Numerical modelling of the cancer destruction during hyperthermia treatment

Ewa Majchrzak and Marek Paruch*

Department of Strength of Materials and Computational Mechanics, Silesian University of Technology
ul. Konarskiego 18A, 44-100 Gliwice, Poland
e-mail: ewa.majchrzak@polsl.pl, marek.paruch@polsl.pl

Abstract

Electromagnetic field induced by two external electrodes and temperature field resulting from electrodes action in 3D domain of biological tissue is considered. To assure the optimum conditions of destruction the magnetic nanoparticles are introduced to the tumor-domain analyzed. External electric field causes the heat generation in tissue domain. The distribution of electric potential in domain considered is described by the Laplace equation, while the temperature field is described by the Pennes equation. These problems are coupled by source function being the additional component in Pennes equation and resulting from the electric field action. The problem is solved by means of the boundary element method. In the final part of the paper the examples of computations are shown.

Keywords: heat transfer, electromagnetic field, boundary element method, numerical analysis, biomechanics

1. Introduction

Hyperthermia (42-46°C) is a condition which occurs when the body produces or absorbs more heat than it can dissipate. It is usually due to excessive exposure to heat. Hyperthermia can also be created artificially by medical devices and it may be used as a therapeutic method to obtain an artificial increase in temperature of the certain types of cancer tissue, such as skin cancer [9]. It has been well established that temperature above 42°C will cause necrosis of living cells. A very important problem is to apply the heat directly to the tumor region in order to avoid the damage of healthy tissue surrounding this domain. Great efforts have recently been made to use magnetic nanoparticles for the concentrated heat deposition at the tumor region located inside the human body. The particles should be made from the material assuring the appropriate magnetic properties and biological compatibility. Here the iron oxides magnetite (Fe_3O_4) and maghemite (γ–Fe_2O_3) can be applied [3].

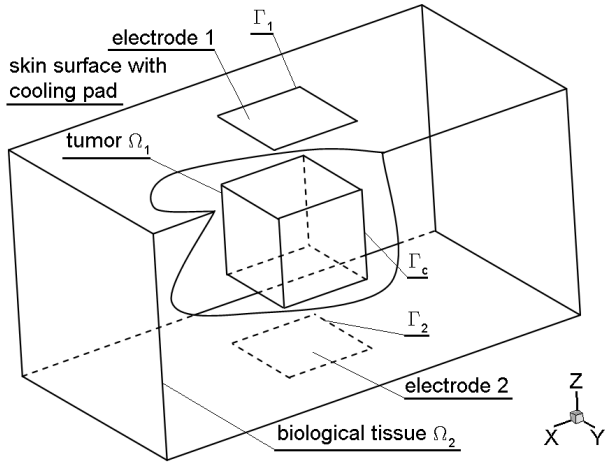

Figure 1: Action of electric field on biological tissue

In Figure 1 a typical radio frequency (RF) hyperthermia system is shown [3]. The mathematical model of the process analyzed consists of two parts [3, 4, 5]. The electric part concerns the Laplace equation to obtain the electric field distribution. The thermal part is connected with the bioheat transfer equation to obtain the temperature distribution. In the bioheat transfer equation the additional source term associated with the heat generation caused by electric field distribution appears.

2. Electromagnetic field

Because the wavelength of the radio frequency (RF) current in tissues is much greater that depth of a human body, the quasistatic electric field approximation can be applied. The quasistatic electric field is irrotational, so the electric potential can be introduced. The potential $\varphi_e(x, y, z)$ inside the healthy tissue (e = 1) and tumor region (e = 2) is described by the system of Laplace equations

$$(x, y, z) \in \Omega_e: \quad \varepsilon_e \nabla^2 \varphi_e(x, y, z) = 0 \tag{1}$$

where ε_e [$C^2/(Nm^2)$] is the dielectric permittivity of sub-domains Ω_e. At the interface Γ_c of the tumor and healthy tissue (Figure 1) the ideal electric contact is assumed

$$(x, y, z) \in \Gamma_c: \quad \begin{cases} \varphi_1(x, y, z) = \varphi_2(x, y, z) \\ -\varepsilon_1 \dfrac{\partial \varphi_1(x, y, z)}{\partial n} = -\varepsilon_2 \dfrac{\partial \varphi_2(x, y, z)}{\partial n} \end{cases} \tag{2}$$

On the external surface of tissue being in a contact with the electrodes the following condition is given

$$\begin{aligned} &(x, y, z) \in \Gamma_1: \quad \varphi_1(x, y, z) = U \\ &(x, y, z) \in \Gamma_2: \quad \varphi_2(x, y, z) = -U \end{aligned} \tag{3}$$

where U [V] is the electric potential of the electrode relative to the ground. On the remaining external boundary of tissue the ideal electric isolation is assumed : $-\varepsilon_1 \partial \varphi_1(x, y, z) / \partial n = 0$.

