Important new results in exact topology optimization

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Abstract

New exact analytical solutions will be presented for some often used truss topology optimization problems, which have important implications for fundamental general properties of optimal topologies. The analytical results will be confirmed by highly accurate numerical calculations.

Keywords: topology optimization, Michell truss, minimum weight design

1. Introduction

In this lecture new exact analytical solutions for three important classes of least-weight (Michell [5]) trusses will be reported:

(a) Doubly symmetric rectangular domain boundary, two supports, two point loads, both symmetric, see Fig. 1(a).

(b,c) Square or rectangular domain, roller and pin (Fig. 1(b)), or two pins (Fig. 1(c)) at corners. Only one point load acts in each case considered.

Although these problems are often used as benchmark examples in numerical topology optimization, surprisingly the exact analytical solutions have not been known previously for such basic boundary, support and load conditions.

For the class of problems under (a), there exist three types of optimal topologies depending on the relative magnitude of various dimensions.

The class of problems under (b) can be handled by extending results published in a recent paper by Sokół and Lewiński [10]. For some of the load positions the solution is a subset of an 'MBB beam' (Lewiński et al. [3]).

The class of problems under (c) may have many types of optimal topologies depending on the position of the load.

The very important general implications of the above results are the following.

2. Problem class (a)

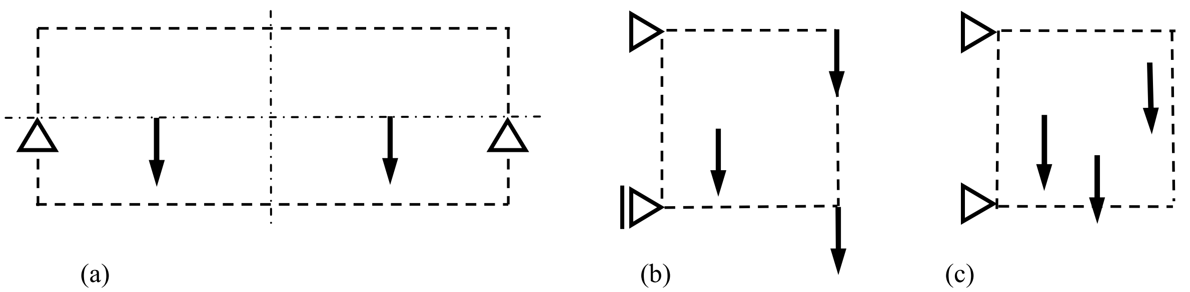
The recent paper by Sokół and Lewiński [10] considered optimal topologies for trusses over a half-plane, with supports and loads along the boundary. The present study considers trusses over the full plane, or over a rectangle that is symmetric about the line containing the supports and loads. This investigation therefore complements the results in the above paper.

In deriving the optimal topologies for the present class of problems, we make use of symmetry and skew symmetry theorems in a paper by Rozvany [7].

Over the middle part of the optimal topologies for this problem class, there are no members apart from concentrated chords along the domain boundaries. Melchers [4] explained the difficulties in finding the optimal adjoint strain fields for such 'memberless' subdomains. However, in this paper the exact adjoint strain fields satisfying all optimality criteria will be given for those subdomains.

3. Problem class (b)

The results in the paper by Sokół and Lewiński [10] can be extended to this case, with some additional derivations. The solutions may also be derived using results in a paper by Lewiński et al. [3]



(a) (b) (c)

Figure 1: Classes of problems considered

4. Problem class (c)

This is the most interesting case, because for certain load positions the optimal topology consists of two non-orthogonal bars. This seemingly contradicts Hemp's [1] orthogonality principle, but the underlying inconsistency was explained in a paper by Rozvany [6]. The adjoint strain field actually consists of two R-type regions. This is unusual, because most researchers use T-type regions for the adjoint fields. If the load is further away from the supports, then the optimal topology consists of a standard Michell cantilever (Lewiński et al. [2]).

5. Numerical confirmation of the results

All analytical solutions presented will be verified numerically by the method developed by Sokół [9]. Due to a linear programming formulation of the optimization problem, the method presented in [9] assures finding the global optimum; hence it may be considered as a reliable verification tool for results obtained by analytical methods. The computational program developed in [9] enables prediction of new exact solutions to Michell's problem with high accuracy.

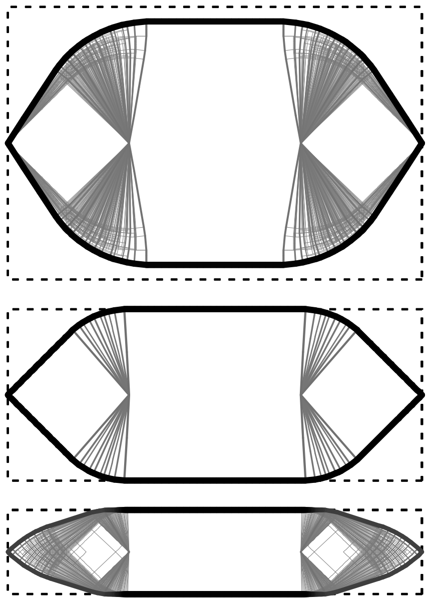


Figure 2: Numerical layouts of problem class (a) for decreasing height/length ratio of rectangular domain

For example, three main types of the optimal topologies of the problem class (a) were derived according to different height to length ratios of the rectangular domain. The numerical layouts presented in Fig. 2 were obtained for very dense ground structures with the number of potential bars exceeding one hundred million. The volumes predicted numerically are very close to the exact ones, with the relative error less than 0.05%.

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