Non-parametric identification of chaotic systems

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Abstract

Non-linearity is a common feature shared by mechanical systems, whereas their linear behaviour is an exception. The irregular, chaotic behaviour finds out in many processes which are appeared in different natural and technical objects. The author has suggested applying the phase trajectory mappings for estimation of parameters of dissipative characteristics in the planes «acceleration–displacement» and «acceleration–velocity». Compared to the trajectories in the plane «velocity– displacement», the phase trajectories require less geometrical constructions for identification of the models of dissipative characteristics in dynamic systems and, thus, upgrade the estimation accuracy.

Keywords: identification, chaotic system, phase trajectories in extended phase space, phase trajectories mappings

1. Introduction

Prediction of dynamic behavior of mechanical systems is currently a topical issue. It is inextricably linked with the problems of identification of such systems. The subject of this research is principally non-linear dynamic systems. It the prime objective of this research is to develop methods for identification of models of essentially non-linear mechanical systems by recording chaotic processes occurring in the systems Ref. [2,3] chaotic behavior is observed in a great number of processes which occur in various natural and technical objects. A specific feature of dynamic systems in the investigated class consists in their large sensitivity to initial conditions. The concept of dynamic chaos, the fundamentals of which were formulated in 70s-80s of the 20th century, allows us to assume that, at least in few cases, a complicated time behavior can be represented by a rather simple mathematical model.

2. Introduction

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3. Identification of dynamic models of mechanical systems

In the past two decades, the issues of construction of mathematical models and prediction of dynamic behaviour of structural elements proceeding from recorded experimental data have attracted considerable interest.

In spite of intensive investigations into the above mentioned matter, which have been undertaken in the scientific centers in different countries (supported by numerous publications on theoretical research and experiments, a number of specialized conferences Ref. [1, 5] as well as the important results obtained, there is no, so far, the only universal effective approach, which would allow for correct determination, prediction and analysis of dynamic properties in construction elements. Most of the methods of structural identification are based on the use of special types of outer excitation for a wide range of frequencies, such as symmetric monoharmonic excitation and rectangular impulse. These types of excitation are often unrealizable in mechanical systems. The methods based on the Fourier transformation do not allow classifying and localizing non-linearity Ref. [6,7] and are inapplicable to investigating stochastic processes Ref. [1,5]. It should be also noted that the application of Winer series and Hilbert transformation for identification of non-smooth non-linear dynamic characteristics is unjustified. Ref.[5].

4. Phase trajectories of oscillations of nonlinear systems in the expanded phase space

Dynamic behaviour of mechanical systems is usually presented as oscillating processes in various graphic forms such as time processes, the Lissajous patterns and hodograph. Such patterns of presentations enable to determine the type of a process and to perform numerical estimations of its characteristics, but do not disclose any properties of the governing system. Unlike them classic phase trajectories have the row of advantages.

A phase space in classic mechanics is represented as a multidimensional space. The number of measured values for a phase space is equal to the doubled number of degrees of freedom of the system being investigated Ref. [2]. The state of the system is presented as a point in the phase space, and any change in the system state in time is depicted as the displacement of the point along a line called a phase trajectory. The image on phase plane $(y,\ \dot{y})$ is a more vivid presentation because it depicts inharmonious oscillations particularly well. Each phase trajectory represents only one definite clearly defined motion. A disadvantage of phase trajectories $(y,\ \dot{y})$ consists in the fact that they do not provide for the immediate presentation of oscillating process in time. However, this drawback is compensated by a significant advantage. The geometric presentation of a single phase trajectory or a set of trajectories allows coming to important conclusions about the oscillation characteristics. It is, foremost, true with the oscillations, which are described with nonlinear differential equations.

As is has been shown by the investigations of several authors Ref. [3, 4], the expansion of a phase space by taking into account the phase planes $(y,\ddot{y})$ and $(\dot{y},\ddot{y})$ substantially promotes the efficiency in analyzing a dynamic system behaviour. Hereby, we pass on to a three-dimensional phase space confined with three co-ordinate axes, i.e. displacement, velocity and acceleration. An interest taken into accelerations in dynamic systems is conditioned by the fact that these accelerations are more sensitive to high-frequency components in oscillating processes.