The electric field inside the tissue is described by equation

$$\mathbf{E}_e(x, y, z) = -\left[\frac{\partial \varphi_e(x, y, z)}{\partial x} \quad \frac{\partial \varphi_e(x, y, z)}{\partial y} \quad \frac{\partial \varphi_e(x, y, z)}{\partial z} \right]^{\mathrm{T}} \tag{4}$$

* This work was supported by Grant No N N501 3667 34 from the Polish Ministry of Science and Higher Education.

Heat generation Q_1 [W/m^3] due to the electromagnetic dissipated power in healthy tissue depends on the conductivity σ_1 [S/m] and the electric field $\mathbf{E}_1$ [3]

$$Q_1(x, y, z) = \frac{\sigma_1 \left|\mathbf{E}_1(x, y, z)\right|^2}{2} \tag{5}$$

The tumor region with embedded magnetic particles is treated as a composite and due to the assumed homogeneity of Ω_2 the mean value of electrical conductivity σ_2 of this sub-domain can be approximated as $1/\sigma_2 = (1-\Theta)/\sigma_2' + \Theta/\sigma_3$, where σ_2', σ_3 are the electrical conductivities of tumor and particles, respectively, and $\Theta = n\frac{4}{3}\pi r^3 / V_t$ is the concentration of particles (*n* is the number of particles, *r* is the radius of particle, V_t is the tumor volume).
The amplitude of the magnetic field intensity can be expressed as [3]

$$\left|\mathbf{H}(x, y, z)\right| = \frac{1}{1+N(\chi)} \frac{\left|\mathbf{E}_2(x, y, z)\right|}{\mu_0 \pi f R} \tag{6}$$

where $\mu_0 = 4\pi \cdot 10^{-7}$ [Tm/A] is the dielectric constant permeability of free space, f [Hz] is the frequency of electromagnetic field, R [m] is the radius of the magnetic induction loop, $N(\chi)$ is the demagnetizing factor ($N(\chi) = 1/3$ for a spherical composite) and χ (here: $\chi = \chi' + i\chi''$ [3]) is the susceptibility of the magnetic nanoparticles.
Heat generation by superparamagnetism (SPM) is given by [3]

$$P_{SPM}(x, y, z) = \mu_0 \pi f \chi'' \left|\mathbf{H}(x, y, z)\right|^2 \tag{7}$$

and then for $(x, y, z) \in \Omega_2$ one has

$$Q_2(x, y, z) = \Theta P_{SPM}(x, y, z) + (1-\Theta)\frac{\sigma_2 \left|\mathbf{E}_2(x, y, z)\right|^2}{2} \tag{8}$$

3. Temperature field

The temperature field in the healthy tissue and the tumor region with embedded magnetic nanoparticles is described by the system of Pennes equations [3, 4, 5, 8]

$$\lambda_e \nabla^2 T_e(x, y, z) + k_e\left[T_B - T_e(x, y, z)\right] + Q_{met\,e} + Q_e^E(x, y, z) = 0 \tag{9}$$

where $e = 1, 2$ correspond to the healthy tissue and tumor region, respectively, T_e denotes temperature, λ_e [W/(mK)] is the thermal conductivity, $k_e = G_{Be}c_B$ (G_{Be} [1/s] is the perfusion rate, c_B [J/(m^{3}K)] is the volumetric specific heat of blood), T_B is the supplying arterial blood temperature, $Q_{met\,e}$ [W/m^3] is the metabolic heat source, $Q_e^E(x, y, z)$ [W/m^3] is the heat source connected with the electromagnetic field action. It should be pointed out that thermal conductivity λ_2 of tumor region with nanoparticles can be calculated as follows $1/\lambda_2 = (1-\Theta)/\lambda_2' + \Theta/\lambda_3$, where λ_2', λ_3 are the thermal conductivities of tumor and nanoparticles, respectively.

At the interface Γ_c between the tumor and healthy tissue the ideal contact is assumed

$$(x, y, z) \in \Gamma_c: \begin{cases} T_1(x, y, z) = T_2(x, y, z) \\ -\lambda_1 \dfrac{\partial T_1(x, y, z)}{\partial n} = -\lambda_2 \dfrac{\partial T_2(x, y, z)}{\partial n} \end{cases} \tag{10}$$

At the $\{x, y\}$ and $\{x, z\}$ surfaces (skin surface - c.f. Figure 1) of tissue domain the convection condition is assumed

$$-\lambda_1 \frac{\partial T_1(x, y, z)}{\partial n} = \alpha_w\left[T_1(x, y, z) - T_w\right] \tag{11}$$

where α_w [W/(m^{2}K)] is the heat transfer coefficient between the skin surface and the cooling water, T_w is the cooling water temperature. On the remaining boundaries the adiabatic condition $-\lambda_1 \partial T_1 / \partial n = 0$ can be taken into account. This condition results from the consideration that at the positions far from the center of the domain the temperature field is almost not affected by the external heating [3].

