Enhanced coupled elasto-plastic-damage model to describe cyclic concrete behaviour

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Abstract

The paper presents results of two-dimensional FE simulations of the concrete behaviour under quasi-static cyclic loading using enhanced coupled elasto-plastic-damage continuum model. Attention is paid to strain localization and stiffness degradation under tensile bending failure. To ensure the mesh-independence, to properly reproduce strain localization and to capture a deterministic size effect, constitutive formulation includes a characteristic length of micro-structure by means of a non-local theory. Numerical results are compared with corresponding cyclic laboratory tests on concrete specimens under bending. In addition, the concrete behaviour under a tension-compression-tension condition is analyzed.

Keywords: cyclic loading, damage mechanics, elasto-plasticity, non-local theory, stiffness degradation, strain localization

1. Introduction

An analysis of concrete elements is complex mainly due to their stiffness degradation during cyclic loading caused by strain localization in the form of cracks and shear zones. The determination of the width and spacing of strain localization is crucial to evaluate the material strength at peak and in the post-peak regime. To take into account a reduction of the concrete strength, irreversible (plastic) strains and stiffness degradation, a combination of plasticity and damage theories is physically very appealing since plasticity considers the first two properties and damage takes into account a loss of the material strength and stiffness deterioration.

The aim of the paper is to show the capability of an improved enhanced coupled elasto-plastic damage continuum model to describe strain localization and stiffness degradation in concrete elements subjected to quasi-static cyclic loading.

2. Constitutive model for concrete

The coupled model combines elasto-plasticity with a scalar damage assuming a strain equivalence hypothesis by Pamin and de Borst [7]. The elasto-plastic deformation is defined in terms of effective stresses according to a general elasto-plastic relationship

$$f_p = F\left(\sigma_{ij}^{eff}\right) - \sigma_y\left(\kappa_p\right), \tag{1}$$

wherein F – the elasto-plastic failure function defined in effective stresses σ_{ij}^{eff}, σ_y – the uniaxial tension yield stress, and κ_p – the hardening parameter equal to plastic strain in uniaxial tension/compression. Two criteria are used in an elasto-plastic regime: a linear Drucker-Prager criterion with a non-associated flow rule in compression and a Rankine criterion with an associated flow rule in tension (Marzec et al. [5]) defined by effective stresses

$$\sigma_{ij}^{eff} = C_{ijkl}^{e}\varepsilon_{kl} . \tag{2}$$

Next, the material degradation is calculated within damage mechanics, independently in tension and compression using one equivalent strain measure by Mazars [6] (ε_i - principal strains)

$$\tilde{\varepsilon} = \sqrt{\sum_i \left\langle \varepsilon_i \right\rangle^2} . \tag{3}$$

In tension, the standard exponential evolution function is chosen

$$D_t = 1 - \frac{\kappa}{\kappa_0}\left(1 - \alpha + \alpha e^{-\beta(\kappa-\kappa_0)}\right), \tag{4}$$

wherein α and β are the material parameters.
In turn, in compression, the definition by Geers [2] was adopted

$$D_c = 1 - \left(1 - \frac{\kappa_0}{\kappa}\right)\left(0.01\frac{\kappa_0}{\kappa}\right)^{\eta_1} - \left(\frac{\kappa_0}{\kappa}\right)^{\eta_2} e^{-\delta\left(\kappa-\kappa_0\right)}, \tag{5}$$

where η_1, η_2 and δ are the material constants.

Equation 3 contributes to a differentiation of the stiffness degradation under tension and compression. Thus, damage under compression starts to develop later than under tension (that is consistent with experiments). The stresses are obtained according to

$$\sigma_{ij} = \left(1 - D\right)\sigma_{ij}^{eff} , \tag{6}$$

with the term '1-*D*' defined as follows (Abaqus [1])

$$\left(1 - D\right) = \left(1 - s_c D_t\right)\left(1 - s_t D_c\right) , \tag{7}$$

with two splitting functions s_c and s_t

$$s_c = 1 - a_c\left(1 - w\left(\sigma^{eff}\right)\right) \quad \text{and} \quad s_t = 1 - a_t w\left(\sigma^{eff}\right), \tag{8}$$

where a_t and a_c are the scale factors and $w\left(\sigma^{eff}\right)$ denotes the stress weight function. For simple cyclic tests, the scale factors a_t and a_c can be equal to 0 and 1, respectively. Thus, the splitting functions are $s_t = 1$ and $s_c = w\left(\sigma^{eff}\right)$. For a uniaxial case, the stress weight function is

$$w\left(\sigma^{eff}\right) = \begin{cases} 1 \;\; if \;\; \sigma_{ii}^{\;eff} > 0 \\ 0 \;\; if \;\; \sigma_{ii}^{\;eff} < 0 \end{cases} . \tag{9}$$

