Numerical analysis of convergence of the remeshing method based on the grid generator

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Abstract

The paper presents an analysis of convergence of the adaptation algorithm [1] developed by the author. The main feature of the considerd algorithm is an application of mesh generator with mesh size function [1, 2]. The proposed method uses a sequence of meshes obtained with successive modification of the mesh size function. In consequtive steps of the algorithm the values of the mesh size function taken at nodes are so modified that at points with greatest values of error indicators the values of mesh size functions are the most diminished. Having the values of the mesh size function at nodes the new mesh size function is defined by the linear interpolation. The process is performed till the error indicator obtains satisfied value. The error indicator is found at every node as a measure of discontinuity of derivatives values at the node, which are calculated at elements coming from the node.

The presented numerical analysis of convergence suggests the linear dependence between number of degrees of freedom and error norm. In further development it is planned to generalize the method to apply anisotropic meshes. The proposed method was applied to the problems, in which the solutions is unknown. The obtained results were in consistence with physical interpretations [1].

Keywords: adaptivity, error estimation, finite element methods, finite difference methods, fluid mechanics, plasticity

1. Introduction

The presented paper is an continuation of the paper [1], where the algorithm of adaptation was introduced and applied [2]to potential problem. As the approximated solution is not known the effieciency of the algorithm was tested by comparison of two solutions on different size meshes. The present paper analysis of convergence is based on the consideration of rate of convergence on benchmark solution taken from the Ref. [4]. The adapted mesh and graph of depenence between error norm and number of degrees of freedom were compared with those obtained in [4, 5].

2. Mesh generation with mesh size function

For the purpose of adaptation an algorithm of unstructured [1, 3] grid generation over plain domains with the mesh size function has been investigated. The topology and appropriate data structure for plane domain triangulation are connected with different types of curves, which are pieces of the boundary. The curve is represented by its ends and type, i.e. all the available curves having the representation in the computer data structure are topologically equivalent with straight line segment.

3. Algorithm of Adaptation

The algorithm of remeshing can be divided into the following steps:

1. Preparation of the information about the geometry and boundary conditions of the problem to be solved,
2. Fix an initial mesh size function,
3. mesh generation taking into account a mesh size function,
4. Approximated solution to the problem on the generated mesh,
5. Evaluation of error indicator at every element,
6. Calculation of nodal error indicators values by using averaging method,
7. Evaluation of the new mesh size function by using the errors indicators, which are calculated at every point,
8. if error not satisfactory go to point 3,
   else
   end of computations.

In the examples solved by the author of the paper it was suffiecient to make from 4 to 7 steps of adaptation.

4. Error indicators and Mesh Size Function Evaluation

The modification of the mesh size is performed at every adaptation step for the realization of the next. The main idea of this part of the algorithm relies on reduction of the values of the mesh size function by an appriopriately chosen function. The chosen function is continuous, linear on every element and has the smallest value at node where the value of the errors indicator is maximal and greatest where the value the error is minimal. It increases when error decreases. To describe the algorithm of the mesh size function modification it is necessary to find the values of the error indicator at the nodes. For every node P_i a weighted averaged value of the indicator is defined as follows:

$$\tilde{e}_i = \frac{\sum_{k \in L_i} area(T_i) e_i}{\sum_{k \in L_i} area(T_i)}. \tag{1}$$

In such a way a set of values of the errors at every nodal point is given.

$$\alpha = \min_{k=1,2,\ldots,N_{NOD}} \tilde{e}_k, \tag{2}$$

$$\beta = \max_{k=1,2,\ldots,N_{NOD}} \tilde{e}_k, \tag{3}$$

where N_{NOD} is the number of nodes. Obviously, $\alpha \leq \tilde{e}_k \leq \beta$ for $k = 1, \ldots, N_{NOD}$.