4. Boundary element method – electric field

The boundary integral equations corresponding to the equations (1) can be expressed as [1, 2, 7]

$$B_e(\xi_1, \xi_2, \xi_3)\varphi_e(\xi_1, \xi_2, \xi_3) + \iint_\Gamma \psi_e(x, y, z)\varphi_e^*(\xi_1, \xi_2, \xi_3, x, y, z)\,d\Gamma = \iint_\Gamma \varphi_e(x, y, z)\psi_e^*(\xi_1, \xi_2, \xi_3, x, y, z)\,d\Gamma \tag{12}$$

where (ξ_1, ξ_2, ξ_3) is the observation point, the coefficient $B_e(\xi_1, \xi_2, \xi_3)$ is dependent on the location of source point (ξ_1, ξ_2, ξ_3), $\psi_e(x, y, z) = -\varepsilon_e \partial\varphi_e(x, y, z)/\partial n$.
Fundamental solutions of the problem discussed have the following form

$$\varphi_e^*(\xi_1, \xi_2, \xi_3, x, y, z) = \frac{1}{4\pi\varepsilon_e r} \tag{13}$$

where r is the distance between points (ξ_1, ξ_2, ξ_3) and (x, y, z). Differentiating the function $\varphi_e^*(\xi_1, \xi_2, \xi_3, x, y, z)$ with respect to the outward normal $\mathbf{n} = [\cos\alpha, \cos\beta, \cos\gamma]$ the function $\psi_e^*(\xi_1, \xi_2, \xi_3, x, y, z)$ is obtained

$$\psi_e^*(\xi_1, \xi_2, \xi_3, x, y, z) = -\varepsilon_e \frac{\partial\varphi_e^*(\xi_1, \xi_2, \xi_3, x, y, z)}{\partial n} = \frac{d}{4\pi r^3} \tag{14}$$

where

$$d = (x-\xi_1)\cos\alpha + (y-\xi_2)\cos\beta + (z-\xi_3)\cos\gamma \tag{15}$$

The boundaries of the domains are divided into N_1 and N_2 boundary elements as shown in Figure 2.

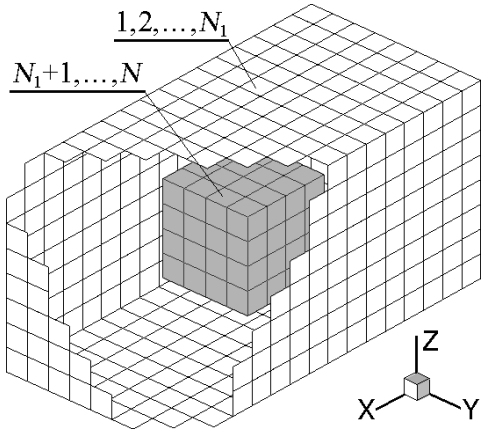

Figure 2: Boundary elements

For constant boundary elements [1, 7] one obtains the following approximation of equations (12):

- for healthy tissue

$$\sum_{j=1}^{N_1} G_{ij}^1 \psi_j^1 = \sum_{j=1}^{N_1} H_{ij}^1 \varphi_j^1, \quad i = 1, 2, ..., N_1 \tag{16}$$

- for tumor region

$$\sum_{j=N_1+1}^{N} G_{ij}^2 \psi_j^2 = \sum_{j=N_1+1}^{N} H_{ij}^2 \varphi_j^2, \quad i = N_1+1, N_1+2, ..., N \tag{17}$$

where

$$G_{ij}^e = \iint_{\Gamma_j} \varphi_e^*(\xi_{1i}, \xi_{2i}, \xi_{3i}, x, y, z)\, d\Gamma_j, \quad e = 1, 2 \tag{18}$$

and (for $i \neq j$)

$$H_{ij}^e = \iint_{\Gamma_j} \psi_e^*(\xi_{1i}, \xi_{2i}, \xi_{3i}, x, y, z)\, d\Gamma_j, \quad e = 1, 2 \tag{19}$$

while $H_{ii}^e = -\sum_{\substack{j=1 \\ j \neq i}}^{N} H_{ij}^e, \quad e = 1, 2$.

