Discrete optimization of structures subjected to dynamic loads using graph representation

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Abstract

A relatively simple method of finding discrete minimum structural weight is proposed. It is based on a tree graph, representing discrete values of the structural volume. In the proposed method, the number of analyses is limited to the order of two. The paper is illustrated with an example containing up to 42^{38} combinations.

Keywords: structural optimization, problem oriented optimization, discrete optimization, combinatorial optimization, graphs, structural static and dynamic analyses, time domain, frequency domain

1. Introduction

The design consists often in assigning to all structural members elements from the catalogue, assuring the minimum weight and fulfilment of imposed constraints. Such a process is known, as Discrete Structural Optimization (DSO).

In the present paper, a relatively simple algorithm, is proposed. The algorithm is based on the notion of a graph-tree, representing the volumes of structural members and of the whole structure [1]. The graph is explained in [2].

The algorithm is based on the assumption that the volume obtained from the continuous minimum solution is a lower bound for discrete minimum weights. In this study, the idea of applying the continuous minimum solution, as a starting point in a graph, representing structural volume, is extended to dynamic cases.

The algorithm starts with a tree graph containing only two branches. If one or more constraints are violated, some adjustment of the cross section areas is performed. The same procedure is performed, after enlarging the catalogue to four, six etc available parameters. The process of enlarging the catalogue is ended, when for two successive graphs, the obtained smallest discrete values are the same.

The paper is illustrated with a minimum weight design 160 bar tower truss, containing 42^{38} combinations.

2. The optimized structure

The structure under consideration is of a given topology and composed of elements, denoted with subscript j=[1, 2, …, j^0]. The design consists in assigning to the j-th structural member a parameter, which is taken from lists of k^0 available parameters, such, as: thickness of a metal sheet h^k, cross section areas (CSA) A^k, and/or moments of inertia I^k of a beam. Without loss of generality, in further consideration only discrete optimum design of trusses is discussed. Parameters, in the list are denoted by superscript k=[1, 2, …, k^0]. With above notation $A_j^{k_j}$ means that *k-th* cross section area from the list is assigned to j-th structural member.

All structural members are made of a linear elastic material. Small displacements and stresses, within elastic range are assumed for the whole structure. The structure is subjected to q^0 multiple static and r^0 dynamic loads. The most important part of notations is as follows:

k_j - the number of the CSA assigned to j-th structural member;

$A_j^{k_j}$ $-$ k_j -th CSA from the list assigned to j-th design variable

Find discrete cross section areas, taken from a list of available profiles,

to minimize: $$W = \sum_{j=1}^{j^0} A_j^{k_j} l_j \qquad (1)$$

subjected to equality constraints in the form of equilibrium and motion equations and inequality constraints imposed on sizes, stresses (including buckling) displacements and eigenfrequencies.

3. Graph representation of the structural volume

All possible discrete values of structural volume W

$$W = \sum_{j=1}^{j^0} A_j^{k_j} l_j \qquad (2)$$

can be represented by a graph with the tree structure, given in [2].

Inspecting the graph represented (Fig. 1), it can be seen, that extreme vertices, belonging to the same layer of the graph (sub graph), represent the smallest and the largest volumes from all possible volumes included in the layer.

It is assumed, that the structural volume, obtained from continuous minimum solution, constitutes a lower bound for all values of discrete cost functions, fulfilling given constraints. It means then, that discrete minimum volume W_{min} is not smaller than the continuous minimum volume V,

$$V \le W_{min} \qquad (3)$$

This important graph property, is applied in finding the discrete minimum of the structural volume.

