# hp-HGS twin adaptive strategy for inverse resistivity logging measurements

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Abstract
In this paper we present the *hp*-HGS twin adaptive strategy for solving inverse problems related to resistivity logging measurements simulations. The Hierarchical Genetic Search (HGS) algorithm is a multi deme genetic algorithm, constructing a tree of genetic populations, searching with increased accuracy while going down the tree. The *hp*-HGS extension of the HGS algorithm presented in this paper, the fitness of each individual is estimated by using direct problem solution obtained with goal oriented *hp*-adaptive Finite Element Method (*hp*-FEM). The direct problem solution at the root populations is computed with lowest possible accuracy, while the leaf populations utilize maximum available accuracy that can be obtained from goal-oriented *hp*-FEM technology. The algorithm is testes on several numerical examples of the 3D DC borehole resistivity measurements simulations.

*Keywords: computational mechanics, conference, instructions, formatting, camera-ready paper*

## 1. Introduction

In this paper we utilize the *hp*-HGS (Hierarchical Genetic Search) [1, 2] strategy to solve the inverse problem of finding unknown resistivities of formation layers. We utilize the *hp* adaptive goal-oriented Finite Element Method (goal-oriented *hp*-FEM*)* for solving the direct problem [3, 4].

## 2. Direct and inverse problems of 3D DC borehole resistivity measurement simulation

### 2.1. Computational domain

The problem geometry can be described by using cylindrical coordinates $(\rho,\varphi,z)$. The following sources, receivers, and materials are used

a) four (one current and three voltage) 2 × 5-cm ring electrodes located 8 cm from the axis of symmetry and moving along the vertical direction (*z* axis). Voltage (collector) electrodes are located 100, 125, and 150 cm above the current (emitter) electrode, respectively,
b) borehole: a cylinder $\Omega_A$ of radius 10 cm surrounding the axis of symmetry $\Omega_A=\{(x,\varphi,z):\rho\le 10\,\mathrm{cm}\}$ with resistivity $R=0.1\,\Omega\cdot m$,
c) upper formation: a subdomain $\Omega_B$ defined by $\Omega_C=\{(x,\varphi,z):\rho>10\,\mathrm{cm},3\,\mathrm{m}\le z\}$ with resistivity $R=100\,\Omega\cdot m$,
d) formation material 1: a subdomain $\Omega_C$ defined by $\Omega_C=\{(x,\varphi,z):\rho>10\,\mathrm{cm},2\,\mathrm{m}\le z\le 3\mathrm{m}\}$ with resistivity $R_1\,\Omega\cdot m$,
e) formation material 2: a subdomain $\Omega_D$ defined by $\Omega_D=\{(x,\varphi,z):\rho>10\,\mathrm{cm},0\,\mathrm{m}\le z\le 2\,\mathrm{m}\}$ with resistivity $R_2\,\Omega\cdot m$,
f) formation material 3: a subdomain $\Omega_E$ defined by $\Omega_E=\{(x,\varphi,z):\rho>10\mathrm{cm}\text{ or }-2m\le z\le 0\,\mathrm{m}\}$ with resistivity $R_3\,\Omega\cdot m$,
g) lower formation: a subdomain $\Omega_F$ defined by $\Omega_C=\{(x,\varphi,z):\rho>10\,\mathrm{cm},z<-2m\}$ with resistivity $R=1000\,\Omega\cdot m$.

### 2.2. Variational problem formulation

Find $u\in V$ the electrostatic scalar potential such that

$$b(u,v)=l(v)\quad\forall v\in V \tag{1}$$

$$b(u,v)=\int_{\Omega}\sum_{i=1}^{3}\sigma\frac{\partial u}{\partial x_i}\frac{\partial v}{\partial x_i}dx \tag{2}$$

$$l(v)=\int_{\Omega}\sum_{i=1}^{3}\frac{\partial J}{\partial x_i}v\,dS+\int_{\Gamma_N}g\,v\,dS \tag{3}$$

where

$$V=\left\{v\in L^2(\Omega):\int_{\Omega}\|v\|^2+\|\nabla v\|^2dx<\infty:\mathrm{tr}(v)=0\text{ on }\Gamma_D\right\} \tag{4}$$

and $J$ denotes a prescribed, impressed current source, $\sigma$ is the conductivity, and the electrostatic scalar potential $u$ is related to the electric field $E$ by $E=-\nabla u$.