The boundary condition (2) on the contact surface (healthy tissue – tumor region) written in the form

$$\begin{cases} \varphi_j^1 = \varphi_j^2 = \varphi_j \\ \psi_j^1 = -\psi_j^2 = \psi_j \end{cases} \tag{20}$$

is introduced to the systems of equations (16), (17). So, one obtains

$$\sum_{j=1}^{N_1} G_{ij}^1 \psi_j = \sum_{j=1}^{N_1} H_{ij}^1 \varphi_j, \quad i = 1, 2, ..., N_1 \tag{21}$$

and

$$-\sum_{j=N_1+1}^{N} G_{ij}^2 \psi_j = \sum_{j=N_1+1}^{N} H_{ij}^2 \varphi_j, \quad i = N_1+1, N_1+2, ..., N \tag{22}$$

Joining together the systems of equations (21), (22) and using the matrix convention one has

$$\begin{bmatrix} \mathbf{G}^1 & -\mathbf{H}_c^1 & \mathbf{G}_c^1 \\ \mathbf{0} & -\mathbf{H}_c^2 & -\mathbf{G}_c^2 \end{bmatrix} \begin{bmatrix} \boldsymbol{\psi}^1 \\ \boldsymbol{\varphi} \\ \boldsymbol{\psi} \end{bmatrix} = \begin{bmatrix} \mathbf{H}^1 \boldsymbol{\varphi}^1 \\ \mathbf{0} \end{bmatrix} \tag{23}$$

Next, the remaining boundary conditions (3) and $-\varepsilon_1 \partial \varphi_1(x, y, z) / \partial n = 0$ should be introduced to the system of equations (23). This system allows one to determine the 'missing' boundary values of functions φ_j^e, ψ_j^e.

Next, the values of functions φ_e at the internal points $(\xi_{1i}, \xi_{2i}, \xi_{3i})$ can be determined using the formula

$$\varphi_i^e = \sum_{j=1}^{N} H_{ij}^e \varphi_j^e - \sum_{j=1}^{N} G_{ij}^e \psi_j^e, \quad e = 1, 2 \tag{24}$$

It should be pointed out that in order to determine the electric field inside tissue (equation (4)) the partial derivatives $\partial \varphi_e(x, y, z) / \partial x$, $\partial \varphi_e(x, y, z) / \partial y$, $\partial \varphi_e(x, y, z) / \partial z$ must be known. One of the possibilities is the application of equation (12) for internal nodes (ξ_1, ξ_2, ξ_3) $(B_e(\xi_1, \xi_2, \xi_3) = 1)$ and then

$$\frac{\partial \varphi_e(\xi_1, \xi_2, \xi_3)}{\partial \xi_1} = \iint_\Gamma \varphi_e(x, y, z) \frac{\partial \psi_e^*(\xi_1, \xi_2, \xi_3, x, y, z)}{\partial \xi_1} d\Gamma - \iint_\Gamma \psi_e(x, y, z) \frac{\partial \varphi_e^*(\xi_1, \xi_2, \xi_3, x, y, z)}{\partial \xi_1} d\Gamma \tag{25}$$

$$\frac{\partial \varphi_e(\xi_1, \xi_2, \xi_3)}{\partial \xi_2} = \iint_\Gamma \varphi_e(x, y, z) \frac{\partial \psi_e^*(\xi_1, \xi_2, \xi_3, x, y, z)}{\partial \xi_2} d\Gamma - \iint_\Gamma \psi_e(x, y, z) \frac{\partial \varphi_e^*(\xi_1, \xi_2, \xi_3, x, y, z)}{\partial \xi_2} d\Gamma \tag{26}$$

and

$$\frac{\partial \varphi_e(\xi_1, \xi_2, \xi_3)}{\partial \xi_3} = \iint_\Gamma \varphi_e(x, y, z) \frac{\partial \psi_e^*(\xi_1, \xi_2, \xi_3, x, y, z)}{\partial \xi_3} d\Gamma - \iint_\Gamma \psi_e(x, y, z) \frac{\partial \varphi_e^*(\xi_1, \xi_2, \xi_3, x, y, z)}{\partial \xi_3} d\Gamma \tag{27}$$

where

$$\frac{\partial \varphi_e^*}{\partial \xi_1} = \frac{x - \xi_1}{4\pi\varepsilon_e r^3}, \quad \frac{\partial \varphi_e^*}{\partial \xi_2} = \frac{y - \xi_2}{4\pi\varepsilon_e r^3}, \quad \frac{\partial \varphi_e^*}{\partial \xi_3} = \frac{z - \xi_3}{4\pi\varepsilon_e r^3} \tag{28}$$

and

$$\frac{\partial \psi_e^*}{\partial \xi_1} = \frac{1}{4\pi}\left[\frac{3(x - \xi_1)d}{r^5} - \frac{\cos\alpha}{r^3}\right],$$
$$\frac{\partial \psi_e^*}{\partial \xi_2} = \frac{1}{4\pi}\left[\frac{3(y - \xi_2)d}{r^5} - \frac{\cos\beta}{r^3}\right], \tag{29}$$
$$\frac{\partial \psi_e^*}{\partial \xi_3} = \frac{1}{4\pi}\left[\frac{3(z - \xi_3)d}{r^5} - \frac{\cos\gamma}{r^3}\right]$$

Applying discretization of the boundary of domain, numerical calculations of partial derivatives are not difficult to obtain. These derivatives are determined at the internal nodes.

