Gaussian mixture model for time series-based structural damage localization

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Abstract

In this paper, a time series-based damage localization algorithm is proposed using a set of Gaussian mixture models (GMMs) and expectation maximization (EM) framework for learning a structural damage classifier. The measured vibration time series from the structure are modeled as autoregressive (AR) processes. The first AR coefficients are used as a feature vector in the classifier. To test the efficacy of the damage localization algorithm, it has been tested on the pseudo-experimental data obtained from the FEM model of the ASCE benchmark structure. Results suggest that presented approach is able to localize primarily severe damage.

Keywords: artificial intelligence, dynamics, inverse problems, soft computing, structural monitoring

1. Introduction

Vibration based damage detection and localization methods using various pattern recognition algorithms have recently received noticeable attention in the field of structural health monitoring (SHM) [2].

There are two types of vibration based methods: model based (using a global structural analysis) and non-model based which is used in this paper. This work examines the use of a set of Gaussian mixture models (GMMs) to solve the damage localization problem. The problem is defined using the simulation data from the IAPR-ASCE SHM Task Group benchmark structure [3]. Previous approaches for solving this benchmark problem applied different machine learning and pattern recognition algorithms [2, 5, 6].

2. Description of the damage localization algorithm

Damage localization is based on the premise that structural damage causes changes in measured vibration signals [5]. The analysis here is limited to linear stationary signals.

2.1. Autoregressive time series modeling

The autoregressive (AR) model of order p for the acceleration signal $x_{\mathrm{acc},i}(t)$ from sensor i^{th} is given by

$$x_{\mathrm{acc},i}(t) = \sum_{k=1}^{p} \alpha_{ik} x_{\mathrm{acc},i}(t-k) + \epsilon(t), \tag{1}$$

where α_{ik} is k^{th} AR coefficient and $\epsilon(t)$ is the residual term.

The AR coefficients generally contain information about the dynamic characteristics of the structure (modal natural frequencies and damping ratios). Thus, changes to a structure's stiffness matrix as result of permanent damage will change the AR coefficients. It turns out that it is sufficient to use only the first AR coefficient to pick up changes in structural stiffness resulting from damage [5].

Using time series modeling of a structure's acceleration response and the autoregressive (AR) coefficients α_{i1} as a features vector $\mathbf{x} = \{\alpha_{11}, \alpha_{21}, ..., \alpha_{I1}\}$, it is possible to build a classifier which is able to localize the damage in a structure.

2.2. Damage localization as classification problem

The problem of structural damage localization is considered here as a multi-class classification problem. The classifier is built on the base of N input-output data pairs $\{\mathbf{x}^n, \mathbf{t}^n\}_{n=1}^{N}$, where the input vector is defined as the feature vector $\mathbf{x}$ and the target vector $\mathbf{t} = \{t_1, t_2, ..., t_k, ..., t_K\}$ uses 1-of-K coding scheme. So if $t_k = 1$ actual damage pattern is k.

Having built the classifier, it is used to localize possible damage for a new feature vector $\mathbf{x}^{N+1}$ describing current structural stiffness. So the classifier predicts a corresponding target vector $\mathbf{t}^{N+1}$ which consists of probabilities of considered damage patterns to be the actual damage case.

2.3. Gaussian mixture model and EM algorithm

The Gaussian mixture model (GMM) is popular model for clustering and continuous multimodal density modeling tasks. It can also be used for classification.

A Gaussian mixture model is defined as a superposition of K Gaussian densities and has the following form [1]:

$$p(\mathbf{x}) = \sum_{k=1}^{K} \pi_k \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k). \tag{2}$$

Each Gaussian component of the mixture $\mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k)$ has its own mean $\boldsymbol{\mu}_k$ and covariance $\boldsymbol{\Sigma}_k$. The parameters π_k are called mixing coefficients satisfying $\sum_{j=1}^{K} \pi_k = 1$ and $0 \leq \pi_k \leq 1$. Therefore these parameters satisfy the requirements to be probabilities.

One way to set the values of the Gaussian mixture distribution is to use maximum likelihood approach, maximizing the log of the likelihood function. It can be done with iterative optimization techniques like conjugate gradient method or alternatively using a powerful framework called expectation maximization [1].