It is precisely diagram $\ddot{y}(y)$ that enables to define the type and the level of non-linearity in a system. The geometric presentation of an individual phase trajectory or of a family of trajectories allows coming to important conclusions about the properties of a model of the system being studied.

Incorporation of phase trajectories on planes $(y,\ddot{y})$ and $(\dot{y},\ddot{y})$ enhances the capabilities of classic methods of the qualitative theory due to their extension onto the class of inverse problems of dynamics.

The major difficulty of formation of phase trajectories $\ddot{y}(y)$ and $\ddot{y}(\dot{y})$ consists in the necessity to exclude parameter of time t from the appropriate dependencies. It's not always possible to perform this operation analytically. Accepting consistently appropriate pairs of parameter values $y(t)$ and $y(t)$ or $y(t)$ and $\dot{y}(t)$, it is possible to obtain phase characteristic data.

Proceeding from the results of analytical and experimental investigations, the authors have performed the generalization of phase trajectory properties in the planes “acceleration – displacement” and “acceleration – speed” for the systems, which are subjected to external excitations Ref. [3].

5. Experimental investigation of a dynamic behaviour of a bulked rod

5.1. Model

The both the bench and the model were designed and made the solution of noted above problem. The body of a bench is a fixed part, which one allows to fix of all remaining members. The bench represents a system of collar beams, racks and braces. The parts of a bench were connected between themselves on bolts, that allows to change spacing interval between racks and collar beams. The special attention was given to a centre-of-gravity position of members, with the purpose to avoid of appearance of eccentricities in an assembly time of model, mounting of racks, collar beams and reference parts.

Rod was fabricated from a band of a spring steel of length $l_1 = 2\ m$. The cross section sizes are $\mathrm{b}\times\mathrm{h} = 0.05\times 0.056\ m$. Spacing interval between support was $l\ = 1.955\ m$.

The modulus of elasticity of a material was determined by step static loading of a straight-line rod on midpoint on its length. The deviation of the obtained value $E = 2.07\ 10^5\,MPa$ from normative $E = 2.1\ 10^5\,MPa$ was 1.4 %, that is permissible. The rod was loaded with additional freights with weight 1kg with the purpose of increasing of the inertial characteristics. They were arranged on equal spacing intervals concerning midpoint of the length rod. The initial longitudinal force of compression was applied to the flexible rod at the stage of model assembly. The value of the axial compression force was greater than the value of the Euler force. After compression the straight-line shape of a centreline of a rod became unstable.

The parameters of an outer excitation have an important influence to oscillatory regimes of the rod systems. Special attention was given to a capability to adjust and fix parameters of an outer excitation at realization of experiment

The engine of a direct current was used as the generator of an outer excitation to change the frequency of rotations. Weight of the engine was 2kg. The generator was attached to a rod by means of the clamp. In experiment the engine rotation speed was fixed taking into account a possibilities of installation of the sub - and ultraharmonic oscillations, and as it is as a matter of convenience constructions of amplitude-frequency characteristics. The frequency of external excitation was fixed by one of ports multiway registered equipment.

The value of amplitude of an external monoharmonic excitation depend on rotation speed of the engine and was determined under the formula

$$P = m_i\, a = m_i\, \omega^2\, R_i\,, \tag{1}$$

where m_i is weight of an eccentric, a is centrifugal acceleration, ω is engine speed, R_i is radius of rotation of an eccentric.

In experiment two eccentrics having the following characteristics $R_1 = 0.114\ m$, $m_1 = 0.358kg$ and $R_2 = 0.107\ m$, $m_2 = 0.125kg$ were used

The complete set of a measuring - recording equipment was used for investigation of forced oscillations of a flexible rod. The means of registration, transformation and storage of signals, and also personal computer were included on it. The application of the computer had allowed to automize a procedure of numerical processing, and had enabled to apply standard graphic packages for representation of signals.