5. Boundary element method – temperature field

The Pennes equation (9) can be written in the form

$$(x, y, z) \in \Omega_e: \quad \lambda_e \nabla^2 T_e(x, y, z) + Q_e(x, y, z) = 0 \tag{30}$$

where

$$Q_e(x, y, z) = Q_{perf\,e} + Q_{met\,e} + Q_e^E(x, y, z) \tag{31}$$

where $Q_{perf\,e}, Q_{met\,e}, Q_e^E(x, y, z)$ are the heat sources connected with perfusion, metabolism and electromagnetic dissipated power, respectively.

The boundary integral equations corresponding to the equations (30) can be expressed as follows [1, 2, 7]

$$B_e(\xi_1, \xi_2, \xi_3) T_e(\xi_1, \xi_2, \xi_3) + \iint_\Gamma q_e(x, y, z) T_e^*(\xi_1, \xi_2, \xi_3, x, y, z)\, d\Gamma = \iint_\Gamma T_e(x, y, z) q_e^*(\xi_1, \xi_2, \xi_3, x, y, z)\, d\Gamma + \iiint_\Omega Q_e(x, y, z) T_e^*(\xi_1, \xi_2, \xi_3, x, y, z)\, d\Omega \tag{32}$$

where

$$T_e^*(\xi_1, \xi_2, \xi_3, x, y, z) = \frac{1}{4\pi\lambda_e r} \tag{33}$$

and

$$q_e^*(\xi_1, \xi_2, \xi_3, x, y, z) = -\lambda_e \frac{\partial T_e^*(\xi_1, \xi_2, \xi_3, x, y, z)}{\partial n} = \frac{d}{4\pi r^3} \quad (34)$$

while $q_e(x, y, z) = -\lambda_e \partial T_e(x, y, z)/\partial n$.

To solve the equations (32), not only the boundary but also the interior of the domains considered should be discretized.

For constant boundary elements and constant internal cells one obtains the following systems of equations

- for healthy tissue

$$\sum_{j=1}^{N_1} W_{ij}^1 q_j^1 = \sum_{j=1}^{N_1} Z_{ij}^1 T_j^1 + \sum_{l=1}^{L_1} P_{il}^1 Q_l^1, \quad i = 1, 2, ..., N_1 \quad (35)$$

- for tumor region

$$\sum_{j=N_1+1}^{N} W_{ij}^2 q_j^2 = \sum_{j=N_1+1}^{N} Z_{ij}^2 T_j^2 + \sum_{l=L_1+1}^{L} P_{il}^2 Q_l^2, \quad i = N_1+1, N_1+2, ..., N \quad (36)$$

where

$$W_{ij}^e = \iint_{\Gamma_j} T_e^*(\xi_{1i}, \xi_{2i}, \xi_{3i}, x, y, z)\, d\Gamma_j, \quad e = 1, 2 \quad (37)$$

and

$$Z_{ij}^e = \iint_{\Gamma_j} q_e^*(\xi_{1i}, \xi_{2i}, \xi_{3i}, x, y, z)\, d\Gamma_j, \quad i \neq j$$
$$Z_{ii}^e = -\sum_{\substack{j=1 \\ j \neq i}}^{N} Z_{ij}^e, \quad e = 1, 2 \quad (38)$$

while

$$P_{il}^e = \iiint_{\Omega_l} T_e^*(\xi_{1i}, \xi_{2i}, \xi_{3i}, x, y, z)\, d\Omega_l, \quad e = 1, 2 \quad (39)$$

The systems of equations (35), (36) are coupled by boundary condition (10), which can be written as

$$\begin{cases} T_j^1 = T_j^2 = T_j \\ q_j^1 = -q_j^2 = q_j \end{cases} \quad (40)$$

Finally one obtains

$$\begin{bmatrix} \mathbf{W}^1 & -\mathbf{Z}_c^1 & \mathbf{W}_c^1 \\ \mathbf{0} & -\mathbf{Z}_c^2 & -\mathbf{W}_c^2 \end{bmatrix} \begin{bmatrix} \mathbf{q}^1 \\ \mathbf{T} \\ \mathbf{q} \end{bmatrix} = \begin{bmatrix} \mathbf{Z}^1\mathbf{T}^1 + \mathbf{P}^1\mathbf{Q}^1 \\ \mathbf{P}^2\mathbf{Q}^2 \end{bmatrix} \quad (41)$$

The remaining boundary conditions should be introduced to the system of equations (41). The solution of (41) allows one to calculate the 'missing' boundary temperatures T_j^e and heat fluxes q_j^e. Next, the temperatures at the internal nodes are calculated by means of formula [1, 7]

$$T_i^e = \sum_{j=1}^{N} Z_{ij}^e T_j^e - \sum_{j=1}^{N} W_{ij}^e q_j^e + \sum_{l=1}^{L} P_{il}^e Q_l^e, \quad e = 1, 2 \quad (42)$$

In the paper the external boundary of the tissue has been divided into 640 constant boundary elements, the interface Γ_c of the tumor and tissue has been divided into 96 boundary elements. To solve the Pennes equation in the interiors of Ω_1 and Ω_2, the $L_1 = 960$ and $L_2 = 64$ internal cells have been distinguished, respectively.