Thus, for pure tension $w\left(\sigma^{eff}\right) = 1$ and for pure compression $w\left(\sigma^{eff}\right) = 0$. For a multiaxial case, the stress weight function is determined with the aid of principal effective stresses (Lee and Fenves [4])

$$w\left(\sigma^{eff}\right) = \frac{\sum\left\langle \sigma_{ii}^{eff} \right\rangle}{\sum\left|\sigma_{ii}^{eff}\right|} . \tag{10}$$

To capture properly strain localization within continuum mechanics, a characteristic length of micro-structure is included. by means of a non-local theory [5]. When using a coupled elasto-plastic damage model, non-locality is applied in damage (since softening is not allowed in elasto-plasticity). The equivalent strain measure in Eq.3 is replaced by its nonlocal counterpart:

$$\bar{\varepsilon}\left(x_k^a\right) = \int_V \omega\left(r\right)\tilde{\varepsilon}\left(x_k\right)dV , \tag{11}$$

where x_k^a – the coordinates of the considered (actual) point, x_k – the coordinates of the surrounding points, r – the distance between material points, ω – the weighting function and V – the body volume.

3. FE-calculations

The comparative numerical simulations were performed with a concrete notched beam under four-point cycling bending (Hordijk [3]). The calculated force-displacement curve exhibits good agreement with experimental outcomes (Fig.1). A calculated stiffness decrease is almost the same as in the experiment. The magnitude of a stiffness reduction is well reflected due to differentiation of the stiffness degradation under tension and compression.

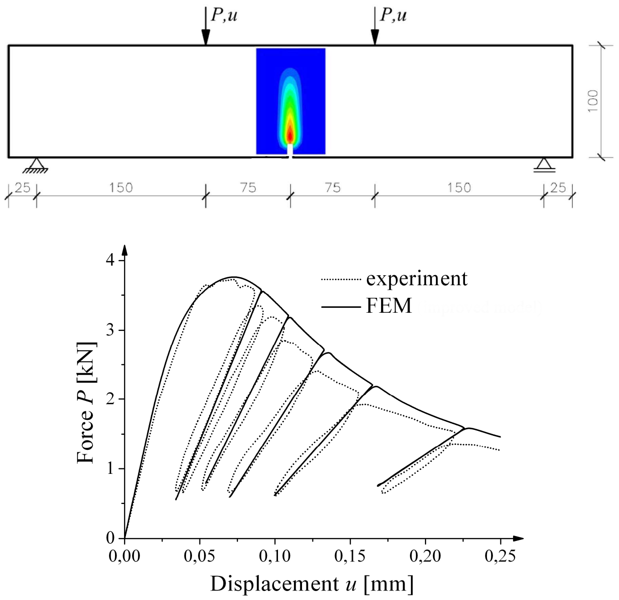

Figure 1: Assumed boundary conditions, calculated localized zone above notch and calculated load-displacement curve compared with experiment [3] for concrete beam

Fig.2 demonstrates the 1D load-displacement diagrams under cyclic uniaxial tension, cyclic uniaxial compression and tension-compression-tension with the model. The results for a 1D cyclic uniaxial tension-compression-tension test with and without plastic strains are presented in Fig.3. A different stiffness degradation during compression and tension is obtained (it is stronger in tension). A recovery of the compressive stiffness upon crack closure as the load changes between tension and compression is well reflected. Moreover, the tensile stiffness is not recovered when the load varies between compression and tension. For a better adjustment of the theoretical results to the experimental material behaviour, the scale factors should lie between 0 and 1. An evident difference between a pure damage model (without plastic strains) and coupled one (with plastic strains) for one uniaxial load cycle is also indicated.

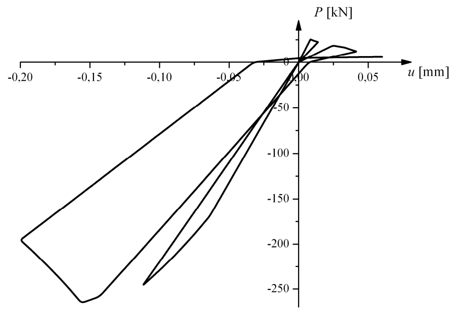

Figure 2: Response of coupled elasto-plastic-damage model for concrete under: a) cyclic uniaxial tension, b) cyclic uniaxial compression and c) tension-compression-tension

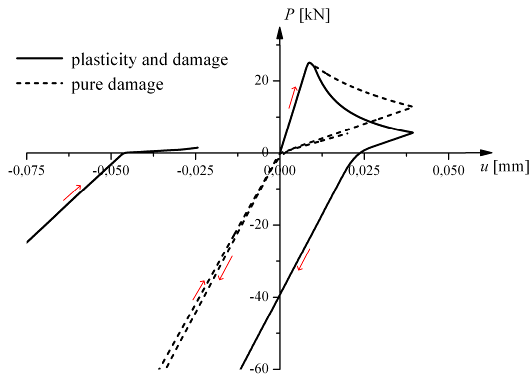

Figure 3: Response of coupled elasto-plastic-damage model during tension-compression-tension test with and without plastic strains

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