The amplitude of the oscillations of points of a rod were measured by wire-resistance strain gauges. One of disadvantage of wire-resistance strain gauges consists in their sensitivity to lateral deformations on accuracy of measurements. For compensation of the given phenomenon the strain gauges will be used in groups - sockets.

The gauges were placed in points arranged on spacing intervals $1/8;\ 3/8;\ 1/2;\ 3/4\ l$ from a fixed bearing for research of oscillation modes of a flexible rod. The calibration of strain gauges was done before application of compressive force on a rod. The scheme of arrangement of measuring equipment is shown on a Fig. 1.

The signals from strain gauges for amplification transmitted on inputs of tensoamplifier device, and then they were transmitted to a magnetograph such as TEACR -71. The application of a magnetograph has allowed to receive records of signals in the continuously form, with a capability of their repeated their subsequent processing.

The measurement of vertical accelerations on midpoint of rod length was executed by means of sensors of vertical accelerations an AC- 2. The signals from the sensor of vertical accelerations were transmitted on the scale amplifier TMA-32, having pass band 0 … 20Hz. The reinforced signal recorded on one of the channels of a magnetograph. An analog signal recorded by a magnetograph was transformed to the discrete form by an analog-digital converter.

The frequency of sampling of a signal was adopted 200 Hz. The signals, converted in the digital file, were stored on a hard disk of the computer. The special software was used for primary data processing. It is included the multiplying and division of the sensors indications into transfer factors, deduction of allowances for a zero offset.

5.2. Analysis of free oscillations. Definition of natural frequencies and decrement of oscillations

The investigated model is essentially non-linear asymmetrical system. It has two stable non-adjacent equilibrium state. In experiment of free oscillations were excited by two ways.

At first way the generator of oscillation was stopped, and the second way the rod subjected to impact load on a midpoint of the its length. In experiment, characteristic of free oscillations, namely the natural frequencies were determined separately for each of equilibrium states.

There are some basic ways of definition of decrements of oscillations exist. So, the width of resonance characteristics also called hodograph of frequency is used for this purpose for the systems with several degrees of freedom. In case of system with one degree of freedom the transient regimes of oscillations analyzed. The first two periods of oscillations were not considered at definition of dynamic characteristics of free oscillations. It caused with essential influence of transients on it. The remaining part of the oscillogram is presenting common regularity. One of the typical records of free oscillations is shown on a Fig. 2. As it is seen from the given figure, the “large” free oscillations concerning all three equilibrium state are unstable.

The transition to small oscillations relative one of two non-adjacent steady equilibrium state with time is watched. The natural frequencies of oscillations concerning each of oscillating behaviours have compounded $\omega_l = 12.5\ rad/s$ and $\omega_u = 13.9\, rad/s$, and decrement of oscillations - $\delta = 0.074$.

5.3. Definition of an oscillation mode

The oscillation modes were measured by means of a signal recording during all experiment. The shapes of bending oscillations were determined by a simultaneous record of signals all of strain gauges. The analyses of the oscillograms demonstrate that the oscillation phases are identical in all points of a rod, and the oscillation amplitude increase from the end sections to the middle.

Therefore, rod during oscillations takes the form of one half-wave of a sine. Thus, for a given type and parameters of an outer excitation in an investigated mechanical system the oscillations only of first mode are possible.

6. Mapping phase trajectories of chaotic oscillations

At dynamic tests of a flexible rod on operating of a periodic outer excitation the frequency ranges corresponding to some stable and chaotic regimes of oscillations were found Ref. [1] (see. Fig. 3). The chaotic oscillations represented by cascade of bifurcations of a period doubling

The solution of a problem of identification does not have the special difficulties if the oscillatory process is simple and studied enough. The situation is different if we deal with chaotic oscillations. The more often description of such processes is executed on the basis of statistical relations, in spite of the fact that their description by the way e of differential equations are known.