6. Results of computations

The 3D domain of dimensions 0.04×0.04×0.08 [m] has been considered. The heating area is described as $\{0.032 \le x \le 0.048, 0.012 \le y \le 0.028, z = 0$ [m]$\}$,$\{0.032 \le x \le 0.048, 0.012 \le y \le 0.028, z = 0.04$ [m]$\}$ and the voltage applied on these surfaces is different for three-cases: 5V and –5V (variant 1), 10V and –10V (variant 2) and 15V and –15V (variant 3), respectively. The tumor region corresponds to $\Omega_2 = \{0.032 \le x \le 0.048, 0.016 \le y \le 0.032, 0.016 \le z \le 0.032$ [m]$\}$ as shown in Figure 1.

At first, the temperature distribution in the tissue with a tumor subjected to electric field action in the case of 'natural' situation (the particles are not introduced) has been considered. For healthy tissue the following parameters have been assumed: thermal conductivity $\lambda_1 = 0.5$ [W/(mK)], perfusion rate $G_{B1} = 0.0005$ [1/s], metabolic heat source $Q_{met1} = 4200$ [W/m^3], blood temperature $T_B = 37$°C, volumetric specific heat of blood $c_B = 4.2$ [MJ/(m^{3}K)] [3]. It has been revealed that existence of malignant tumor often leads to very different blood perfusion and abnormally high capacity of metabolic heat source in the tumor region [3, 6, 8]. The following parameters are thus given for a highly vascularized tumor situated in the skin tissue: $G_{B2} = 0.002$ [1/s], $Q_{met2} = 42000$ [W/m^3], $\lambda_2 = 0.75$ [W/(mK)] [3]. On the skin surface the Robin boundary condition (equation (11): $\alpha_w = 45$ [W/(m^2 K)], $T_w = 20$°C) has been accepted.

Additionally it is assumed that the frequency of electromagnetic field is equal to $f = 1$ [MHz] and the radius R of magnetic induction loop is equal to 0.01 [m]. The following values of parameters have been used [3]: electric conductivity of healthy tissue $\sigma_1 = 0.4$ [S/m], electric conductivity of tumor region $\sigma_2 = 1.2 \cdot \sigma_1$ [S/m], dielectric permittivities $\varepsilon_1 = 2000 \cdot \varepsilon_0$ and $\varepsilon_2 = 1.2 \cdot \varepsilon_1$ ($\varepsilon_0 = 8.85 \cdot 10^{-12}$ [C^2/(Nm^2)]), respectively.

The nanoparticles are made from iron oxides maghemite γ-Fe_2O_3 which electric conductivity equals to $\varepsilon_3 = 25000$ [S/m], thermal conductivity: $\lambda_3 = 40$ [W/(mK)] and $\chi'' = 18$ [3].

In Figure 3 the temperature distribution in domain of tissue and tumor without electric field and nanoparticles is shown. Figures 4, 5 and 6 illustrate the temperature field in the tissue subjected to the electric field for different values of electrodes voltage. Figures 7, 8 and 9 illustrate the temperature distribution under the assumption that in tumor region $n = 10^{13}$ nanoparticles with radiuses $r = 10^{-8}$ (iron oxide Fe_3O_4) are embedded.

