Modeling cracks in concrete elements with XFEM

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Abstract

The paper describes the eXtended Finite Element Method (XFEM) to simulate cracks in quasi-brittle materials (e.g. concrete). Fundamental relationships and basic discrete constitutive laws are given. Criteria to determine the crack's growth condition and its direction are provided. Aspects of numerical implementation are discussed.

Keywords: concrete, cracks, failure, finite element methods

1. Introduction

Modelling of quasi-brittle materials is a quite demanding task due to the presence of cracks. At the beginning of the loading, a region with several microcracks is formed. Later these microcracks create a macrocrack.

In general there are two basic approaches to simulate a solid body with cracks. The first idea is based on continuum description, which is also valid after a macrocrack creation. The material can be described using e.g. elasto-plastic, damage mechanics or coupled constitutive laws. These formulations usually include softening, so they have to be equipped with a characteristic length of the microstructure. It can be done by means of a Cosserat, non-local or gradient theory.

Alternatively, a crack can be regarded as a macrocrack with a displacement jump (by omitting a microcrack phase). The oldest solutions used interface elements defined along element edges. The modern proposals allow one to locate cracks in the interior of finite elements. One possibility is to use elements with embedded discontinuities. Another solution, called XFEM, is based on a partition of unity concept [4]. It adds global degrees of freedom in some nodes during calculations to describe jumps in a displacement field. This technique was originally used to simulate brittle materials [1, 5] and was extended to handle also cohesive cracks in quasi-brittle materials [8].

2. Theory background

2.1. Displacement field

In a body crossed by a discontinuity (Fig. 1), a displacement field $\boldsymbol{u}$ can be decomposed into a continuous part $\boldsymbol{u}_{cont}$ and discontinuous part $\boldsymbol{u}_{disc}$. For a standard XFEM displacement field is defined as [1, 8]

$$\boldsymbol{u}(\boldsymbol{x},t)=\hat{\boldsymbol{u}}(\boldsymbol{x},t)+\Psi(\boldsymbol{x})\tilde{\boldsymbol{u}}(\boldsymbol{x},t) \quad (1)$$

with continuous functions $\hat{\boldsymbol{u}}$ and $\tilde{\boldsymbol{u}}$ and a generalised step function Ψ (other definitions can be used here)

$$\Psi(\boldsymbol{x})=\begin{cases}1 & \boldsymbol{x}\in\Omega^+\\ -1 & \boldsymbol{x}\in\Omega^-\end{cases} \quad (2)$$

2.2. Standard and shifted-basis enrichments

In a finite element format, Eqn (1) can be written as

$$\boldsymbol{u}(\boldsymbol{x})=\boldsymbol{N}(\boldsymbol{x})\boldsymbol{a}+\Psi(\boldsymbol{x})\boldsymbol{N}(\boldsymbol{x})\boldsymbol{b} \quad (3)$$

where $\boldsymbol{N}$ contains shape functions, $\boldsymbol{a}$ – standard displacements in nodes and $\boldsymbol{b}$ – enriched displacements (jumps) in nodes. Only nodes which belong to cracked elements have to be enriched. The alternative formulation [2] called shifted-basis enrichment will be used here. It assumes the following definition of the displacement field

$$\boldsymbol{u}(\boldsymbol{x})=\boldsymbol{N}(\boldsymbol{x})\boldsymbol{a}+(\boldsymbol{\Psi}(\boldsymbol{x})-\boldsymbol{\Psi}(\boldsymbol{x}_I))\boldsymbol{N}(\boldsymbol{x})\boldsymbol{b} \quad (4)$$

with diagonal matrices $\boldsymbol{\Psi}(\boldsymbol{x})$ and $\boldsymbol{\Psi}(\boldsymbol{x}_I)$ containing $\Psi(\boldsymbol{x})$ and $\Psi(\boldsymbol{x}_I)$, respectively ($\boldsymbol{x}_I$ is the position of the node *I*). Starting from a weak form of equation of motion, force and stiffness matrices can be obtained ([8]).

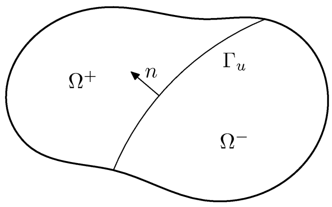

Figure 1: Body crossed by a discontinuity

3. Constitutive laws

3.1. Bulk material

In an (uncracked) continuum, usually a linear elastic constitutive law between stresses $\boldsymbol{\sigma}$ and strains $\boldsymbol{\varepsilon}$ is assumed

$$\boldsymbol{\sigma}=\boldsymbol{D}^e:\boldsymbol{\varepsilon} \quad (5)$$

where $\boldsymbol{D}^e$ is a linear elastic material matrix.

