Cyclic loading of a cracked sheet: approaches and results

Larisa V. Stepanova[1]

[1]*Department of Mathematical Modelling in Mechanics, Samara State University*
Ak. Pavlov 1, 443011 Samara, Russia
e-mail: lst@ssu.samara.ru

Abstract

Two approaches to cyclic loading of cracked bodies are presented. The first one is based on the classical Dugdale scheme of the plastic flow localization. This approach takes into account the stress and displacement fields caused by localized plastic flow near a mode I crack in a sheet under plane stress conditions. The loading, unloading and second loading of the cracked sheet are considered. It is shown that the second loading induces the origin of two plastic flow zones localized near the crack tip and at the point, where the maximum residual tensile stresses are concentrated. According to the Dugdale scheme of the plastic flow localization, both the plastic flow zones are modelled as narrow stripes on the line extending the crack. To determine three non-dimentional parameters, characterizing the position of the plastic zones, a non-linear system of equations is obtained. The numeric solution of the system is found. The second approach is based on the continuum damage mechanics considering material deterioration processes near the crack tip. The new asymptotic solution of the stress field and damage variable in the vicinity of the fatigue growing crack tip in the coupled formulation of the problem is obtained.

Keywords: Dugdale model, plastic flow zone, second loading of the cracked sheet, fatigue crack growing, damage mechanics

1. Introduction

The elastic-plastic analysis of a through-the-thickness mode I crack in a sheet under plane stress conditions was for the first time given by Dugdale in his classical work (Ref. [1]). According to the experimental observations it was assumed that the plastic flow zone located at the crack tip was a narrow stripe extending the crack. Since then the Dugdale model has been used for the analysis of the stress-strain state and for the estimation of the crack opening displacement in plates. The Dugdale model has been discussed in the numerous publications (Ref. [2]–[4]). Investigations of cracks in elastic-plastic solids are based on the consideration of the single plastic zone near the crack tip. Hence these results are not applicable to stable cracks in elastic-plastic solids with the considerable residual stresses that can be induced by cyclic loading. The residual stresses can significantly alter the actual stress field in the loaded elastic-plastic solid. This can particularly appear in originating a number of distant plastic flow zone. In this paper the second loading of the residually prestressed cracked plate is studied. The results obtained continue the investigation realized in Ref. [4].

2. Second loading of the cracked plate

It is assumed that the repeating loading of the cracked plate with the residual stress distribution can result in the origin of two plastic flow zones. The first one is contiguous to the crack tip, where as the second one is localized at a distant point where the maximum residual stresses are concentrated. Both the yielded zones are modelled by narrow stripes on the line extending the crack. The boundary value problem is reduced to the determination of the single analytical in the upper half-plane function – the Kolosoff-Muskhelishvivli complex potential. For three non-dimensional parameters characterizing the position of the segment-like plastic flow zones to be determined a non-linear system of algebraic equations is obtained. This rather complicated system is derived from the normal stress continuity at the elastic-plastic transition points. The numerical solution of the system for different values of the tensile stresses at infinity is found. The numerical solutions for the elastic-plastic problem of the cracked plate with the central crack by finite element method are either obtained. The results of the analytical solution based on the classical Kolosoff-Muskhelishvili complex potentials and the finite element modelling of the second loading of the cracked plate are in good agreement.

3. The asymptotic study of fatigue crack growth based on damage mechanics

With the appearance and development of continuum damage mechanics it becomes possible to describe the phenomenon of fatigue failure in the framework of continuum damage mechanics. The phenomenon of crack growth in materials undergoing deformations coupled with damage has been investigated extensively over the past twenty years. Some of the essential aspects of the considered set of two-dimensional crack problems and the results obtained can be highlighted. 1. The damage gives significant influence on the stress and strain (strain rate) fields near the crack tip. 2. While the solutions of linear fracture mechanics and the Hutchinson-Rice-Rosengren (HRR) - field of non-linear fracture mechanics always show the stress singularity at the crack tip for any finite value of the stress exponent, the preceding material damage in front of the crack tip decreases the singularity, and may give non-singular stress field. 3. The totally damage and (or) active damage zone (process zone) need be modelled in the crack tip region.

In this paper the mechanical behaviour around a growing fatigue crack tip for plane stress and plane strain conditions is analyzed by the light of continuum damage mechanics.

The aim of the present paper is to derive the asymptotic stress, strain and continuity fields at the tip of a crack that grows in a linear elastic material undergoing damage. To do this the asymptotic analysis permitting an analytical insight into the issue concerned and making some essential characteristics of the problem readily understood is used. The rather simple method employed hitherto by Zhao and Zhang (Ref. [5]) provides some interesting results which enable us to evaluate the influence of the material deterioration on the stress-strain state in the vicinity of the crack tip.