The following parameters varrying in interval [0,1] are used for the mesh size reduction:

λ - a value indicating the greatest mesh size function reduction,

μ - a value indicating the smallest mesh size function reduction,

5. Numerical rate of convergence

As the model problem the problem from the Ref. [5] is assumed:

$$-\triangle u = f, \tag{4}$$

in square domain Ω, $\Omega = [0,1] \times [0,1]$, with u=0 on $\partial\Omega$. The function is assumed to correspond to an exact solution

$$u = x(a-x)y(a-y)\arctan[a(\xi - \xi_0)]. \tag{5}$$

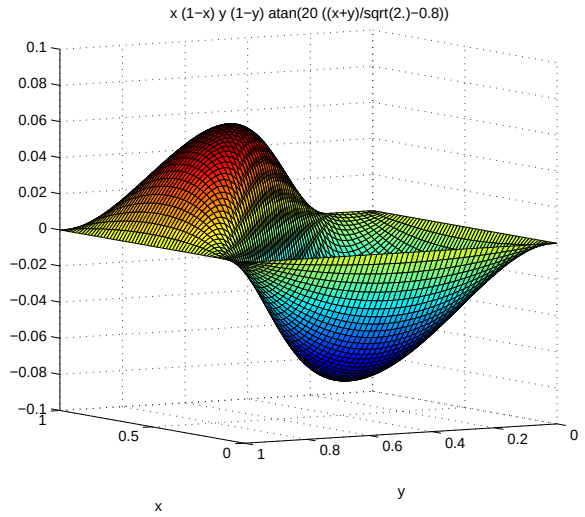

Figure 1: The strict solution to the problem 4

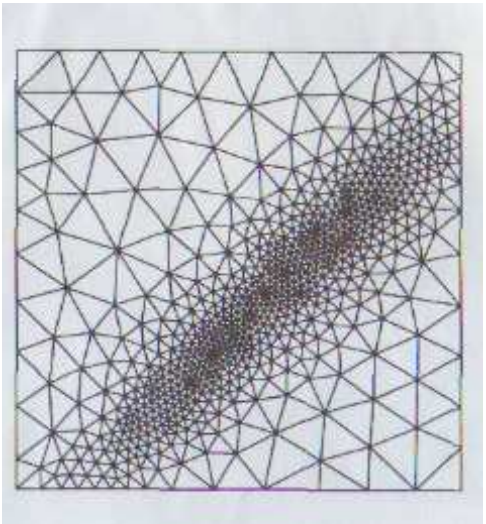

Figure 2: The adapted mesh from the Ref. of O.C Zieniewicz and J.Z. Zhu (1406 elements) [4]

The surface representing the exact solution is shown in the Fig. 1. In Fig. 2 adapted mesh for the problem 4 obtained by O.C Zieniewicz and J.Z. Zhu is presented. The adapted mesh after 4 iteration steps for that same problem is shown in Fig. 3 and in the Fig.4 the graph representing the rate of the convergence of the proposed method is drawn.

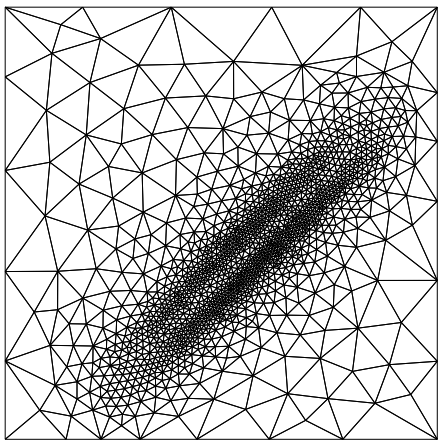

Figure 3: The adapted mesh obtained by the author (1515 elements)

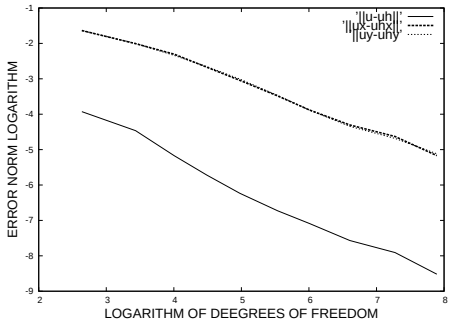

Figure 4: The curve of the remeshing convergence

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