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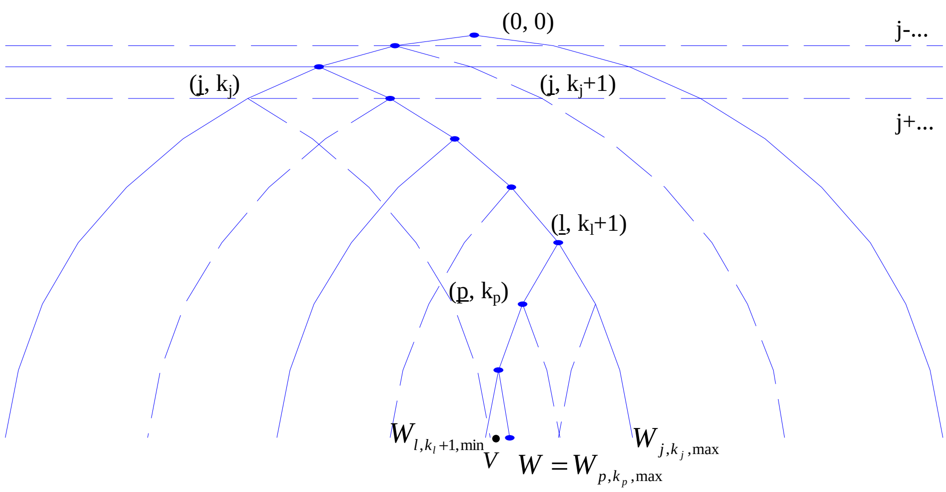

Figure 1: Search for smallest feasible discrete cost function

4. The outline of the algorithm

- Find parameters C_j (cross section areas) of structural members, or linking groups of members, solving continuous minimum weight problem.
- Take, for each j-th structural member $A_j^{k_j}$ and $A_j^{k_j+1}$, such, as the value C_j obtained from the continuous solution is included within the interval limited by their values:

$$A_j^{k_j} \le C_j \le A_j^{k_j+1} \tag{4}$$

Farther steps can be deducted from Fig. 1. Detailed steps are given in [2].

5. Numerical example – 160 bar 3D truss

The example deals with the minimum weight design of 160-bar truss (Fig.2) made of rolled pipes with sizes taken from IS808. The data for the truss are taken from the paper [3]. The truss members are linked in 38 independent groups of design variables. The structure is subjected to eight, different sets of loads. Buckling constraints for compression members are considered. The CSA and radii of gyration, for the 42 prescribed discrete sections are given in Table 1.

Table 1: CSA (cm^2) and radii of gyration (cm) for 160 bar truss

k	A^k	r^k	k	A^k	r^k	k	A^k	r^k
1	1.84	0.47	15	9.40	1.35	29	33.90	2.33
2	2.26	0.57	16	10.47	1.36	30	34.77	2.97
3	2.66	0.67	17	11.38	1.45	31	39.16	2.54
4	3.07	0.77	18	12.21	1.55	32	43.00	2.93
5	3.47	0.87	19	13.79	1.75	33	45.65	2.94
6	3.88	0.97	20	15.39	1.95	34	46.94	2.94
7	4.79	0.97	21	17.03	1.74	35	51.00	2.92
8	5.27	1.06	22	19.03	1.94	36	52.10	3.54
9	5.75	1.16	23	21.12	2.16	37	61.82	3.96
10	6.25	1.26	24	23.20	2.36	38	61.90	3.52
11	6.84	1.15	25	25.12	2.57	39	68.30	3.51
12	7.44	1.26	26	27.50	2.35	40	76.38	3.93
13	8.06	1.36	27	29.88	2.56	41	90.60	3.92
14	8.66	1.46	28	32.76	2.14	42	94.13	3.92

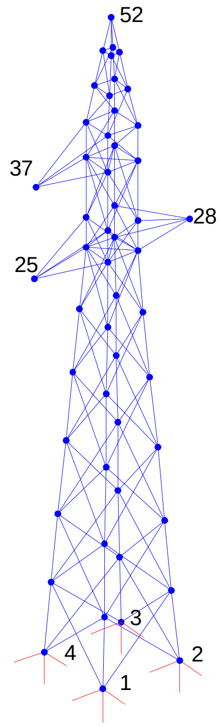

Figure 2: 160 bar truss structure

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