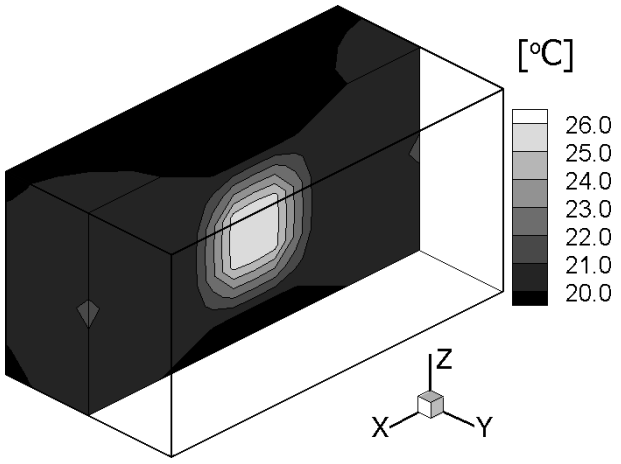

Figure 3: Temperature distribution without electric field and nanoparticles

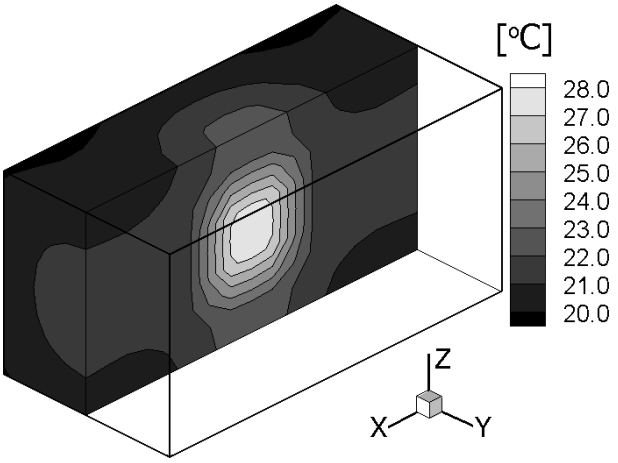

Figure 4: Temperature distribution in the domain subjected to the electric field (domain without nanoparticles), U = 5V

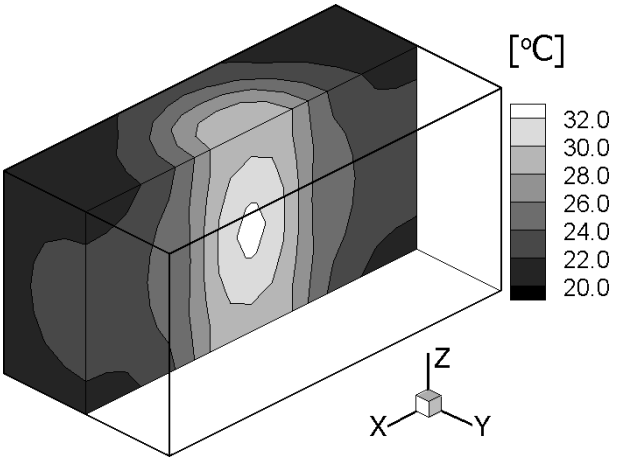

Figure 5: Temperature distribution in the domain subjected to the electric field (domain without nanoparticles), U = 10V

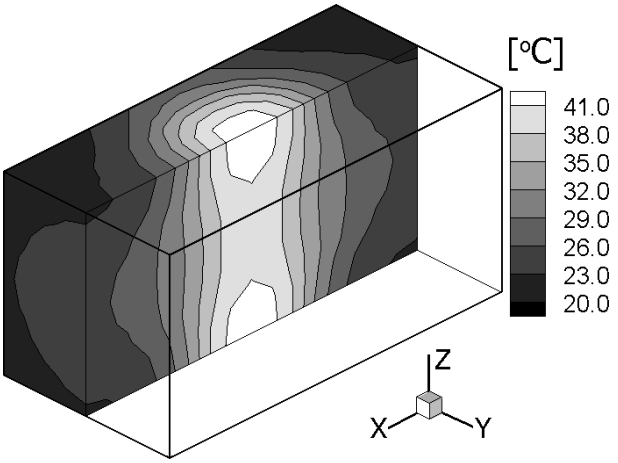

Figure 6: Temperature distribution in the domain subjected to the electric field (domain without nanoparticles), U = 15V

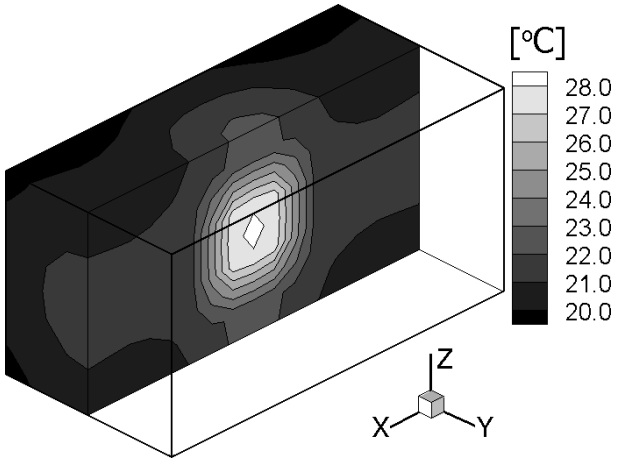

Figure 7: Temperature distribution in the domain subjected to the electric field (domain with nanoparticles), U = 5V