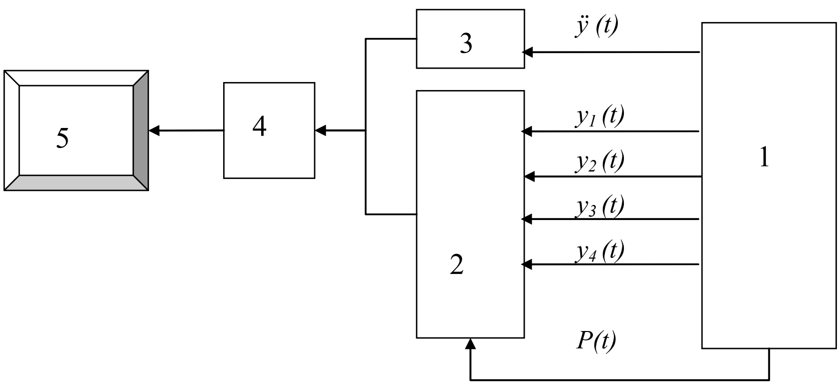

Figure 1: The scheme of arrangement of measuring equipment: 1- models; 2 – tensoamplifier device; 3- scale amplifiers; 4- magnetographs; 5 - personal computer.

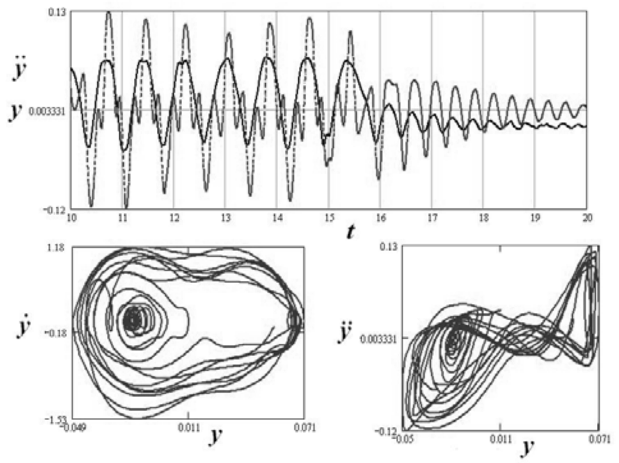

Figure 2: Time processes and phase trajectories of a free oscillation of a rod

The main peculiarity of chaotic systems is imposibillity to predict their behaviour on a long term of time. The unessential error of initial conditions results that the process passes on other trajectory after short time. Processes in such systems change depending to dissipation of energy in a system.

Let's take method of nonparametric identification suggested in Ref. [7]. It can be applied for the wide class of mechanical systems with one degree of freedom showing non-linear property of real systems. The method is based on usage of the information about accelerations Ref. [2], displacement and also outer excitation governed to system, and by methods of the qualitative theory.

Let's assume, that the studied physical model can be described by a second order differential equation of a view

$$m\ddot{y} + h(y, \dot{y}) + r(y) = P(t) \quad (2)$$

where m is weight of one meter of length of a rod, $h(y,\dot{y})$ is dissipative force, $r(y)$ is elastic force.