3.2. Activation/direction criteria

To activate (create or extend) a crack, the following condition has to be fulfilled at least in one integration point in the element at the front of the crack tip (Rankine criterion)

$$\max\{\sigma_1,\sigma_2,\sigma_3\} > f_t \tag{6}$$

where $\sigma_1,\sigma_2,\sigma_3$ are principal stresses and f_t is a tensile strength. The direction of the extension is perpendicular to the direction of the maximum principal stress. To smooth the stress field around the crack tip, non-local stresses $\boldsymbol{\sigma}^*$ can be taken to determine the crack direction [8]. These stresses are calculated according to

$$\boldsymbol{\sigma}^* = \int \boldsymbol{\sigma}\, w\, dV \tag{7}$$

over a hemi circle at the front of the crack tip with a weight function *w* defined as

$$w = \frac{1}{(2\pi)^{3/2} l^3} exp\left(-\frac{r^2}{2l^2}\right) \tag{8}$$

Here *l* is an averaging length (usually equal to 3 times the average element's size) and *r* is a distance between a point and the crack tip. It should be noted that this operation does not introduce non-locality (as a length of the microstructure) in the model. It only smoothes stress field around the crack tip.

3.3. Discrete cohesive law

Discrete cohesive law binds tractions ***t*** with displacement jumps $[[\boldsymbol{u}]]$ at the discontinuity. One of the simplest possibilities assumes the following format of the loading function

$$f([[u_n]],\kappa) = [[u_n]] - \kappa \tag{9}$$

and the formula for a normal component of traction vector

$$t_n = f_t \exp(-\beta\kappa) \tag{10}$$

with a history parameter κ and a coefficient β. During loading, the tangent stiffness is calculated and in unloading phase, the secant stiffness is used. In a compressive regime, elastic stiffness recovery is assumed. In a tangent direction, a linear relationship between a displacement jump and traction is defined with the stiffness T_s.

4. Implementation details

The inclusion of enriched displacements ***b*** requires several modifications in the standard FE code. The final number of extra degrees of freedom ***b*** is unknown at the beginning, but it can grow during calculations. Therefore special techniques are required to handle extra data. If an essential boundary condition has been specified at a node with enriched degrees of freedom, an additional condition $b = 0$ has to be added at this node.

Some modifications in the FE iteration algorithm have to be introduced. A new crack segment can be defined only at the converged configuration. End points of a crack segment can be placed only at element's edges. After defining a new segment, a current increment has to be restarted.

Finally, a new scheme for calculating strains, stresses, internal forces and stiffness in a cracked element is required. Due to the arbitrary location of a discontinuity segment inside the element, new coordinates of integration points have to be defined. To determine these coordinates a sub-division algorithm has to be proposed. Figure 2 shows auxiliary sub-triangles in 6-node triangle. This element requires in total 23 integration points [3, 8].

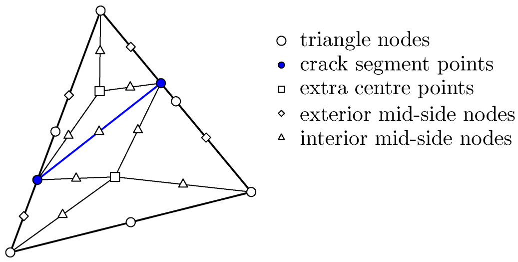

Figure 2: Sub-triangles in 6-node triangle

5. Research plan

The XFEM algorithm described above has been successfully implemented in the Plaxis program to simulate the behaviour of soils like clays [3]. Six node triangular elements were used. Different discrete laws (Mohr-Coulomb), activation and direction criteria for shear zone's growth were defined. Biaxial test and retaining wall problem were simulated.

The goal of the current research is to adapt/redefine the existing implementation and use the XFEM approach to simulate the behaviour of cracked concrete elements. During the first phase simple tests like uniaxial tension and uniaxial bending will be simulated. The mesh insensitivity will be examined. The performance of different finite elements and various integrations schemes will be investigated. During the second phase, the possibility of modelling curved cracks will be tested. Experiments by Nooru-Mohamed [6] and Schlangen [7] will be reproduced.

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