Multiple-point beam-to-beam contact finite element

Przemysław Litewka

Institute of Structural Engineering, Poznan University of Technology
ul. Piotrowo 5, 60-965 Poznań, Poland
e-mail: przemyslaw.litewka@gmail.com

Abstract

In this paper a new contact finite element for beams with circular cross-sections is presented. The element is a development of the previously formulated beam-to-beam contact finite elements to be used in cases when beams get in contact at very acute angles. In such situation if beam deformations in the vicinity of the contact point are taken into account the contact is not point-wise but it extends to a certain area. To cover such a case in a more realistic way four additional contact points are introduced to accompany the original single contact point. The central point is located using the orthogonality conditions for the beam axes and the positions of four extra points are defined by a shift of the local co-ordinates. This shift depends on beam geometries and the angle between tangent vectors at the central contact point.

Keywords: beam-to-beam contact, orthogonality conditions, multiple-point contact

1. Introduction

Contact between beams is a special case in the 3D contact analysis. This topic was started in Ref. [7] and continued in Refs. [8, 5, 6] where contact without and with Coulomb friction for beams of circular and rectangular cross-sections was covered. The further development concerned inclusion of thermal and electric coupling, see Ref. [1]. Some subsequent research was also devoted to the smoothing procedures for 3D curves representing axes of beams in contact, e.g. Ref. [4]. A rigorous approach to the question of point-wise contact was also suggested in Ref. [3]. There the authors focused their interest on the closest-point projection procedure which for the beam-to-beam contact leads to orthogonality conditions, see Ref. [7].

The key assumption in all these analyses was that the closest points can always be uniquely located for two beams, at least locally. In other words, the point-wise contact between beams was assumed. However, such an approach may fail in some special cases, for instance when irregular assemblies of fibre-like objects are considered Ref. [2]. Hence, the general approach must include possibilities when the contacting beam-like objects form acute angles and are parallel or conforming.

The new beam-to-beam contact finite element presented in this paper is developed to model in a more precise way situations when contact between beams should not be considered as point-wise. The main assumption for this new element is that locally there still exists the unique solution for the closest-point projection equations. It is therefore not suitable for the case of conforming/parallel beams – this case requires a completely different approach. In the present proposal, in addition to the main central contact points found using the orthogonality conditions four extra points are involved. The problem of finding their location is discussed in Section 2. Section 3 presents some important details of the contact formulation for the additional points.

2. The additional contact points in the element

Let us consider contact between beams denoted by *m* and *s* with axes presented in Fig. 1 The beams have circular cross-sections with radii r_m and r_s.. The presented multiple-point beam-to-beam contact element involves one pair of central contact points C_{mn} – C_{ms} and four additional ones. The location of the central points is found using the orthogonality conditions. This issue was exactly described in Refs. [7, 4] and as result one gets the local co-ordinates of closest points on each beam – ξ_{mn} and ξ_{sn} ($-1 \le \xi_i \le 1$).

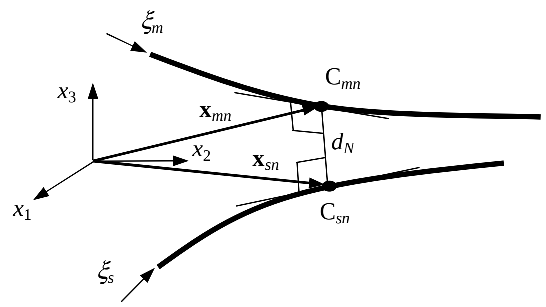

Figure 1: The closest points on contacting beams

The additional contact pairs are found in such a way, that two points per beam: C_{m+n}, C_{m-n} and C_{s+n}, C_{s-n} are first defined by a positive and negative shift of the respective local co-ordinate with respect to the co-ordinates of the central point:

$$\begin{aligned} \xi_{m-n} &= \xi_{mn} - \xi_{\Delta m} \\ \xi_{m+n} &= \xi_{mn} + \xi_{\Delta m} \\ \xi_{s-n} &= \xi_{sn} - \xi_{\Delta s} \\ \xi_{s+n} &= \xi_{sn} + \xi_{\Delta s} \end{aligned} \tag{1}$$

The shift values come from the geometric considerations presented in Figs. 2 and 3. The distances between the centre contact point on the beam axis and the edge of the overlap region in the plan view (Fig. 2) are

$$\begin{aligned} a_m &= \frac{r_s}{\sin\varphi} \\ a_s &= \frac{r_m}{\sin\varphi} \end{aligned} \tag{2}$$

The distance of the shifted contact point from the central one is assumed as one half of a_m and a_s, respectively. This yields the following shift values for the local co-ordinates

$$\xi_{\Delta m} = \frac{\frac{1}{2}a_m}{\frac{1}{2}l_m} = \frac{r_s}{l_m \sin\varphi}$$
$$\xi_{\Delta s} = \frac{\frac{1}{2}a_s}{\frac{1}{2}l_s} = \frac{r_m}{l_s \sin\varphi} \quad (3)$$

where l_m and l_s are the lengths of the respective beam finite elements.