2.4. Damage localization using Gaussian mixture models

In this paper, a set of Gaussian mixture models is used for structural damage localization. Each mixture is responsible for modeling one, defined a priori, damage case. This multi-mixture

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Table 1: Learning (L) and testing (T) success ratios (in %) of damage localization for six and three damage cases (columns) and for different number of input variables (rows)

# of inputs	six damage cases		three damage cases	
	learning [%]	testing [%]	learning [%]	testing [%]
16D	68	66	99	98
3D	52	50	95	94
2D	51	50	93	92

approach was selected because feature vectors for each damage case may form two or more clusters. It is illustrated in Fig. 1, where two-dimensional feature vectors form two clusters for each damage case.

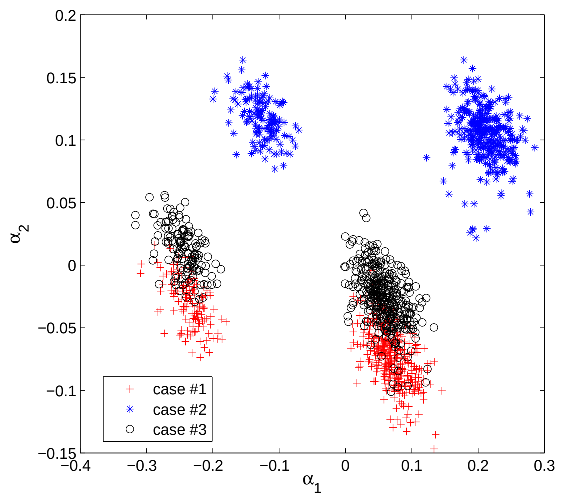

Figure 1: Clusters of feature vectors for three damage cases

3. Experiments and results

The presented algorithm is validated using data generated from the numerical simulator of the ASCE benchmark steel frame [3]. This benchmark is a standardized simulation tool for development and comparison of algorithms for SHM. The laboratory structure is a 4-story, 2 bay by 2 bay, built in the Earthquake Engineering Research Laboratory at the University of British Columbia (UBC). Diagram of the structure is shown in Fig. 2.

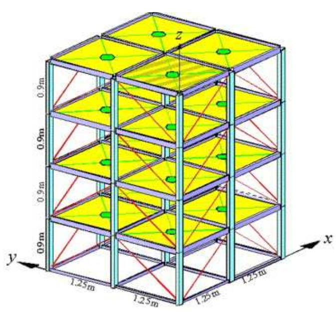

Figure 2: Diagram of the ASCE benchmark structure [3].

The structure is modeled as a 120 degree of freedom system. The structure has 16 accelerometers and output is given in the form of 16 acceleration time histories. The data generation scripts in MATLAB are available on the web at http://mase.wustl.edu/asce.shm. These programs allow to simulate the benchmark structure's response to ambient vibrations in presence of damage.

Structural damage of the benchmark structure is simulated mainly by removing braces (loss of stiffness). There are defined a priori six damage cases. For example the first damage case is defined as a removal of all braces on the first floor. This case can be treated as an example of severe damage. In the second damage case one of the braces on the first floor is removed and it is an example of minor damage.

Using data generation scripts acceleration time histories were generated for six damage cases. Then these time series were modeled with AR model and the feature vectors were formed. After preparation of datasets for learning and testing, a set of Gaussian mixture models were trained with EM algorithm using Netlab toolbox for MATLAB [4]. Final GMMs were tested using testing feature vectors from testing dataset. Results for this experiment are presented in Tab. 1 in the first two columns and the first row. In this case the results are rather poor because the classifiers have problems with recognizing minor damage cases. After merging four minor damage cases into one, the results are superior as can be seen in in Tab. 1 in the last two columns and first row. Finally, the sixteen-dimensional feature vectors were preprocessed using principal components analysis (PCA) and modeled with GMMs. Preprocessed feature vectors for three damage cases are shown in Fig. 1. The results for 3D and 2D feature vectors are presented in Tab. 1 in the last two rows.

The presented GMM-based algorithm for damage localization is very effective in recognition of severe damage and inefficient for localizing minor damages.

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