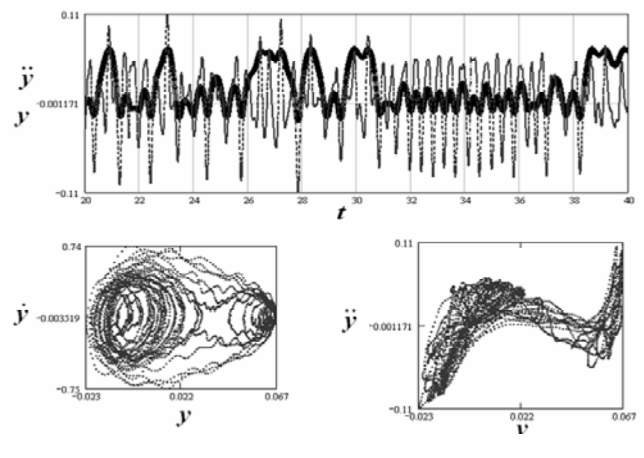

Figure 3: Chaotic oscillations of a model

7. Phase trajectories mappings of nonlinear systems in the expanded phase space

To study forced oscillations in the flexible rod, at the mounting stage of the model the rod was subjected to initial longitudinal compression. After compression the straight-line shape of the rod center-line portion became unstable. When the flexible rod was subjected to dynamic testing under periodic actions of the outer excitations, it was revealed that there are both frequency ranges, which envelop several stable regimes of oscillations, and time processes of chaotic oscillations Ref. [9]. Let's take a fairly simple method Ref. [8] for the nonparametric identification. This method makes use of information about displacements and accelerations Ref. [8] as well as the outer excitation affecting the system. Let us assume that the physical model under investigation can be described by a second order differential equation as follows:

$$m\ddot{y} + h(y, \dot{y}) + r(y) = P(t) \quad (3)$$

where m is the mass per meter of a rod, $h(y,\dot{y})$ is dissipative force, $r(y)$ is elastic force. One of the objectives of the present investigations is to obtain a comprehensive description of h and r using an outer periodic excitation and by this strategy to study the system

$$m\ddot{y} + h(y, \dot{y}) + r(y) = P(t),\ P(t+T) = P(t) = c\ . \quad (4)$$

Let us denote a sequence of points by $\{\Pi_k\} = \{y_k,\ \dot{y}_k,\ \ddot{y}_k\}$, $k = 1,\ldots,\ n$, describing the measured displacement, velocity and acceleration in the system (1) at the discrete moments of time $t = t_k = t_0 + kT$, where T is a period of the outer excitation.

8. Results

By first, ignoring the effects of the energy dissipation, we can assume that the characteristic of the elastic force can be determined from the relationship:

$$r(y_k) = c - m\ddot{y}_k\ . \quad (3)$$

The time processes of acceleration and displacements having the lengths equal to 252 periods of outer excitations were adequately processed and used in the mapping construction of the phase trajectories. The estimation of the acceleration and displacement values were performed at discrete moments of time meeting condition $c = F(t_o) = F(t_k)$ Fig 4.

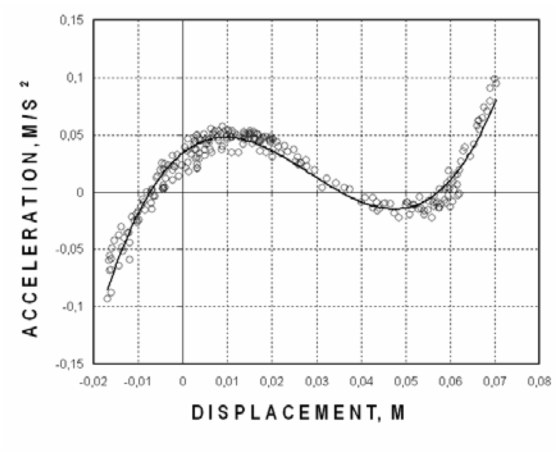

Figure: 4 Mappings of phase trajectories of chaotic oscillations

By averaging the data obtained in the experiment, a polynomial trend was derived. The trend curve has the shape of an asymmetrical cubic parabola intersecting the axis of the displacements in points $y_1 = 0,06\,m$, $y_2 = 0,039\,m$ and $y_3 = -\ 0,006\,m$, agreeing closely with the values of the coordinates $y_b = 0,058\,m$, $y_a = 0,034\,m$ and $y_c = -\ 0,006\,m$ of the rod state. To verify statistical reliability of the polynomial trend obtained, the multiple factor of determination was calculated and found equal to $R^2 = 0,835$.

9. Conclusions

The suggested method for analysis of the determinately chaotic processes provides fresh approach to data processing. The most significant feature of this method is that, in spite of its simple appearance, it enables to obtain maximum information about an investigated process or a phenomenon. The applicability of the suggested method is limited by noise levels, measuring errors or duration of a process under study.

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