* The work presented in this paper has been supported by Polish MNiSW grant no. 519 447 739

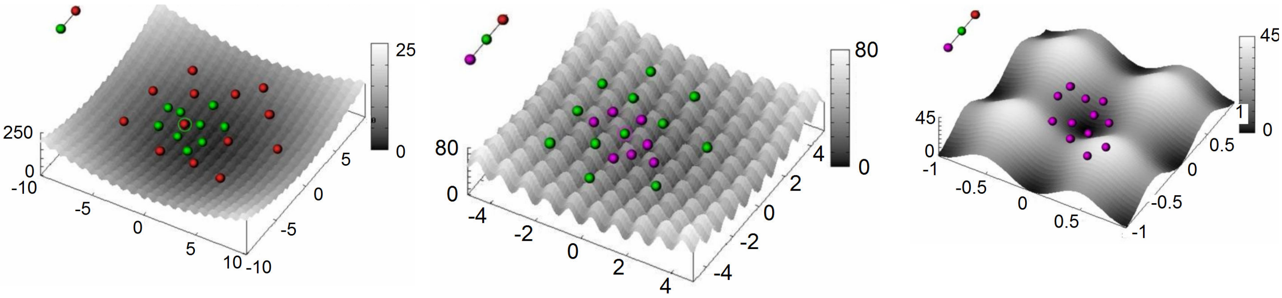


Figure 1: Left panel: root population (red) Middle panel: second level population (green) Right panel: leaf population (pink)

### 2.3. *Direct problem formulation*

Given the resistivities $R_i\ \Omega \cdot m$ for $i = 1,2,3$ compute the logging curve. The logging curve contains values of the electric potential scalar field for 50 positions of the receiver electrodes.

### 2.4. *Inverse problem formulation*

The inverse problem consists in finding unknown resistivities $R_i\ \Omega \cdot m$ for $i = 1,2,3$ for three formation layers with known geometry, based on given logging curve (50 values of the electric potential scalar field).

## 3. *hp*-HGS strategy

The HGS strategy is a set of concurrent evolutionary processes. The dependency between the processes has a tree structure. A single process can be summarized by the following algorithm

```
IF (j=1) create population for root P^0;
T := 0;
REPEAT
  IF (global stop condition) STOP;
  FORALL  i ∈ P^t compute fitness f_j(i);
  IF (branch stop condition) STOP;
  Perform selection based on
    computed fitness f_j;
  Execute genetic operations;
  IF (NOT (t mod K)) select
    best fitted individual x* from P^t;
  IF (((NOT prefix comparision(x*)) AND
  (j < m)) sprouting procedure(x*);
  t := t+1;
UNTIL(false)
```

HGS strategy starts with a single root process performing search with low accuracy (see left panel in Figure 1). In the places found by the root level process, several second level processes are created by the sprouting procedure. The second level processes perform search with higher accuracy (see middle panel in Figure 1). Finally, the third level processes are created at the results of the second level search, and they perform high accuracy search. In the *hp*-HGS algorithm, the fitness of each individual is estimated by executing *hp* adaptive goal oriented FEM algorithm, with the required accuracy depending on the level of the HGS tree. Root level direct problem are solved with low accuracy, leaf level direct problems are solved with highest accuracy.

## 4. Preliminary numerical results

We conclude the presentation with some numerical results. We performed the simulation of the 3D DC borehole resistivity measurements problem up to five levels of the HGS tree. The algorithm has found the requested resistivities based on the provided logging curve. We have forced the *hp*-HGS to ask for the accuracy listed in Table 1 for individuals from populations from different levels of the HGS tree. We also list here the execution time and the Euclidean distance between the exact logging curve and the curve obtained from the goal-oriented *hp*-FEM algorithm. By using the table we can estimate the execution times of the *hp*-HGS algorithm for different accuracies.

Table 1. Relation between *hp*-FEM accuracy, quality of the solution and the execution time

| *hp*-FEM accuracy | Euclidean distance to exact logging curve | Execution time |
|---|---|---|
| 10 | $1.84*10^{-3}$ | 2min 26sec |
| 1 | $1.36*10^{-3}$ | 3min 49sec |
| $10^{-1}$ | $1.99*10^{-4}$ | 9min 41sec |
| $10^{-2}$ | $4.97*10^{-5}$ | 20min 19sec |
| $10^{-3}$ | $4.31*10^{-6}$ | 1h 3min 52sec |

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