Such an adoption is in accordance with the assumption of small strains and exclusion of extreme cases when one beam would wind itself about the other one, see Ref. [5].

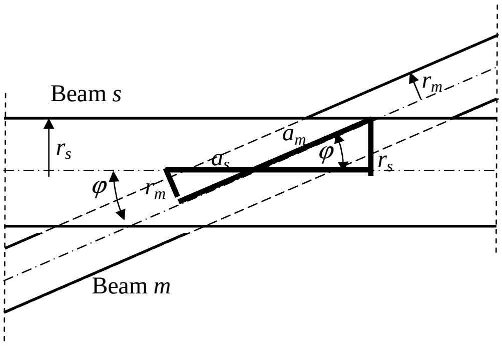

Figure 2: Plan view of contacting beams

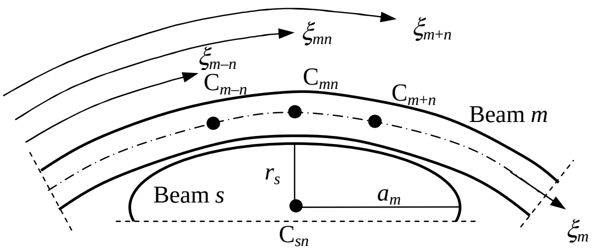

Figure 3: Section A-A across the beam s at the contact point

3. Details of contact formulation for the additional points

The formulation of contact between the central closest points given in Refs. [7, 8, 5, 6, 4] remains unchanged. For the additional points the similar formulation is used. However due to the fixed location of the additional points which are tied to the central point some issues can be simplified.

The first step in the formulation is the closest point projection. Since the local co-ordinate of an additional point is fixed on one beam, say C_{m+n} on the beam m, it is only necessary to use one orthogonality condition to determine the local co-ordinate of the contact point on the second beam – in this case s. This yields the following expression for the local co-ordinate update in the iterative process of solving the non-linear equation

$$\Delta\xi_s = \frac{-(\mathbf{x}_m - \mathbf{x}_s)\cdot\mathbf{x}_{s,s}}{\mathbf{x}_{s,s}\cdot\mathbf{x}_{s,s} + (\mathbf{x}_m - \mathbf{x}_s)\cdot\mathbf{x}_{s,ss}} \quad (4)$$

where $\mathbf{x}_i$ stands for the position vector of a point on the beam i and the subscripts after comma denote partial derivatives with respect to local co-ordinates ξ_i.

Due to the fact that the local co-ordinate of the additional point, say C_{m+n} on the beam m, is tied to that for the central point C_{mn} by (1), the expressions for the variation, the linearization and the linearization of the variation of this co-ordinate, $\delta\xi_{mn}$, $\Delta\xi_{mn}$ and $\Delta\delta\xi_{mn}$, are common for these points. Hence, the derivation of kinematic variables for contact and friction is simplified, too. For instance, the linearization of the variation of the normal gap g_N (see Ref. [5]) at C_{m+n} is given as

$$\begin{aligned}\Delta\delta g_N = {} & (\Delta\delta\mathbf{u}_{m+n} - \Delta\delta\mathbf{u}_{s+n})\circ\mathbf{n} + \\ & + (\delta\mathbf{u}_{m+n,m}\Delta\xi_{mn} - \delta\mathbf{u}_{s+n,s}\Delta\xi_{s+n})\circ\mathbf{n} + \\ & + (\Delta\mathbf{u}_{m+n,m}\delta\xi_{mn} - \Delta\mathbf{u}_{s+n,s}\delta\xi_{s+n})\circ\mathbf{n} + \\ & + (\mathbf{x}_{m+n,mm}\Delta\xi_{mn}\delta\xi_{mn} - \mathbf{x}_{s+n,ss}\Delta\xi_{s+n}\delta\xi_{s+n})\circ\mathbf{n} + \\ & + \frac{1}{g_N}(\delta\mathbf{u}_{m+n} + \mathbf{x}_{m+n,m}\delta\xi_{mn} - \delta\mathbf{u}_{s+n} - \mathbf{x}_{s+n,s}\delta\xi_{s+n}) \\ & \times(\mathbf{1} - \mathbf{n}\otimes\mathbf{n})(\Delta\mathbf{u}_{m+n} + \mathbf{x}_{m+n,m}\Delta\xi_{mn} - \Delta\mathbf{u}_{s+n} - \mathbf{x}_{s+n,s}\Delta\xi_{s+n}).\end{aligned} \quad (5)$$

where $\mathbf{x}_i$ stands for the displacement vector at a point on the beam i and $\mathbf{n}$ denotes the unit normal vector.

4. Conclusions

The presented enhanced beam-to-beam contact finite element with five contact points allows to model in a more realistic way the contact situations when contacting beams form acute angles in the plan view. It is also worth to note that the presence of multiple contact points spreads the possible inaccuracy in meeting the contact constraints on a certain area, thus allowing to reduce the penalty parameter value. The numerical results will be presented at the conference.

References

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