A Cartesian coordinate system is used with its origin attached to the moving crack tip and its x-axis pointing the direction of the crack growth; the coherent polar coordinate frame (r, θ) is introduced (Fig. 1).

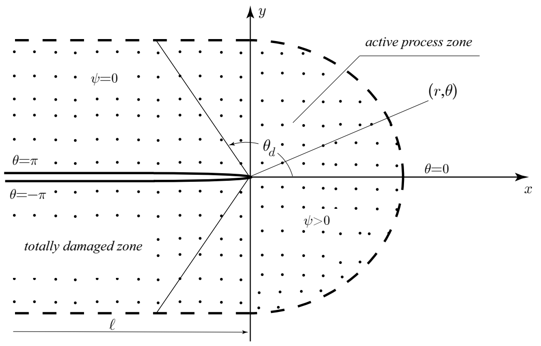

Figure 1: Geometry of the growing fatigue crack tip

In this paper the general constitutive equations, including the damage coupled stress-strain relations of linear elastic materials and the damage evolution are used for analysis of fatigue crack growth. It is supposed that the continuity parameter evolutes according to (see Ref. [5])

$$\frac{d\psi}{dN} = \begin{cases} -c\left(\dfrac{\sigma_e}{\psi}\right)^m \dfrac{1}{\psi^{n-m}} & (\sigma_e \geq \sigma_{th}\psi), \\ 0 & (\sigma_e < \sigma_{th}\psi), \end{cases} \qquad (1)$$

where ψ is the continuity parameter, $\sigma_e = (3s_{ij}s_{ij}/2)^{1/2}$, $s_{ij} = \sigma_{ij} - \sigma_{mm}\delta_{ij}/3$ is the stress deviation tensor, N is the number of cycles, c, m, n, γ are positive material parameters.

4. Asymptotic solutions

The problems of plane stress and plane strain for mode I growing crack are considered under small-scale damage conditions, the equilibrium requirement can be satisfied by introducing the Airy stress function. The asymptotic presentation of the Airy stress function and the continuity field in the vicinity of the crack tip can be written as

$$\chi(r,\theta) = r^{\lambda+2} f(\theta), \ \psi(r,\theta) = r^{\mu} g(\theta).$$

A series of numerical results are exhibited for different pairs of m and n with $\nu = 0.3$. The integrations of nonlinear system of ordinary differential equations are performed through a fifth-order Runger-Kutta-Fehlberg method. Moreover the numerical results allowed to find the exact analytical presentation of the stress and damage fields near the crack tip. The asymptotic distributions of the stress and damage fields in the neighborhood of the crack tip are found in the form

$$\sigma_{\theta\theta}(r,\theta) = \kappa r^{\lambda} 2 (\cos\theta)^{2+\mu}, \sigma_{r\theta}(r,\theta) = \kappa r^{\lambda} \sin 2\theta \cos^{\mu}\theta, \quad (2)$$

$$\sigma_{rr}(r,\theta) = \kappa r^{\lambda} \sin^2\theta \cos^{\mu}\theta, \psi(r,\theta) = \kappa^{m/(n+1)} r^{\lambda} \cos^{\mu}\theta, \quad (3)$$

where $\lambda = \mu = 1/(1+n-m)$. The angular distributions of the stress and continuity fields near the crack tip are presented in Fig. 2 (points correspond to the numerical solution, solid line corresponds to the analytical solution Eqn. (2), (3)).

5. Conclusions

In the first part of the paper a numerical analysis of the plastic strain localization for the second loading of the cracked plate is performed for any specified value of the loading tensile stress.

In the second part the effects of material damage on the asymptotic stress and elastic strain fields of mode I cracks under plane stress and plane strain conditions were analyzed on the basis of continuum damage mechanics. The asymptotic stress, strain and integrity fields are found by solving a two-point boundary value problems of non-linear differential equations. It is established that the stress and strain have no singularity in the vicinity of the crack tip. It is shown that the active process zone is formed ahead of the moving fatigue crack tip where as the totally damaged zone is adjacent to the crack surfaces. The asymptotic fields in and around the damage zone could be quite different from the obtained in Ref. [5]. The analytical presentation for the stress and damage fields near the crack tip is obtained.

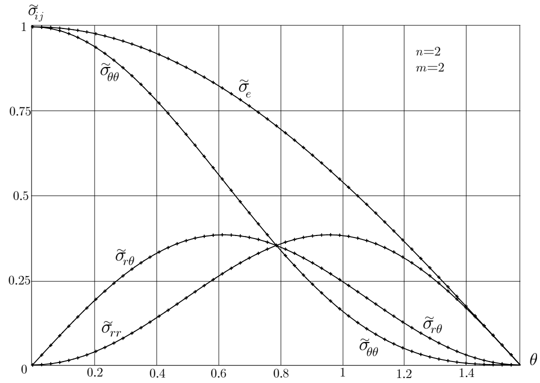

Figure 2: Normalized stress distribution

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