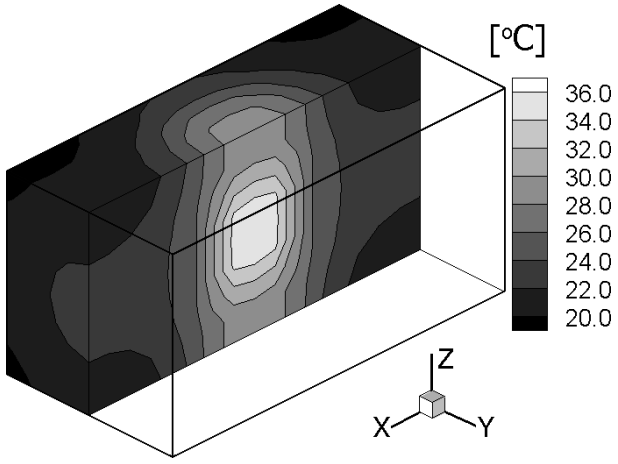

Figure 8: Temperature distribution in the domain subjected to the electric field (domain with nanoparticles), U = 10V

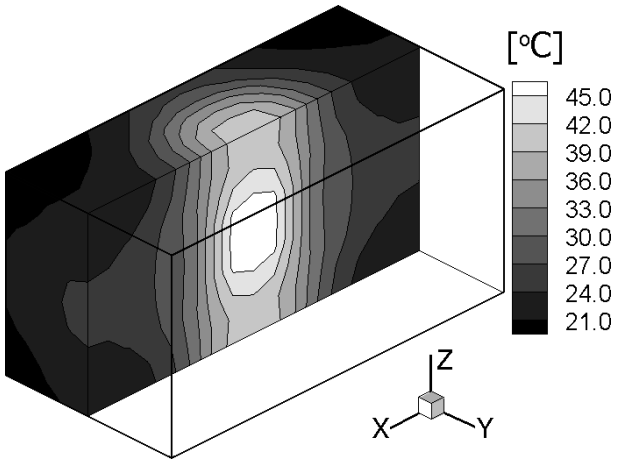

Figure 9: Temperature distribution in the domain subjected to the electric field (domain with nanoparticles), U = 15V

Conclusions

The boundary element method has been applied to solve the coupled problem connected with the biological tissue heating. The 3D mathematical model based on the Pennes equations supplemented with the equations determining the electric field due to the external electrodes action are considered.

The changes of the voltage of external electrodes cause the changes of temperature in the entire domain considered, but the possibilities of temperature field control (e.g. a concentration of maximum temperature at the central point of tumor region) are rather unrealizable, because the maximum temperature we could observe in the neighbourhood to the electrodes (Figure 6). On the other hand, too small voltage not allows one to obtain the hyperthermia state (see: Figure 4, 5).

So, the idea consisting in the introduction of nanoparticles to the tumor region [3] (for the concentrated energy deposition at the target tissue) is very effectiveness. We obtain the maximum temperature exactly in the center of tumor (see: Figure 7, 8, 9) and for good choice of electrodes voltage and number of nanoparticles (Figure 9) the tumor could be destroyed.

The application of the method discussed requires a credible information concerning the tumor position and its dimensions. In a such case the methods of numerical simulation can be a very effective tool for a proper choice of electric field parameters and cooling conditions on a skin surface.

References

[1] Brebbia, C.A. and Dominguez, J., *Boundary Elements, an Introductory Course*, Computational Mechanics Publications, McGraw-Hill Book Company, London 1992.

[2] Jabłoński, P., *Boundary element method in the analysis of electromagnetic field*, Publ. of the Czestochowa University of Technology, Czestochowa, 2003 (in Polish).

[3] Lv, Y.G., Deng, Z.S. and Liu, J., 3D numerical study on the induced heating effects of embedded micro/nanoparticles on human body subject to external medical electromagnetic field, *IEEE Transactions on Nanobioscience*, Vol. 4, No 4, pp. 284-294, 2005.

[4] Majchrzak, E., Dziatkiewicz, G. and Paruch M., The modelling of heating a tissue subjected to external electromagnetic field, *Acta of Bioengineering and Biomechanics*, Vol. 10, No 2, pp. 29-37, 2008.

[5] Majchrzak, E. and Paruch, M., Numerical modelling of tissue heating by means of the electromagnetic field, *Scientific Research of the Institute of Mathematics and Computer Science*, Czestochowa University of Technology, 1(9), pp. 89-97, 2010.

[6] Majchrzak, E. and Mochnacki, B., Analysis of thermal processes occurring in tissue with a tumor region using the BEM, *Journal of Theoretical and Applied Mechanics*, Vol. 1, No 40, pp. 101-112, 2002.

[7] Majchrzak, E., *Boundary Element Method in Heat Transfer*, Publ. of the Czestochowa University of Technology, Czestochowa, 2001 (in Polish).

[8] Majchrzak E. and Paruch M., Identification of electromagnetic field parameters assuring the cancer destruction during hyperthermia treatment, *Inverse Problems in Science and Engineering*, Vol. 19, No 1, pp. 45-58, 2011.

[9] Wang H., Dai W. and Bejan A., Optimal temperature distribution in a 3D triple-layered skin structure embedded with artery and vein vasculature and induced by electromagnetic radiation, *International Journal of Heat and Mass Transfer*, 50, pp. 1843-1854, 2007.