CL experiments on such trajectories. In the practical implementation of the method, special attention should be paid to the following aspects:

1. *Developing of effective methods of deformation processes classification.*

The CLC method procedure can be constructive only if there is an effective method of deformation processes classification, allowing to significantly restrict the number of processes which need to be reproduced within CL experiments. Since the last time, A.A. Ilyushin pupils developed a new approach to the classification of deformation processes based on advanced formulations of the postulate of isotropy and the retardation principle (vector delaying)[2]. In this approach, the set of observed processes is extended to the finite strains and conditions that allow to reduce the dimension of representative trajectories in the strains space are formulated.

2. *Availability of powerful computational resources.*

Thereat, limited computing resources were a significant deterrent to the development of the CLC method. Today, as a

result of the development of powerful and affordable software and hardware, the influence of this factor is neglected.

3. *The presence of an experimental CL machine.*

Availability of complex loading experiments is very limited nowadays. In some cases, to analyze the convergence of the method a virtual machine was used as a CL machine, which reproduces the material response with one or more alternative theories [3]. Additionally, in a variety of particular problems, CL machine may be replaced by relatively simple test benches.

Thus, nowadays we have all the factors the presence of which would rely on the successful development of this area.

### 2. Development of the Complex Loading plus Computer method

The CLC method ideology can be successfully applied outside the classical plasticity as fairly universal method for solving complex, strongly nonlinear boundary value problems of mechanics and, especially, those where it is necessary to ensure high solution reliability. Within recent years, direct computer simulation becomes the most widely used method to solve problems in mechanics. In many cases this approach is justified, the more so when dealing with complex, nonlinear problems, the quality of computer simulation is usually evaluated by comparison with full-scale tests. However, there are a lot of problems for which the full-scale experiments are limited, their implementation is economically impractical or even impossible. These problems include, in particular, modeling of various kinds of emergency situations that may lead to catastrophic consequences. To obtain reliable solutions in such cases it is reasonable to use approaches developed on the basis of the CLC method.

The variant of the CLC method is proposed to the solution of dynamic boundary value problems. It does not impose any significant restrictions on the classes of thermomechanical processes which arise in the problem, fairly general boundary

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conditions, including contact, are allowed, as well as local and macro-fracture.

Adaptation of the method for solving such problems requires a minor adjustment of some basic concepts: time-averaged process - thermo-mechanical state (TMS) is introduced instead of the deformation process (trajectory). The stress state (triaxiality) increment for the characteristic time is observed as the main parameter characterizing the complexity of the TMS. TMS classification is carried out by two major parameters - strain rate and triaxiality. Experimental benches using Kolsky method and dynamic punch tests [4, 5] are used as CL machines.

As an illustration to this approach, consider the procedure of boundary value problem solution, modeling the consequences of fan blade out event.

Boundary problem is solved by the direct simulation on the virtual test bench by successive iterations, in accordance with the Fig. 1.

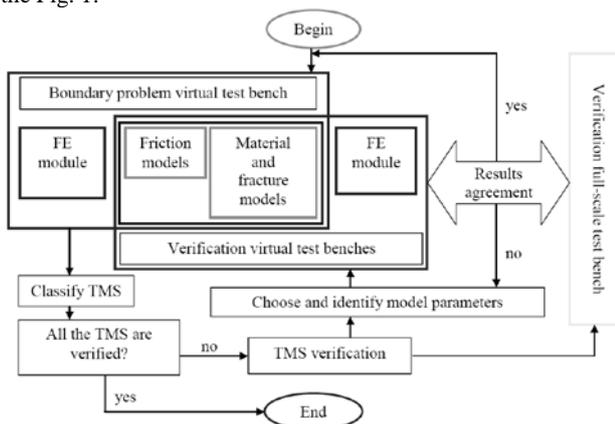


Figure 1. Boundary problem solution scheme.

Since the nature of fracture and fragmentation of structural elements are of greatest interest in this problem, below we consider the procedure of agreement and improvement of the local failure model based on the results of full-scale and virtual verification experiments.

At the first step a simple failure criterion by maximum equivalent plastic strain was used and the solution of the problem was obtained by direct computer modeling. The critical areas in terms of failure risk were then determined and characteristic TMS parameters were measured: equivalent plastic strain, strain rate, triaxiality ratio. Subsequent classification revealed the characteristic TMS classes, which corresponded to the values of the triaxiality ratio in the range of -1 to 1 and the strain rate of 500 s<sup>-1</sup> to 5000 s<sup>-1</sup>. Furthermore, TMS, characterized by the triaxiality values in the range [-0.7, -0.5] were reproduced in the verification experiments on tension and fracture of smooth cylindrical specimens by the Kolsky method. TMS in the range [0.3, 0.5] were simulated in the verification experiments on compression and fracture of cylindrical specimens. To obtain the material response in the remaining cases punch tests were used. Within the comparison of full-scale and virtual tests results various quantitative and qualitative characteristics were evaluated, such as impactor residual velocity, plug mass and velocity, shape and number of cracks - in ballistic tests, elongation (contraction) of the sample, the contraction in the neck, the nature of fracture - in tension and compression test. According to the test results, the second iteration step was conducted, using the local failure criterion based on the Gurson [6] and Johnson-Cook [7] failure models. The proposed relations were used to describe fracture

dependence on TMS, defined by different triaxiality values (viscous or brittle fracture).

Figure and Table shows the results of full-scale and virtual tests agreement.

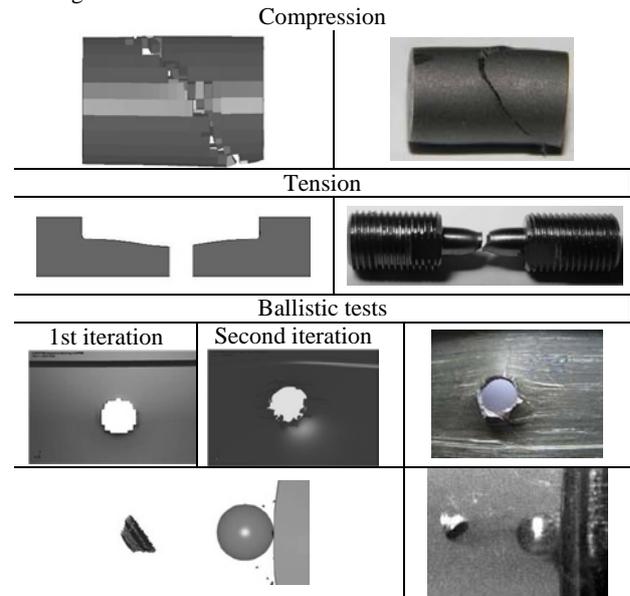


Figure 2: Full-scale and virtual experiments agreement.

Table 1. Agreement of virtual and full-scale ballistic tests

No		Impactor velocity, m/s	Residual velocity, m/s	Plug velocity, m/s	Plug mass, g
1	Full-scale	290	0	—	0,35
	1st iteration			No plug	
	2nd iteration			86	0,3
2	Full-scale	333	149	240	0,33
	1st iteration		143	No plug	
	2nd iteration		146	276	0,35

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